# The Application of the Wavelet Transform of Travelling Wave Phenomena for Transient Based Protection

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*Abstract* – The key point of transient based protection is the ability to extract the transient components from the sampled waveforms and then process these to detect quickly the occurrence of a power system fault and then accurately determine the fault's location in the power system network.

This paper presents an analysis of the characteristics of fault generated travelling wave transient phenomena using the wavelet transform. The characteristics of both the forward and reverse travelling waves are extracted using the Modulus Maxima (MM) of the wavelet transform. The advantages of the techniques are that they are simple and direct in their analysis and thus are able to determine clearly the characteristics of the travelling waves transients and hence determine the type of fault and its location.

The paper provides the theoretical foundation for fault detection and location using wavelet transforms to analyse transients and illustrates their application.

*Keywords* – Transients, Numeric Relaying, Transmission Lines, Travelling Wave, Wavelets and Modulus Maxima.

#### I. INTRODUCTION .

The accurate extraction and characterisation of travelling wave transient components provides the foundation for the travelling wave based protection and fault location systems. Travelling waves are high frequency signals generated by sudden changes to the power system. By their nature, the Fourier transform and other conventional mathematical methods cannot completely represent these transient signals in both their frequency characteristics and the time that they occur. As a result, the extraction and representation of the characteristics of the travelling waves transients have been a problem and therefore the fault location and protection techniques which use these analysis techniques cannot offer an ideal performance.

The wavelet transform is an analysis method that uses both the time and the frequency domain. It is not only suitable for analysis of travelling wave transients, but also provides a tool for the extraction and representation of travelling wave transients (TWT). Based on the theories describing travelling wave propagation, this paper uses the basic characteristics of the various TWT, and the Wavelet analysis of both the power system voltage and current signals to present a basic analysis of various fault characteristics. It provides the foundation for TWT based protection and fault location techniques.

# II. CHARACTERISTICS OF THE TRAVELLING WAVE TRANSIENTS (TWT).

#### A. Basic Characteristics of TWT

A fault occurring on a transmission line will generate both voltage and current travelling waves. These will travel along the line until they meet a discontinuity on the line, such as fault point and busbar. At this point, both a reflection and a refraction of the wave will occur. This generates additional waves which will propagate through the power system.



Fig.1. Fault generated Travelling Wave on a Single-Phase Transmission Line.

Fig.1 shows a diagram for a solid fault on a single-phase transmission line. The voltage and current travelling wave at both ends of the line 'M' and 'N' can be expressed as:-

$$u_{m}(t) = e(t - \tau_{m}) + \alpha_{m}e(t - \tau_{m}) - \alpha_{m}e(t - 3\tau_{m}) - \alpha_{m}^{2}e(t - 3\tau_{m}) + \dots,$$
  

$$i_{m}(t) = \frac{-e(t - \tau_{m}) + \alpha_{m}e(t - \tau_{m}) + \alpha_{m}e(t - 3\tau_{m}) - \alpha_{m}^{2}e(t - 3\tau_{m}) + \dots,}{Z_{c}},$$
  

$$u_{m+}(t) = \alpha_{m}e(t - \tau_{m}) - \alpha_{m}^{2}e(t - 3\tau_{m}) + \dots,$$
  

$$u_{m-}(t) = e(t - \tau_{m}) - \alpha_{m}e(t - 3\tau_{m}) + \dots,$$
  
(1)

$$u_{n}(t) = e(t - \tau_{n}) + \alpha_{n}e(t - \tau_{n}) - \alpha_{n}e(t - 3\tau_{n}) - \alpha_{n}^{2}e(t - 3\tau_{n}) + \dots,$$

$$i_{n}(t) = \frac{-e(t - \tau_{n}) + \alpha_{n}e(t - \tau_{n}) + \alpha_{n}e(t - 3\tau_{n}) - \alpha_{n}^{2}e(t - 3\tau_{n}) + \dots}{Z_{C}},$$

$$u_{n+}(t) = \alpha_{n}e(t - \tau_{n}) - \alpha_{n}^{2}e(t - 3\tau_{n}) + \dots,$$

$$u_{n-}(t) = e(t - \tau_{n}) - \alpha_{n}e(t - 3\tau_{n}) + \dots$$
(2)

Where:-

m, n represent voltage and current quantities at end M and N respectively;

 $\alpha_m$  and  $\alpha_n$  are the reflection coefficients at ends 'M' and 'N' respectively.

 $\tau_m$  and  $\tau_n$  are the time for the wave to travel to the ends 'M' and 'N' respectively.

Zc is the wave resistance; positive is the forward wave and negative is the reverse wave

e(t) is the super-imposed voltage source of the fault network [1]

As shown in Fig.1 and equations (1) and (2), the basic characteristics of fault generated travelling wave transients can be summarised as:-

1) The wave characteristics change suddenly with the arrival of successive waves at the busbar. This marks the occurring of the fault and the travelling time for the journey from the fault to busbar etc.

2) The magnitude of the sudden change depends on the magnitude of the voltage at the fault instant e(t). For later waves, it also depends on the reflection and refraction coefficients at the discontinuity and the attenuation characteristics of travelling wave[2].

3) The polarity of the sudden change depends on the polarity of the fault voltage at the fault instant and the discontinuous characteristics of the wave impedance. Generally speaking, the polarity of travelling wave has the following characteristics:-

a). Reflected voltage and current waves from the fault point will have the same polarity as the incident waves.

b). The initial voltage or current waves have the same polarity at both ends of the line.

c). For the reflected positive wave from the busbar and the reflected negative wave from the fault point, their initial wave and reflected wave have the same polarity.

The above basic characteristics lay the foundation for the TWT protection. However, considering fault detection, the extraction of the fault generated TWT is further complicated by the coupling between three phases; the refraction from non-metallic fault; the refraction from

healthy line sections and the propagation to the faulted line section; the different propagation velocities for aerial mode and ground mode; the propagation losses of the wave; the shunting effects due to the busbar to earth capacitance; and the presence of interference or noise. All these factors cause complications for techniques used for the travelling wave based fault location[3,6] and protection[4] systems.

# B. Wavelet Transform and Modulus Maxima.

The Wavelet description of travelling wave is derived from the 'Base Wavelet' function and describes the irregular characteristics of the transient travelling wave. Assume the function f(t) represents the travelling wave (voltage, current or direction) and the  $\Psi(t)$  represents base Wavelet, the Wavelet description of travelling wave, or Wavelet transform  $W_s f(t)$  is given in Equ.(3) below[5]:

$$W_s f(t) = f(t) \bullet \Psi_s(t) , \qquad (3)$$

Where  $\psi_s(t) = \psi(t/s)/s$  represents the basic Wavelet; *s* is the scale parameter or scaling factor. The Wavelet is the dyadic Wavelet when  $s = 2^j$   $(j \in \mathbb{Z})$ , the corresponding wavelet transform is the Dyadic Wavelet transform.

The dyadic wavelet transform possesses an important characteristic in that the amplitude of the wavelet transform's 'Modulus Maxima' MM, is not modified by translations. Therefore this transform is suitable for pattern identification and detecting singularities in the signal. Because the B-spline function and its derivatives can be readily calculated, and it has the least support in all of the polynomial spline functions, the quadratic B-spline function has been proved to be an optimal function to handle signals contaminated by noise[5]. This technique is therefore used to extract travelling wave signals.

The modulus maxima, MM, of wavelet transform is defined as  $\forall \varepsilon > 0$ ,  $W_{s,f}(t)$  and is used to extract the maxima values in the measured signals within the time period  $(t-\varepsilon, t+\varepsilon)$ . The MM procedure is used to identify the maximum signals in the monitored data and provides both the value of these maximum signals and the time of their occurrence. These are then stored for the next stage of the analysis. Since the input waveforms are reduced to a table of these measurements, this dramatically simplifies the identification of the discontinuities associated with the travelling wave transients. The volume of data that needs to be processed is dramatically lower than that used by alternative approaches.

The modulus maxima of the results from using the Wavelet transform can completely describe the original function (or signal), and can also be used to reconstruct the original function[5]. In addition, the use of the modulus maxima of wavelet can also eliminate the majority of any noise in the original signals.

C. Procedure for Analysis using Wavelets to describe Travelling Waves.

- 1) Perform Wavelet transform on measured waveforms.
- 2) Obtain the MM of results from using the wavelet transform.
- 3) Store the values of the maxima and the time when they occur.
- 4) Analyse the results of the MM, to identify the travelling wave transients and to check the validity of the results to ensure that they define travelling wave transients associated with power system fault conditions. Other types of signals will also have an MM signature and therefore can be extracted from the record.

# III. THE MODULUS MAXIMA REPRESENTATION OF VARIOUS TRAVELLING WAVES

### A. MM representation of Voltage Travelling Wave.

The network shown in Fig.2 was used to illustrate the technique. This shows a 500kV transmission system with an 'a' phase to earth fault on the line section MN. The fault inception angle is  $45^{\circ}$  and fault path resistance is  $50\Omega$ .



Fig.2. The Test 500kV Transmission Line System.

Fig.3 shows the three phase voltages and results of using the Wavelet transform on the waveforms recorded for the phase 'a' travelling wave voltage signal and its MM, the transformation was carried out 4 times using the different wavelet transforms. From Fig.3, it is observed that:-

1) The Wavelet transforms reach a maxima when the fault travelling wave arrives at the measurement point and is represented by (12341'2')etc.. The sudden change point corresponds to the time when the travelling wave arrives at the busbar.



(a) three phase voltage travelling wave.



(b) 'a' phase voltage for scales,  $2^1$ ,  $2^2$ ,  $2^3$ ,  $2^4$  of wavelet transform  $W_1$ ,  $W_2$ ,  $W_3$  and  $W_4$ 



(c) MIN of a phase voltage corresponding to scales  $M_1$ ,  $M_2$ ,  $M_3$  and  $M_4$ 

Fig.3. Three Phase Voltages and Phase 'a' Wavelet Transform.

2) Different scales of MMs ( $M_1$ ,  $M_2$ ,  $M_3$  and  $M_4$ ) reflect magnitudes and positions of travelling wave components for different frequency bandwidth.

3) The polarity of MM is identical to the polarity of sudden change of the travelling wave.

4) The magnitude of MM depends on two factors:-

(a) the amplitude of the sudden change of the travelling wave;

(b) gradient of the sudden change.

The amplitude of the sudden change due to the travelling wave depends on the instantaneous magnitude of the voltage at the fault point when the fault occurs. The gradient depends on the travelling wave attenuation characteristics, the fault position, system configuration and line parameters. The basic characteristics of these maxima in the MM signal are identical to that associated with travelling wave transients.

The derivation of the MM is based on the Wavelet transform. They can be expressed as:-

$$f(v_a, v_b, v_c, i_a, i_b, i_c) \Leftrightarrow g(\max(w_s f(v_j, i_j))) \quad j = a, b, c \quad (3)$$

Where f is the set of travelling wave and g is the set of MM.

The following points can be seen from the waveforms shown in Fig.3:-

1) The travelling wave appears on both faulted and healthy phases when fault occurs.

2) After the Wavelet transform, the MM process identifies the initial voltage travelling wave ①, the reflection waves from the fault point ③, the reflected waves from remote end busbar ④, the reflected waves from neighbouring line busbar ②, and the ground modal components ①'.

3) The MMs of the reflected wave (3) and the initial wave (1) have the same polarity.

4) The MMs of the reflected voltage wave from the neighbouring busbar and the initial wave have the same polarity.

5) Faults involving earth produce ground modal travelling waves. Because of the different velocities of the ground mode and the aerial mode travelling waves, there are sudden changes in the voltage waveform and corresponding changes in the ground modal travelling wave 1'. The MM produced by the changes will complicate the extraction of the waves.



(a) current signal travelling wave.



(b) Modulus maxima  $M_1$ ,  $M_2$ ,  $M_3$  and  $M_{4.}$ 

# Fig.4 Mode $\alpha$ Current Travelling Wave and the Wavelet Transform.

### B. MM representation of current travelling wave.

Fig.4 shows the waveform of  $\alpha$  modal current and its MMs. With reference to Equation (1), it is observed that:-

1) The magnitude of the initial wave is greater than that of the reflected waves from the fault point and other discontinuity points, such as a busbar. Their MMs also follow this pattern. See point (1) as shown in Fig. 4.

2) The polarity of the MM of the reflected wave from the fault point is the same as that of the initial wave. Its amplitude is  $\alpha_m$  or  $\alpha_n$  times of that of the initial wave.  $\alpha_m$  or  $\alpha_n$  represent the attenuation of these signals. See point ③ as shown in Fig.4.

3) The polarity of the MM of the reflected wave from neighbouring busbar is the same as that from the faulted point, and its amplitude is the inverse of the former (point 2) as shown in Fig.4).

4) The polarity of the MM wave reflected from the remote end busbar is of the opposite polarity to that from the fault point. Its amplitude is proportional to  $\alpha_m \alpha_n$  (point ④ as shown in Fig.4).

There is no ground mode travelling wave using the  $\alpha$  mode current travelling wave and hence there is no equivalent to point  $\bigcirc$  as shown in Fig.3.

# C. MM representation for the forward travelling wave.

The combination of travelling voltage and current waves provides a clear indication of the directionality of a fault. This is very important for power line protection. The forward travelling wave reflects only travelling waves from remote busbar, and can be expressed as:-

$$u_{M.forward}(t) = \frac{1}{2}(u_m(t) + Z_c i_m(t))$$
(5)

When deriving forward travelling wave, both voltage and current use the modal travelling wave in order to cancel the noise interference of the ground modal wave. Figs 5(a) and 5(b) show the Wavelet transform and its MM of the forward travelling wave. As shown in the figure, the reflection wave from neighbouring busbar L, point (2), is strengthened when the forward travelling wave is extracted using MM.

### D. MM representation of reverse travelling wave.

The reverse travelling wave which contains the reflections of the wave from the faulted line, contains information of fault location[3,6] and forms the basis for travelling wave distance measurement[4] and positional protection[7]. The reverse travelling wave at point M can be expressed as:

$$u_{M.inverse}(t) = \frac{1}{2} (u_m(t) - Z_c i_m(t))$$
(6)

Figs 5 (c) and (d) show the reverse travelling waves and the distribution of their MMs. The figure shows the initial travelling wave ①, the reflected wave from the fault point ③ and the reflected wave from the remote busbar ④. Here, the reflected wave from the neighbouring busbar L does not appear, and therefore, the reverse travelling waveforms only contain the waves coming after the initial wave will be the reflected wave either from the fault point or from the remote end busbar[7].

In the system configuration used here, the reflected wave from the faulted point will be of the same polarity as that of the initial wave, and that from the remote busbar will be of the opposite polarity to the initial wave. This can easily be used for fault location, distance and positional protection.



(a) Forward travelling wave at end M.



(b) MM of forward travelling wave,  $M_1$ ,  $M_2$ ,  $M_3$  and  $M_4$ .



(c). Reverse travelling wave at end M.



(d) MM of reverse travelling wave,  $M_1$ ,  $M_2$ ,  $M_3$  and  $M_4$ .

# Fig.5 'α' Modal Travelling Wave and Distribution of MM.

# IV . COMPARISON BETWEEN VOLTAGE AND CURRENT TRAVELLING WAVES.

In order to compare the characteristics of fault voltage and current travelling wave, Fig.6 gives the  $\alpha$  mode of voltage and current travelling waves and the distribution of the forward and reverse MMs at the scale  $2^3$ .



(d) MM of r everse travelling waves.

# Fig.6. Comparison between the MM of Modal $\alpha$ Travelling Waves (scale 2<sup>3</sup>).

It can be seen from the Fig. 6, that the reflection from neighbouring busbar and that from the fault point cannot be clearly identified based on voltage travelling wave, points (2) and (3).

Also, the reflection from the neighbouring busbar and that from the fault point cannot be clearly identified using the current travelling wave if there is an open circuit at the end N busbar [7].

The forward travelling wave is amplified by the wave from the far-end busbar L. In this case the real fault information is not clear when protection is based on using the travelling waves.

The reverse travelling wave only reflects the travelling wave from the fault point. This contains the most important information about the fault and provides the main basis for fault detection and identification.

From the results shown in Fig. 6, it is also shown that all the travelling waves have a common characteristic: the MM of the initial wave has the same polarity as that of the wave reflected from fault point; the amplitude of the MM of the initial wave is also greater than that of the corresponding reflected ones.

#### V. CONCLUSION

The Modulus Maxima of Wavelet transform provides a valuable technique for extracting the characteristics of various transient travelling waves generated by a power system fault.

All the waves have modulus maxima after Wavelet transform, which includes the reflected wave from the fault point, from the neighbouring busbar, from remote end busbar and from noise.

All the travelling waves, such as voltage, current and direction waves, behave differently and this can be used to discriminate between travelling waves generated at different types of discontinuity.

The common characteristic of various waves is that the initial wave has the same polarity of MM as that of the wave reflected from fault point and the amplitude of the MM of initial wave is greater than that of the corresponding reflected ones.

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