An Empirical Formula for the Surge Impedance of a Grounding Conductor along a Reinforced Concrete Pole in a Distribution Line

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Abstract – **The contribution of the surge characteristic of a grounding vertical conductor to an induced voltage across an insulator becomes prominent if the lightning current has a steep front. The present paper derives an empirical formula of a vertical grounding conductor including the effect of a reinforced concrete pole close to the vertical grounding conductor based on a measured result on a reduced-scale model. This empirical formula is expressed in the form of a logarithmical function considering a deviation of the self-surge impedance of the grounding conductor along the reinforced concrete pole from that of a single grounding conductor. The developed formula agrees well with measured results.**

Keywords: Distribution Line, Direct Stroke, Grounding Conductor, Reinforced Concrete Pole, Surge Impedance.

1. Introduction

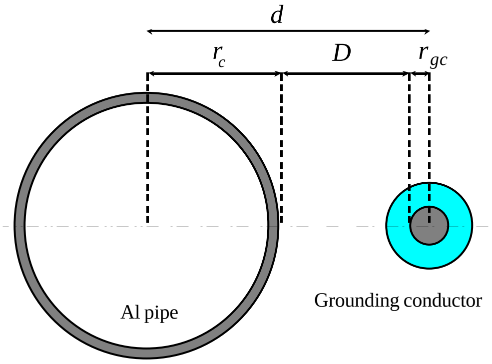
The lightning protection of a power distribution line in Japan is targeted for a lightning hit directly to the distribution line as well as a nearby lightning stroke [1]. Great efforts to establish a rational lightning protection design are made by experimental studies [2], computer simulations [3] and lightning observations [4]. These studies lead to a conclusion that the distribution line can be protected from a direct lightning hit with a comparatively high lightning current by a shielding wire and surge arresters being arranged properly.

A distribution line in Japan usually uses a reinforced concrete pole for bracketing wires and power apertures. The grounding conductor is laid along and isolated from the reinforced concrete pole. The reinforced concrete pole should be treated as a grounding electrode and a conductor for a lightning current [5]. Reference [6] pointed out that the surge impedance of the grounding conductor was affected by the existence of the reinforced concrete pole due to an electromagnetic field scattered by the reinforced concrete pole. A formula of the surge impedance, however, has not been derived.

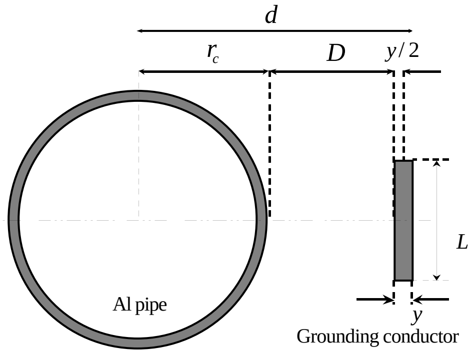
This paper describes measured results of the self- and mutual surge impedances of a grounding conductor and a reinforced concrete pole using a reduced-scale model, and proposes an empirical formula of the self-surge impedance of the grounding close to the reinforced concrete pole based on the measured results.

2. Experimental Setup

A scale model approach is convenient to estimate a surge impedance and a surge velocity because those values on the scale model is proved to coincide with those of the real model [7]. One-seventh reduced-scale model is used in this experiment. A reinforced concrete pole is represented by a 2m long Al uniform pipe with 40mm radius. The thickness of the concrete part of the reinforced concrete pole is about 5% of the pole radius. It has been shown in reference [6] that the concrete part causes a minor effect on the characteristic of a surge and electromagnetic field propagation. The radius of 40mm is one-seventh of the average radius of an actual base-broad reinforced concrete pole. The grounding conductor is represented by a conductor of circular cross section and that of flat shape. Fig. 1(a) shows the cross-section of a grounding conductor system with a circular-shaped conductor for measurement. Fig. 1(b) shows that with a flat-shaped conductor. Tables 1 and 2 give measurement cases illustrated in Fig. 1(a) and in Fig. 1(b), respectively.



(a) Al pipe and circular-shaped grounding conductor



(b) Al pipe and flat-shaped grounding conductor

Fig. 1 Experimental system of a reinforced concrete pole and a grounding conductor

A pulse current is imposed at the top of the vertical conductor through a 6m horizontal coaxial cable. At the cable end, its sheath is connected to the core through a 50Ω resistor. The voltage between the top of a conductor and a horizontal auxiliary wire is measured using a voltage probe (TEKTRONIX P6139A, frequency band: 500MHz). An applied current is obtained from a voltage drop across a 200Ω resistor, which is inserted between the top of the conductor and a current injection cable. This voltage drop is also measured using a voltage probe having the same specification as the above one. The measured signal is transported to a digital oscilloscope (TEKTRONIX TDS644, frequency band: 500MHz).

Table 1 Combination of vertical conductors

Case	r_{gc} [mm]	r_c [mm]	Symbol
1	40	0.2	□
2□	0.2	40	□
3	40	10	□
4	10	40	□
5	0.75	40	□
6	10	0.2	□
7	0.2	10	□
8	10	0.75	□
9	0.75	10	□
10	0.75	20	□

□ Case 2 corresponds to an actual grounding system

Table 2 Combination of a pipe and a flat shape conductor, with the value of constant "*a*" in (7)

L [mm]	r_c [mm]	a	Symbol
15	40	26.99	△
15	20	23.31	□
15	10	19.01	◈
30	40	23.71	□
30	20	19.21	□
45	40	21.23	□
45	20	15.34	□
60	40	19.21	□
60	20	12.29	□
80	40	16.54	□
80	20	9.82	□
120	40	12.77	□
160	40	9.84	▣

A surge impedance of a vertical conductor is defined as the ratio of a voltage and a current at the instance when a reflecting surge from the bottom of the conductor arrives at the top [8]. A noise simulator (TOKIN HIG-3222A) is used as a current source with the rise time of about 2ns, which is fast enough to represent a step current considering that the traveling time on the conductor is about 13ns.

3. Self-Surge Impedance of an Independent Vertical Conductor

A self-surge impedance of an independent vertical conductor of circular and flat shapes is measured as a function of the conductor radius in the circular-shaped case and as a function of the width in the flat-shaped case, respectively.

Fig. 2(a) shows a measured result of the surge impedances of a vertical circular-shaped conductor.

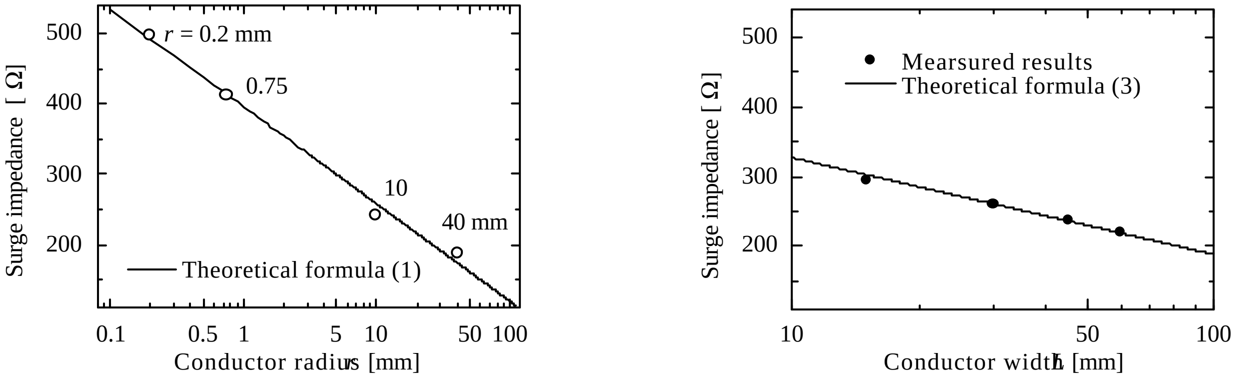
The authors proposed a theoretical formula of the self-surge impedance for the circular-shaped conductor[9]:

$$Z_{sc} = 60 \cdot \ln\left(h / er\right) \tag{1}$$

where h: conductor height [m], r: conductor radius [m],
e: base of natural logarithm.

Fig. 2(a) includes a result of the self-surge impedance calculated by (1). It is clear from Fig. 2(a) that (1) agree well with the measured result.

One of authors proposed the method to estimate the equivalent radius of an arbitrary cross-section conductor [10]. In the case of a flat-shaped conductor, of which the thickness is y and the width is L, the outer and the inner equivalent radii are given in the following equation:



(a) Circular-shaped conductor (b) Flat-shaped conductor

Fig. 2 Measured and theoretical self-surge impedances of an independent vertical conductor

$$\text{Outer radius: } r_0 = (L+y)/p$$
$$\text{Inner radius: } r_1 = \sqrt{r_0^2 - S/p} \qquad (2)$$

where S : cross-section area of flat-shaped conductor (= $L \times y$).

From (1) and (2), the self-surge impedance of a flat-shaped conductor is given by:

$$Z_{sf} = 60 \cdot \ln\{ph/e(L+y)\} \qquad (3)$$

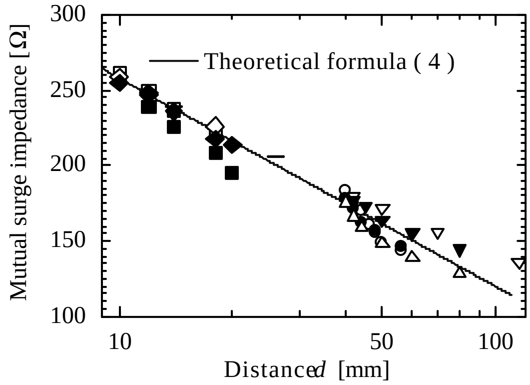
Fig. 2(b) shows a comparison of a measured result with a theoretical result by (3). The theoretical result shows a satisfactory agreement with (3).

4. Mutual Surge Impedance of Multiple Vertical Conductors

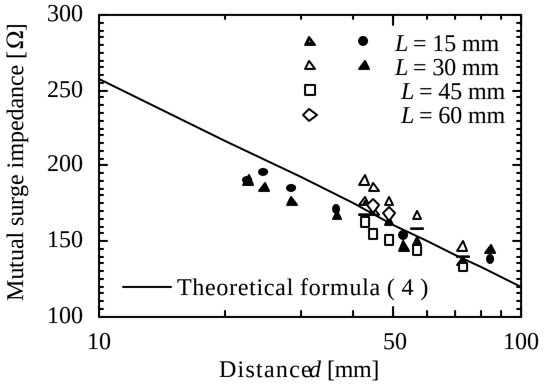
A mutual surge impedance is defined as a top voltage of an induced conductor when a unit step current is injected into an inducing conductor. A theoretical formula of the mutual surge impedance of a multiple vertical conductor system is given in reference [9]:

$$Z_{mc} = 60 \cdot \ln(h/ed) \qquad (4)$$

where d: distance between two conductors as in Fig. 1.



(a) Circular-shaped conductor



(b) Flat-shaped conductor

Fig. 3 Mutual surge impedance of vertical conductors vs. separation distance "d"

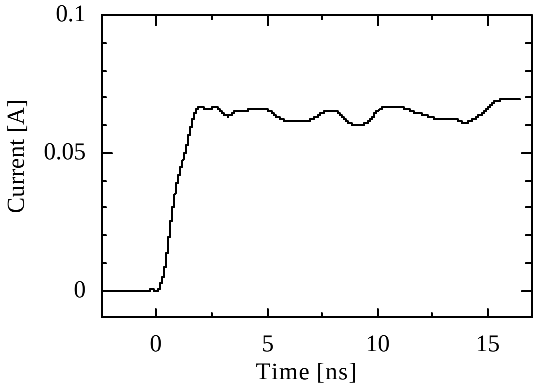
Fig. 3 shows a comparison of a measured result with a theoretical result by (4) in the cases of a circular-shaped and a flat-shaped conductor. From Fig. 3, the theoretical result agrees satisfactorily with the measured result within an error of 10 %. Therefore, (4) is applicable to the grounding system.

5. Self-Surge Impedance of Grounding Conductor Along Reinforced Concrete Pole

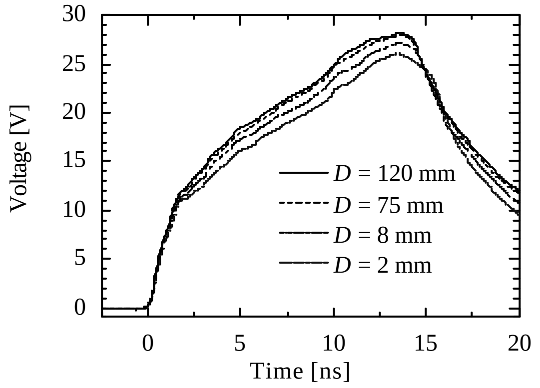
In an actual distribution line, a vertical grounding conductor is laid along a reinforced concrete pole. The self-surge impedance of the grounding conductor may be influenced by the presence of the reinforced concrete pole. The dependence of the self-surge impedance on the distance from the reinforced concrete pole is investigated.

Fig. 4 shows measured waveforms of an applied current and a top voltage of the grounding conductor as a function of the separation distance D in Fig. 1, when the applied current is injected only into the grounding conductor. The waveform of the applied current is almost constant through these experiments.

The waveform of the top voltage decays after 13ns. Considering that the vertical conductor height is 2m, the surge velocity along the conductor is almost the same as the velocity of light in free space. As is observed in Fig. 4, the magnitude of the top voltage decreases as the



(a) Current



(b) Voltage

Fig. 4 Applied current and voltage at the top of a circular-shaped vertical conductor (Case 2) grounding conductor is located closer to the reinforced concrete pole. Thus, the top voltage clearly shows a distance dependency. This characteristic has been observed in the cases of a circular-shaped and a flat-shaped conductor.

Fig. 5 shows a measured result of the self-surge impedance of the circular-shaped grounding conductor as a function of the separation distance *D*.

It is observed in Fig. 5 that the self-surge impedance of the grounding conductor increases logarithmically with the separation distance *D* when the distance is smaller than the radius of the reinforced concrete pole.

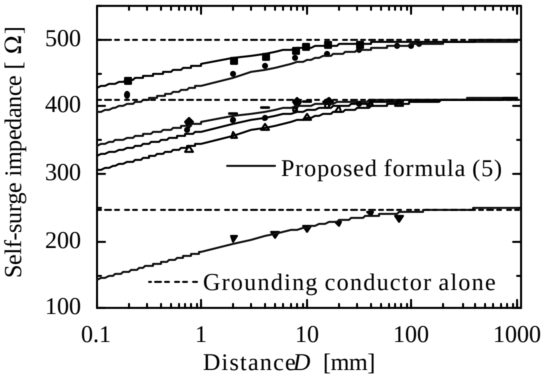


Fig. 5 Self-surge impedance of a circular-shaped grounding conductor vs. separation distance *D*

6. Empirical Formula of Self-Surge Impedance of Grounding Conductor Along Reinforced Concrete Pole

The self-surge impedance is easily calculated using (1) or (3). However, it is clear from Fig. 5 that the self-surge impedance of the independent conductor calculated by (1) or (3) does not agree well with the measured self-surge impedance of a grounding conductor if there is a reinforced concrete pole nearby the conductor. An electromagnetic theory may give an analytical solution of the surge impedance, but no formula has been derived yet. This paper proposes empirical formulas of the self-surge impedances of a circular-shaped and a flat-shaped grounding conductor along the reinforced concrete pole based on the measured results.

Fig. 6 shows a difference between the self-surge impedance of the circular-shaped conductor close to the reinforced concrete pole and that of the conductor alone as a function of r_c / D. The difference of the self-surge impedance decreases linearly with an increase in $\log(r_c / D)$ for $r_c / D > 1$, and converges to zero for a small r_C / D. It is well known that a function $\ln(1 + r_c / D)$ shows a good approximation for such variation. Accordingly, the following empirical formula of the self-surge impedance is derived.

$$Z_{gc} = 60 \cdot \ln(h/er) - k \cdot \ln\{1 + (r_c / D)\} \qquad (5)$$

It is observed in Fig. 6 that the difference is independent of r_g, and the constant k seems to be a function of r_c only.

From Fig. 7, which shows the relation between r_c and k, the constant k is evaluated by a linear function against the conductor radius as follows:

$$k = 0.096 \cdot r_c + 13.95 \qquad (6)$$

where r_c: reinforced concrete pole radius [mm]

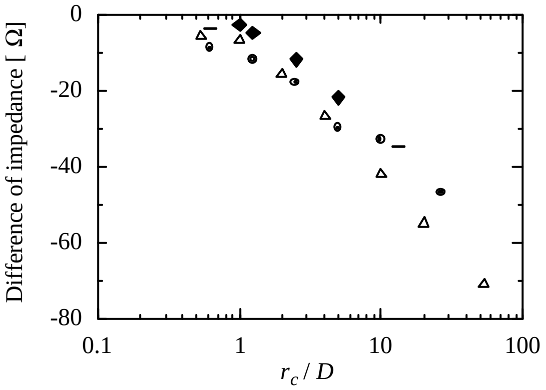


Fig. 6 Difference of the self-surge impedance of circular-shaped grounding conductor with and without a reinforced concrete pole

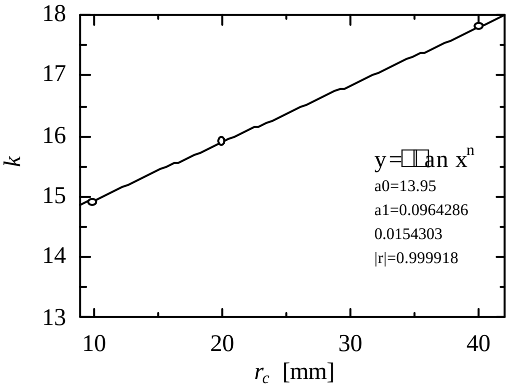


Fig. 7 Relation between reinforced concrete pole radius r_c and constant *k* in (5), for a system including a circular-shaped grounding conductor

The self-surge impedance of a flat-shaped grounding conductor linearly increases as the separation distance increases. It coincides with the surge impedance of the grounding lead conductor alone as the separation distance exceeds the diameter of a reinforced concrete pole as well as the case of a circular shape. Therefore, the self-surge impedance of a flat-shaped grounding conductor may be also expressed by a form similar to (5).

The self-surge impedance of a flat-shaped grounding conductor is written in the following form using (5) and (2).

$$Z_{gf} = 60 \cdot \ln\{ph / e(L + y)\} - a \cdot \ln\{1 + (r_c / D)\} \qquad (7)$$

The value of constant "*a*" in (7), which is determined on the basis of the measured result, is shown in Table 2. The constant "*a*" seems to be dependent on the radius of a reinforced concrete pole and the width of a flat-shaped grounding conductor.

The distance D_L illustrated in Fig. 8 equals to the separation distance D when the width of a flat-shaped grounding conductor is much less than the radius of a reinforced concrete pole. On the other hand, the difference between D_L and D becomes large, when the width of a flat-shaped grounding conductor is large. From Table 2, it is clear that the constant "a" is dependent on the width of the grounding conductor. Therefore, "a" may be expressed in the form of $a = k_0 + k_1 \square f(L,r_C)$, where k_1 is also a constant.

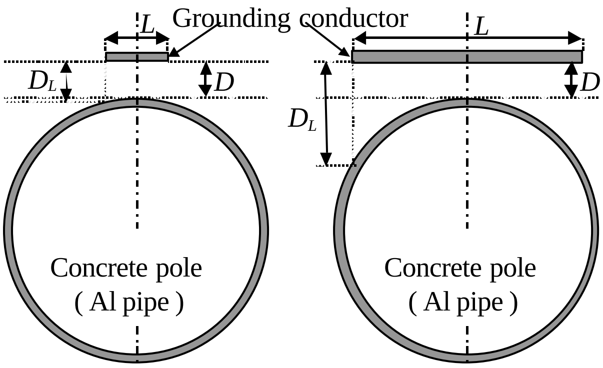


Fig. 8 Two extreme arrangements of a flat-shaped grounding conductor and a reinforced concrete pole

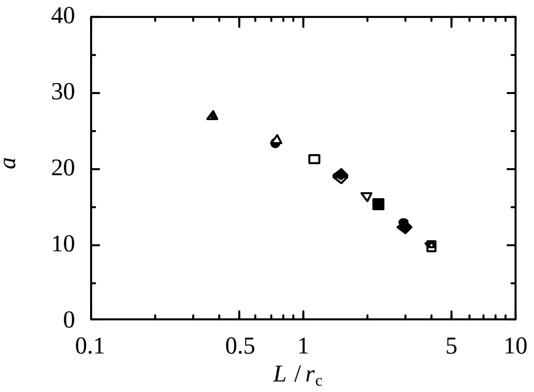


Fig. 9 Constant "a" in (7) vs. L / r_c

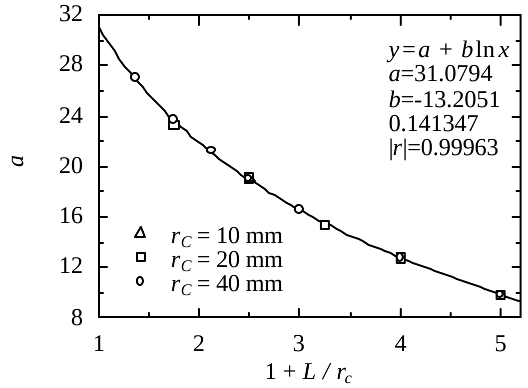


Fig. 10 Constant "a" in (7) vs. $1 + L / r_c$

Fig. 9 is the relation between constant "a" and r_c / L, and shows a characteristic similar to Fig. 6. Thus, $f(L,r_c)$ should converge to zero for $r_c >> L$. Accordingly, "a" is expressed as follows:

$$a = k_0 - k_1 \cdot \ln\{1 + (L / r_c)\} \tag{8}$$

The relation between constant "a" and $1 + L / r_c$ is shown in Fig. 10, and the following formula is obtained from it.

$$a = 31.08 - 13.21 \cdot \ln\{1 + (L / r_c)\} \tag{9}$$

Fig. 5 and 11 show a comparison of the surge impedance calculated from the proposed formula with the measured results. It is clear that the proposed formula shows a good accuracy for any configurations.

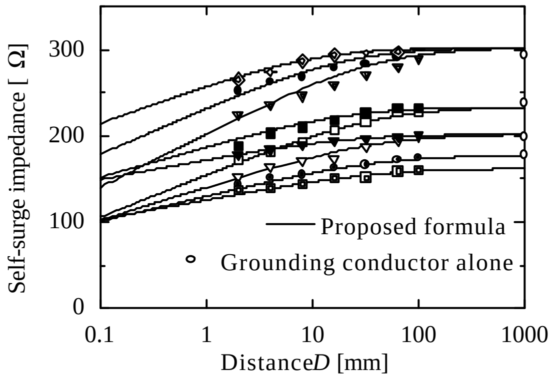


Fig. 11 Self-surge impedance of a flat-shaped grounding conductor vs. distance D

7. Flat- and Circular-Shaped Grounding Conductors

Fig. 12 shows a comparison of the self-surge impedances of a flat-shaped and a circular-shaped grounding conductor. In this comparison, the circular-shaped and the flat-shaped conductor, of which the surge impedances are the same in the independent case, are employed. The radius of the reinforced concrete pole r_c is set 40mm.

When the flat-shaped grounding conductor, of which the width is greater than the pole diameter, its self-surge impedance is higher than that of the circular-shaped case. The reason for this is that the effect of the reinforced concrete pole on the edge of the flat-shaped conductor becomes less. But it is unusual to employ such the wide grounding conductor in practice.

In the case that the width of the flat-shaped conductor is equal to the reinforced concrete pole diameter, the self-surge impedance is almost the same as that of the circular-shaped conductor, independently of D

When the width of the flat-shaped conductor is less than the pole diameter, the self-surge impedance is lower than that of the circular-shaped one. This means that the flat-shaped conductor is more influenced by the proximately placed reinforced concrete pole. Therefore, it is advantageous to employ the flat-shaped conductor with small width in place of a conventional circular-shaped conductor.

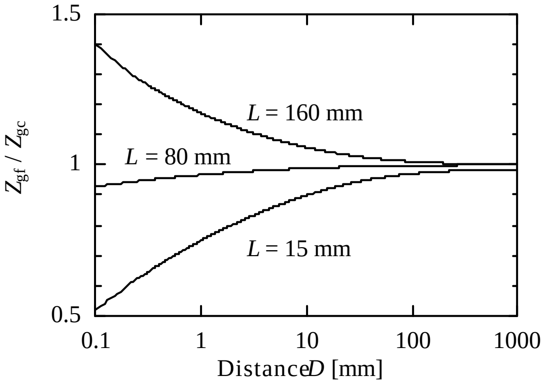


Fig. 12 Ratio of the surge impedances of a flat-shaped and a circular-shaped conductor
(r_c = 40mm, "gf" for flat-shaped, "gc" for circular-shaped)

8. Conclusions

This paper has presented measured results of self- and mutual surge impedances of a grounding system composed of a vertical grounding conductor and a reinforced concrete pole. As a grounding conductor, a circular-shaped and a flat-shaped one are investigated. The measured results show that the self-surge impedance is dependent on the distance between the conductor and the reinforced concrete pole. Calculated results of the self-surge impedance of the independent grounding conductor as well as the mutual surge impedance using theoretical formulas proposed by one of the authors agree satisfactorily with the measured results. On the basis of the measured results, an empirical formula of the self-surge impedance of the grounding conductor along the reinforced concrete pole has been proposed for the circular case and the flat case, respectively. The formula is of a simple expression, and considers the influence of the proximately placed reinforced concrete pole. Calculated results of the self-surge impedance by the proposed formula agree well with the measured results. The proposed formulas of the surge impedances of the grounding system is expected to be useful for an accurate lightning surge analysis in a power distribution line.

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References

[1] S.Yokoyama: "Lightning protection on power distribution line against direct lightning hits", *Trans., IEE Japan*, Vol. 114-B, No. 6, pp. 564-568, 1994 (*in Japanese*)

[2] S.Yokoyama & A.Asakawa: "Experimental study of response of power distribution lines to direct lightning hits", *IEEE Trans. on Power Delivery*, Vol. 4, No. 4, pp. 2242-2248, 1989

[3] Y.Imai, T.Matsuura, K.Itoh, S.Ishibe, M.Azuma & T. Shimomura: "Protective effects of surge arresters on power distribution lines against direct lightning hits to overhead wire", *Trans., IEE Japan*, Vol. 111–B, No. 6, pp. 619-627, 1991 (*in Japanese*)

[4] H.Taniguchi, H.Sugimoto & S.Yokoyama: "Observation of lightning performance on power distribution line by still camera", Proc. 23rd Int. Conf. Lightning Protection, Vol. 1, pp. 119-124, Firenze, Italy, 1996

[5] S.Sekioka, K.Yamamoto & S.Yokoyama: "Surge characteristics of grounding system of power distribution lines in Japan with reference to a reinforced concrete pole", Proc. Int. Conf. Grounding & Earthing, Belo Horizonte, Brazil, 2000

[6] K.Yamamoto, Z.Kawasaki, K.Matsuura, S.Sekioka & S.Yokoyama: "Study on surge impedance of reinforced concrete pole and grounding lead wire on distribution lines by experiment on reduced scale model", Proc. 10th Int. Symp. High Voltage Eng., Vol. 5, pp. 209-212, Montreal, Canada, 1997

[7] A.Abetti: "Transformer model for the determination of transient voltages", *AIEE Trans.*, Vol. 72, pp. 468-480, 1953

[8] M.Kawai: "Studies of the surge response of a transmission tower", *IEEE Trans. Power App. Syst.*, Vol. 83, No. 1, pp. 30-34, 1964

[9] A.Ametani, Y.Kasai, J.Sawada, A.Mochizuki & T. Yamada: "Frequency-dependent impedance of vertical conductors and a multi-conductor tower model", *IEE Proc. Gener., Transm. Distrib.*, Vol. 141, No. 4, pp. 339-345, 1994

[10] A.Ametani & I.Fuse: "Approximate method for calculating the impedance of multiconductors with cross section of arbitrary shapes", Elect. Eng. Japan, Vol. 111, No. 2, pp. 117-123, 1992