Coherency Approach for Dynamic Equivalents of Large Power Systems

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***Abstract* – This paper presents a method of identifying coherent generators on the basis of slow coherency concept, and constructing of dynamic equivalent for large electric power system. We will first describe the slow coherency technique for partitioning a power network into several groups of coherent generators, and second aggregating all the generators and loads belonging to the same group, and eliminating of the rest of the buses by network reduction. The grouping method is illustrated using a 10 machines, 39 bus system (NPCC).**

The different areas are considered as subsystems which are connected by transmission lines, each subsystem will be represented by one dynamic equivalent including a detailed machine model with regulators, turbine and stabilizer and one equivalent load. If higher frequency representation is required, the dynamic equivalent could be associated with an RLC circuit to reflect the frequency dependent impedance.

Keywords – dynamic equivalent, slow coherency method, grouping algorithm , transient stability , EMTP.

I. INTRODUCTION

In power system analysis, the simulation of large number of scenarios with the detailed model is very time consuming. Despite the high capacity of modern computers, the investigation of high dynamic behaviour of large interconnected power systems requires the use of dynamic equivalents for particular parts of the system. It is common practice to represent these parts by some form of reduced order equivalent model by decomposing the entire network into study area and one or more external areas.

Several methods have been developed to determine a coherency-based dynamic equivalent of the power system [1,2]. The benefit of this method is that the resulting dynamic equivalent is composed of physical components. The two-time scale method is the basis of the slow coherency technique for partitioning a power system network into groups of coherent generators. The coherency area identification technique is based on the fact that in multi-machine power system transients following a disturbance, some generators have the tendency to swing together. There are two major approaches to determine the coherent generators, the first is to apply disturbance directly and observe the swing curves of generators and the second is to evaluate coherency properties, which are independent of the disturbance. For finding the slow coherent areas, the latter approach requires the calculations of the eigenbasis matrix of the electromechanical model of the power system. The most linearly independent row of the eigenbasis matrix will become the reference generators. An algorithm is then applied to group non-reference generators to the reference generators. A nodal aggregation technique is finally applied to replace each coherent group by one equivalent generator.

This paper is focused on establishing dynamic equivalent with slow coherency technique. A constant voltage behind transient reactance generator model (classical model) is used in coherency identification and a more detailed model (sub-transient model) of generating units including excitation system, turbine governor and stabilizers is used to aggregate coherent units.

The dynamic equivalent obtained by the proposed method is valid for transient stability analysis with the assumption that the power system is operating only in the fundamental frequency. However, it can also be used in electromagnetic transient simulation to represent parts of the studied system which are far from the interested areas. If it is necessary, a compensation at high frequency can be done by adding a passive network synthesized to reflect the frequency characteristics of the external systems.

To use the dynamic equivalent in electromagnetic transient studies, most of the research efforts have been concentrated on the accuracy of the network equivalent representing the external system. However, the identification of external system boundaries has not received as much attention. The motivation of this paper is therefore to provide rigorous and accurate method to decomposing power systems into an internal system (study system) and external systems which may be used in electromagnetic transient studies.

II. ELECTROMECHANICAL MODEL

The well-known electromechanical model [3] of n-machine power system is:

$$\begin{aligned} &\dot{\delta}_i = \omega_i \\ &M_i.\dot{\omega}_i = P_{mi} - P_{ei} - D_i\omega_i \qquad (1)\\ &for \quad i = 1,...,n. \end{aligned}$$

where δ_i, ω_i, P_{mi}, P_{ei}, M_i, and D_i are respectively the rotor angle, angular speed, mechanical input power, electrical output power, inertia and damping constant of machine i. P_{ei} is given in terms of the angles and admittances as:

$$P_{ei} = V_i^2 Y_{ii}\cos\theta_{ii} + \sum_{j=1, j\neq i}^{n} V_i V_j Y_{ij}\cos(\delta_i - \delta_j - \theta_{ij}) \qquad (2)$$

Where the per unit voltage V_i behind transient reactance is assumed to be constant. Loads are represented by passive impedance. **Y** is the reduced admittance matrix at the internal machines nodes. The mechanical input power is also assumed to be constant.

The linearised electromechanical model of system (1) can be expressed by the state representation :

$$\begin{bmatrix} \Delta\dot{\boldsymbol{\delta}} \\ \Delta\dot{\boldsymbol{\omega}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{K} & \mathbf{D} \end{bmatrix} . \begin{bmatrix} \Delta\boldsymbol{\delta} \\ \Delta\boldsymbol{\omega} \end{bmatrix} \qquad (3)$$

where **0** and **1** are the nxn zero and unit matrices respectively. **D**= diag(D_i). **K** is the matrix of K_{ij} defined as follows:

$$\begin{aligned} K_{ij} &= -\frac{1}{M_i}\frac{\partial \Delta P_{ei}}{\partial \Delta\delta_j} \qquad i \neq j \\ &= -\frac{1}{M_i}\left[V_i V_j Y_{ij}\sin(\delta_i - \delta_j - \theta_{ij})\right] \end{aligned} \qquad (4)$$

$$\begin{aligned} K_{ii} &= -\frac{1}{M_i}\frac{\partial \Delta P_{ei}}{\partial \Delta\delta_i} \\ &= \frac{1}{M_i}\sum_{k=1, k\neq i}^{n}\left[V_i V_k Y_{ik}\sin(\delta_i - \delta_k - \theta_{ik})\right] \end{aligned}$$

At the operating point, the eigenvalues of (3) are of the following three types [4]:

a- a zero eigenvalue corresponding to the motion of all the machine angles,

b- A small negative eigenvalue corresponding to the aggregate speed of all the machines, and

c- (n-1) pairs of lightly damped oscillatory modes (in range of 1/2 to 2 Hz).

Models involving more details such as excitation systems and governors would still contain the above set of eigenvalues modified mostly in the damping and not in their frequencies [4].

Since the small damping constant D_i does not significantly affect the frequencies of the oscillatory modes, they may be neglected. Thus the linear model is a second order system :

$$\Delta\ddot{\boldsymbol{\delta}} = \mathbf{K}.\Delta\delta \qquad (5)$$

Therefore instead of dealing with a system of order 2n, we only need to deal with the nxn matrix **K**.

The matrix **K** is symmetric if **Y** is symmetric which is true for networks without phase shifters. Thus, **K** is diagonalizable and it is similar to : ($\mathbf{M}^{-1/2}$ **.A.** $\mathbf{M}^{1/2}$). Where **A** is a diagonal form of **K** and $\mathbf{M}^{1/2}$ is the square root of the inertia constant matrix **M**.

All the eigenvalues λ_i of **K** are therefore real. For λ_i negative, the eigenvalues of the system (3) are $\pm\sqrt{\lambda_i}$, on the imaginary axis. Then the low frequency modes of system (3) are the slow modes of **K** [4].

III. Slow Coherency approach

The slow coherency technique has been successfully applied for partitioning power systems in groups of coherent generators, it is based on the two time scale method. For finding the slow coherent area, the method requires the calculations of the slow eigenbasis matrix of the electromechanical model of the power system [2].

Using the system state matrix **K** from (5), we compute the eigenvalues of the system. After choosing the number of area corresponding to the r smallest eigenvalues (r is the number of reference generators), then the eigenvector matrix **U** of the r eigenvalues is computed. By applying Gaussian elimination with complete pivoting on **U**, we can obtain the most linearly independent rows. These rows correspond to the eigenvalues of the reference generators of the coherent areas.

To assign the (n-r) other generators to coherent areas, we separate the eigenmatrix **U** into $\mathbf{U_r}$ and $\mathbf{U_{n-r}}$. Matrix $\mathbf{U_r}$ contains eigenvectors of the r reference generators and $\mathbf{U_{n-r}}$ contains eigenvectors of the (n-r) other generators.
U is then ordered as :

$$\mathbf{U} = \begin{bmatrix} \mathbf{U_r} \\ \mathbf{U_{n-r}} \end{bmatrix} \qquad (6)$$

where $\mathbf{U_r}$ is rxr matrix and $\mathbf{U_{n-r}}$ is (n-r)xr matrix.
We solve for **L** in the following equation:

$$\mathbf{U_r^T}.\mathbf{L^T} = \mathbf{U_{n-r}^T} \qquad (7)$$

Matrix **L** is used to assign other generators to the coherent areas according to the largest value in each row of **L**. To include the load buses of the system, reference [5] calculated matrix **G** of direction cosines between each reference generator and all the other buses in the system instead of matrix **L**. By using the eigen matrix , **G** is formed for as :

$$\mathbf{G} = \frac{\mathbf{u_i}.\mathbf{u_j^T}}{\|\mathbf{u_i}\|.\|\mathbf{u_j}\|} \qquad (8)$$

$$for \quad i = 1,...,r \;;\; j = r+1,...,(n+p-r).$$

where r, n and p are the number of areas, machines and load buses respectively. $\mathbf{u_i}$ is the reference row vector and $\mathbf{u_j}$ is non reference row vector of the augmented matrix **U** including load buses. The significance of calculating direction cosines is that it offers a measure of closeness of the reference generator to each load bus in the power system. Hence any large separation between any two load bus row eigenvectors, each with respect to their reference eigenvectors, will constitute a weak link dividing the two areas and defining a boundary of coherent area [5].

This procedure is repeated until all the eigenvectors are assigned a reference and all boundaries have been defined.

IV. NODAL AGGREGATION METHOD

The dynamic equivalent is constructed in the basis of power invariance at the terminal bus of the equivalent generator representing the coherent group[6]. The terminal buses of the coherent generators are replaced by an equivalent bus such that the following two conditions must be satisfied:

1. The current and the voltages at the tie buses does not change.
2. The active and reactive power injection at the equivalent bus must be equal to the sum of injections at the terminal buses of coherent generators: $S_e = \sum S_i$.

Figure 1 visualizes the reduction of an external system into its dynamic equivalent.

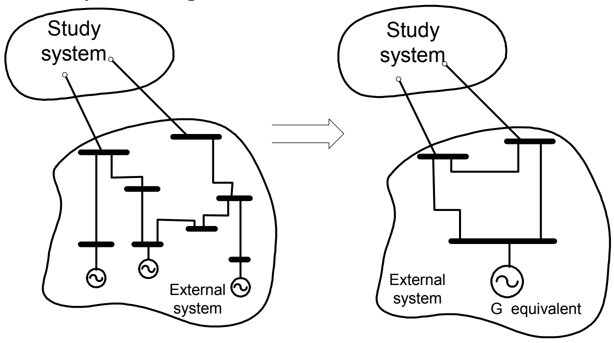

Fig. 1 A schematic partitioned power system

The transformation of the network can be described for the original network by :

$$\begin{bmatrix} \bar{\mathbf{I}}_\mathbf{r} \\ \bar{\mathbf{I}}_\mathbf{c} \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{Y}}_\mathbf{rr} & \bar{\mathbf{Y}}_\mathbf{rc} \\ \bar{\mathbf{Y}}_\mathbf{cr} & \bar{\mathbf{Y}}_\mathbf{cc} \end{bmatrix} . \begin{bmatrix} \bar{\mathbf{V}}_\mathbf{r} \\ \bar{\mathbf{V}}_\mathbf{c} \end{bmatrix} \tag{9}$$

and for the reduced one :

$$\begin{bmatrix} \bar{\mathbf{I}}_\mathbf{r} \\ \bar{I}_e \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{Y}}_\mathbf{rr} & \bar{\mathbf{Y}}_\mathbf{re} \\ \bar{\mathbf{Y}}_\mathbf{er} & \bar{\mathbf{Y}}_\mathbf{ee} \end{bmatrix} . \begin{bmatrix} \bar{\mathbf{V}}_\mathbf{r} \\ \bar{V}_e \end{bmatrix} \tag{10}$$

where the subscripts r, c and e refer to retained buses(tie buses), coherent buses and single equivalent bus respectively. As "e" is a single bus, $\bar{\mathbf{Y}}_\mathbf{re}$ is a column, $\bar{\mathbf{Y}}_\mathbf{er}$ is a row and $\bar{Y}_{ee}$ is a scalar.

The first condition is satisfied for any vector $\bar{\mathbf{V}}_\mathbf{c}$ when :

$$\bar{\mathbf{Y}}_\mathbf{rc} . \bar{\mathbf{V}}_\mathbf{c} = \bar{\mathbf{Y}}_\mathbf{re} . \bar{V}_e \tag{11}$$

This means that the coherent buses are transformed to the equivalent bus using phase shifting transformers with complex ratios vector $\bar{\mathbf{a}}$ expressed as:

$$\bar{\mathbf{a}} = \bar{V}_e^{-1} . \bar{\mathbf{V}}_\mathbf{c} \tag{12}$$

The second condition is satisfied when :

$$\bar{V}_e . \bar{I}_e^* = \bar{\mathbf{V}}_\mathbf{c}^\mathbf{T} . \bar{\mathbf{I}}_\mathbf{c}^* \tag{13}$$

Where the left hand side expresses the injection at the equivalent bus and the right hand side expresses the sum of all coherent buses injections. The admittances of the equivalent network are given by :

$$\begin{aligned} \bar{\mathbf{Y}}_\mathbf{re} &= \bar{\mathbf{Y}}_\mathbf{rc} . \bar{\mathbf{a}} \\ \bar{\mathbf{Y}}_\mathbf{er} &= \bar{\mathbf{a}}^{*\mathbf{T}} . \bar{\mathbf{Y}}_{cr} \\ \bar{Y}_{ee} &= \bar{\mathbf{a}}^{*\mathbf{T}} . \bar{\mathbf{Y}}_\mathbf{cc} . \bar{\mathbf{a}} \end{aligned} \tag{14}$$

Theses admittances of the equivalent branches linking the equivalent bus with tie buses depend on the vector of transformation ratios $\bar{\mathbf{a}}$, and hence on the voltage angles at the equivalent bus.

The same result will be obtained by defining a transformation matrix **T** which converts the parameters and variables from the original system to the reduced one. **T** is defined as :

$$\mathbf{T} = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \bar{\mathbf{a}} \end{bmatrix} \tag{15}$$

Its dimensions are $(n_r+n_c)x(n_r+1)$. n_r and n_c are number of retained buses and coherent buses respectively. The complex vector $\bar{\mathbf{a}}$ is as follows :

$$\bar{\mathbf{a}} = \left[\bar{a}_1 \ldots \bar{a}_{nc} \right]^T \tag{16}$$

the relationship between bus voltages in the original and the reduced network is given by :

$$\bar{\mathbf{V}} = \mathbf{T} . \bar{\mathbf{V}}' \tag{17}$$

where superscript ' refers to bus voltage in reduced system. From the first condition, we obtain current injection in the reduced network as follows:

$$\bar{\mathbf{I}}' = \mathbf{T}^{*\mathbf{T}} . \bar{\mathbf{I}} \tag{18}$$

then :

$$\bar{I}_e' = \bar{\mathbf{a}}^{*\mathbf{T}} . \bar{\mathbf{I}}_\mathbf{c} \tag{19}$$

The admittance of the equivalent network becomes :

$$\bar{\mathbf{Y}}' = \mathbf{T}^{*\mathbf{T}} . \bar{\mathbf{Y}} . \mathbf{T} \tag{20}$$

then :

$$\begin{cases} \bar{\mathbf{Y}}_\mathbf{rr}' = \bar{\mathbf{Y}}_\mathbf{rr} \\ \bar{\mathbf{Y}}_\mathbf{re}' = \bar{\mathbf{Y}}_\mathbf{rc} . \bar{\mathbf{a}} \\ \bar{\mathbf{Y}}_\mathbf{er}' = \bar{\mathbf{a}}^{*\mathbf{T}} . \bar{\mathbf{Y}}_\mathbf{cr} \\ \bar{Y}_{ee}' = \bar{\mathbf{a}}^{*\mathbf{T}} . \bar{\mathbf{Y}}_\mathbf{cc} . \bar{\mathbf{a}} \end{cases} \tag{21}$$

which are the same admittances as defined in equation (14).

V. Aggregation of Generating Units

The generating unit components include synchronous machine, excitation system, turbine-governor system and power system stabilizer.

An accurate method for dynamic aggregation of generating units with detailed representation is developed in reference [7]. The method uses a frequency domain approach where the parameters of the equivalent model are numerically adjusted to obtain a minimal error between its transfer function and the sum of the transfer functions of individual units. This method involves a time consuming process of optimization.

The proposed aggregation procedure is similar to that developed in reference [8]. The method uses a non iterative procedure to estimate the parameters of an equivalent generating unit. The equivalent parameters are determined by structure preservation of the coefficient matrices in time domain representation of machines. The linear aggregation is used to determine the parameters of corresponding equivalent controllers which are mainly gains and time constants. This method requires much less computation time than the frequency domain procedure.

The parameters of aggregated generators determined by this technique ensure that the sub-transient, transient and steady state short circuit behaviors of the equivalent generators are mostly similar to those of the original group of coherent generators.

VI. Test System

The coherent grouping algorithm was applied to the Northeast Power Coordinating Council (NPCC) system as shown in figure 2 [9]. Detailed models are considered for all generators with sub-transient effects on d and q axes and equipped with IEEE-type-1 excitation system, turbine governor system and power system stabilizer.

The natural frequencies of the electromechanical modes of the 10 machines 39 bus system sorted from the lowest to highest frequency are as follows:
0, **0.6227**, **0.9558**, **1.0655**, 1.1999, 1.2751, 1.3443, 1.5444, 1.5571, 1.8406.

To find the coherent groups of generators, the first step is to determine the low modes of oscillation that form the basis of coherency grouping. Since the inter-area modes frequencies are approximately between 0.05 and 1 Hz, then we can chose the first four low frequencies to be inter-area modes. The results of areas identification are given in table I.

Table I Area grouping of NPCC system

Area	I	II	III	IV
Reference gen. n^o.	**9**	**3**	**4**	**1**
Other gen. n^o.	8	2	5, 6, 7	10

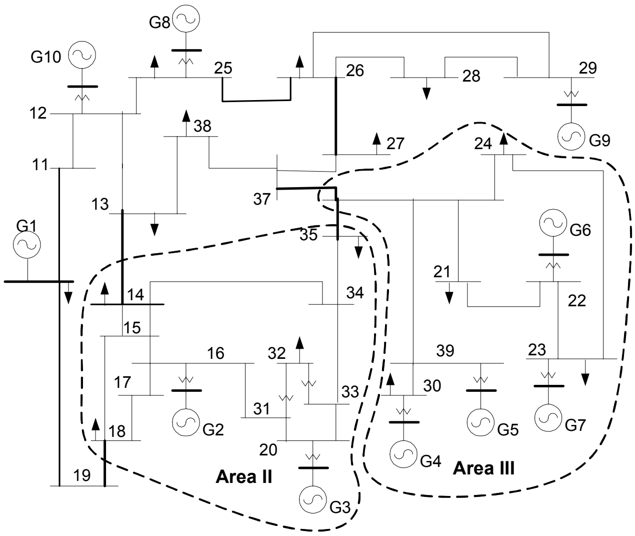

Fig. 2 NPCC test system

VII. EMTP Application

There are various techniques to obtain a reduced equivalent circuit for the external system which gives the same behavior as the actual system for both steady and transient operations. Many reduction technique are employed for power transient studies, which can be carried out in time domain or frequency domain [11], [13].

Most of the research efforts have concentrated in the frequency domain where the external system is replaced by an appropriate passive circuit of lumped R, L and C components whose values are synthesized via an optimal curve fitting over range of frequencies. Response will be transformed to time domain by application of convolution integral. The advantage of frequency domain method is related to ease the modeling of the frequency dependant parameters, and its major disadvantage is modeling of nonlinear characteristics. The time domain method permits the representation of non linear elements, however its major difficulty is modeling frequency dependant parameters [12].

A. Dynamic coherency approach

The study of electromagnetic phenomena in power systems involves a frequency range from dc to many thousands hertz. Phenomena not at the fundamental frequency are usually belong to electromagnetic domain.

For low frequency transients the dynamic equivalent determined by the slow coherency approach should represent any area. However, for high frequency, this representation is not appropriate, therefore, a compensation with frequency dependant impedance is required. This impedance is calculated for positive and zero sequences system and it is connected in series with the dynamic equivalent as shown in Figure 3. This should be sufficient to replicate electrical transient in high frequency.

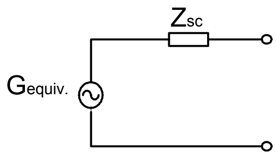

Fig. 3 simplified equivalent circuit

VIII. SIMULATION RESULTS

To show the accuracy and efficiency of the equivalent model, three phase short-circuit fault was applied on line 27-37 at bus 27 and cleared after six cycles(100ms). It is simulated for five seconds.

The reduction consists of replacing coherent generators (4, 5, 6 and 7) by one equivalent. The fault described above is applied on the full system as well as on the reduced system. Transient responses obtained from these simulations are presented in figures 4, 5, 6, 7, 8 and 9.

The comparison of the swing curves of the coherent group in full system and that of equivalent machine in the reduced system is represented in figure 4. These curves are plotted with reference to the rotor angle of machine 1. Figure 5, 8 and 9 give the same kind of comparison but for machine speed, terminal voltages and excitation voltages respectively. The coherent behavior of equivalent and individual machines is confirmed in these figures.

A good matching is also observed in the superimposition of the total active power (Figure 6) and the terminal voltage (Figure 7) of individual machines in coherent group and that of equivalent machine.

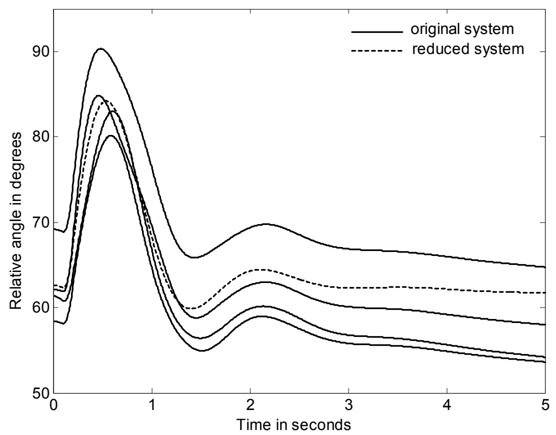

Fig. 4 Swing curves of 4 machine coherent group and its equivalent

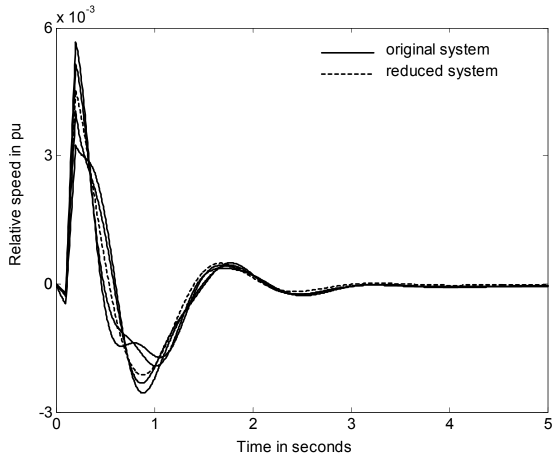

Fig. 5 Speed curves of 4 machines coherent group and its equivalent

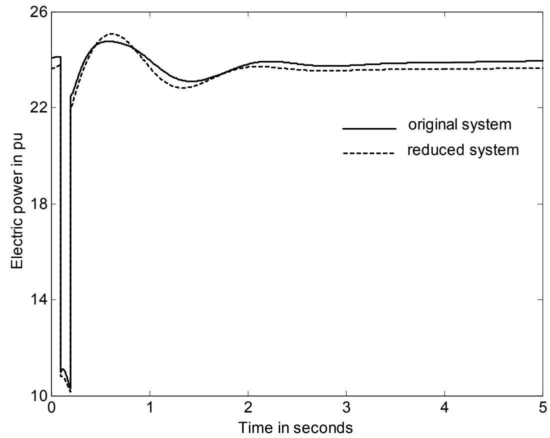

Fig. 6 Comparison of the total generator active power output

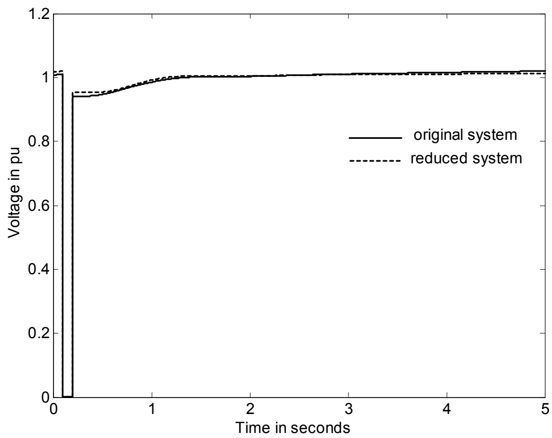

Fig. 7 Comparison of voltage responses at faulted bus

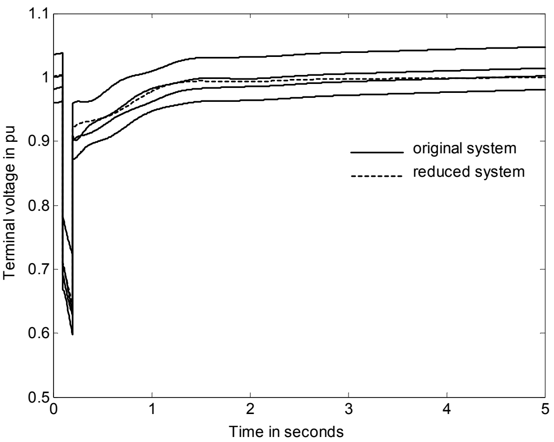

Fig. 8 Terminal voltages of 4 generator coherent group and its equivalent

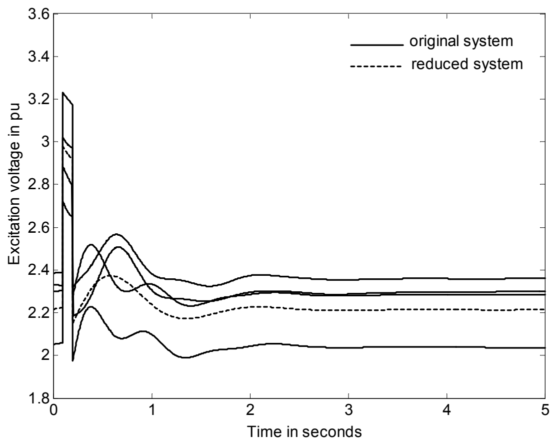

Fig. 9 Excitation voltages of 4 generator coherent group and its equivalent

VII. Conclusion

In this paper we have presented a method for partitioning large power system into study area and one or more external systems on the basis of slow coherency concept. Each area should be represented by one equivalent generator in detailed model with corresponding equivalent controller devices. This method is used for transient stability and electromechanical oscillations studies.

For electromagnetic transient applications, we have proposed to use the coherency dynamic equivalent in association with a frequency dependant impedance which can be synthesized by one of the methods commonly used in EMTP.

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