# Overview of Grounding Electrode Models and their Representation in Digital Simulations

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*Abstract* –**Lightning and surge protection studies require accurate estimation of grounding systems dynamic behavior. For this reason, grounding electrodes need to be accurately modeled and effectively represented in digital simulations. Many attempts have been made to this direction during the last years. These attempts are either based on computer models, which are solved numerically or on analytical expressions for current and voltage distributions under simplifications or special initial conditions. Several solutions handle the equations of propagation, considering frequency dependence of the electrode parameters. Another class of solution methods is based on the principles of electromagnetism. This paper presents a summary of the main works in grounding electrodes modeling and their representation in digital simulations.**

***Keywords*** – Grounding Electrode, Open Ended Transmission Line, Transient Response, Effective Length, EMTP, Electric Dipoles, Method of Moments

## I. INTRODUCTION

A significant number of papers dedicated to modeling of grounding electrodes for digital simulations have been presented during the last 20 years. A large amount of the main works in this field focuses on analysis of the frequency-dependent behavior of grounding electrodes for digital simulations. All grounding electrode models, and their variations can be summarized in the following main categories:

1. Lumped–parameter models that represent transmission systems by lumped elements whose values are independent of frequency[1][2]. This category contains circuit models of the electrode that consist of linear elements.
2. Distributed-parameter models, for which two categories can be distinguished, constant parameter[3][1][2], and frequency-dependent parameter models[4],[5],[2].
3. Electromagnetic models based on frequency domain calculations with subsequent transformation of the solution in time domain using Inverse Fast Fourier Transformation (IFFT)[6][7].
4. Hybrid Models combining the electromagnetic with the distributed parameter model [8], [9].

The first types of models are adequate for steady-state calculations. Models of the second category with constant parameters, consider the electrode as a transmission line divided in a number of series connected – circuits. Most of these methods need to make low frequency, quasi-static approximations. The upper frequency limit of satisfactory accuracy depends on the size of the electrode segments and the electrical characteristics of the surrounding soil[10]. Nevertheless, an error is introduced when phenomena involving high frequencies, such as lightning, are examined.

An improved technique using J.Marti's approach for calculation of transients in transmission lines is presented in [4],[5],[2]. In this approach, voltage and current values along the electrode are calculated using EMTP's frequency dependent transmission line model[12]. The main advantage of this method is incorporation in the model of high frequencies making it suitable for lightning studies.

The third type models are the most accurate ones for transient calculations, as they take into account the distributed nature of parameters and consider their frequency-dependence. Methods of the third category use an Electromagnetic Field approach for the calculation of the response of the grounding system in a wide range of frequencies [7], [6]. These methods, when applied in the analysis of fast transient phenomena, are characterized by increased accuracy because they are based on the principles of electromagnetism and the least neglects possible are made. The fact that a set of equations has to be solved for every single frequency however, increases significantly the required computational time[7].

The model presented in [8] and [9] is a combination of the models of the second and third category and it has the advantages of both models. However, implementation in EMTP[10] requires a preprocessing of data and use of a suitable subroutine written in ATP- MODELS language during calculation. A disadvantage of this model is the large amount of required computational time.

In this paper, a summary of the main works in modelling of grounding electrodes for time domain simulations is presented. All models presented in this paper are computationally efficient and give results of a good accuracy, when contrasted to experimental values, as shown in literature.

Soil ionization phenomena are not incorporated in most of the models presented in literature. It is however possible to consider ionization, by suitably modifying the circuit elements of the electrode models as shown in [13],[14],[15],[16]. Alternatively, a recurrent process will be needed, increasing this way the time for calculations.

## II. GROUNDING ELECTRODE EQUATIONS

### *A. Time Domain Models – Pi Circuits Model*

To construct the $\pi$-circuits model the electrode is divided into elementary segments of equal length. Each segment is represented by a $\pi$-circuit with lumped parameters *R L C* and *G* as it is shown in figure 1. $R_e$ $L_e$ $G_e$ and $C_e$ are per unit length parameters calculated as in [17], [18], [19].

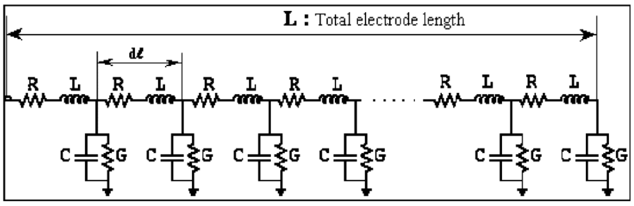


Figure 1 : $\pi$ -circuits model of a grounding electrode

The $\pi$-circuits model can be easily constructed using the EMTP-ATPDraw graphical preprocessing program. It can be directly solved by EMTP for the unknown currents and voltages at any point. The internal calculation process involves the construction of a matrix of the system, and the update of the unknown variable values at every time step.

Division into shorter segments results in higher accuracy. Furthermore, when the number of $\pi$-circuits tends to infinite, the expressions of current and voltage distributions and the input impedance tend to those of the open-ended transmission line, as proven in [20].

Numerical error produced by the $\pi$-circuits model, is due to:

1. Consideration of R, L, C and G as lumped, while they are distributed along the electrode. This type of error can be written in closed form versus frequency, contrasting the impedance of the n $\pi$-circuits' network to the one of the open-ended transmission line. Z "seen" at the injection point of n $\pi$-circuits ladder network is equal to [20], [21]:

$$Z(n) = \frac{\alpha_1 (Z_p - \alpha_1)^n - \alpha_2 (Z_p - \alpha_2)^n}{(Z_p - \alpha_2)^n - (Z_p - \alpha_1)^n} \qquad (1)$$

Where $Z_s = (R + j\omega L) \cdot \frac{\ell}{n}$ and $Z_p = 1 \Big/ \left[ (G + j\omega C) \cdot \frac{\ell}{n} \right]$ the shunt and series impedance per elementary segment, and

$$b = Z_s + 2Z_p, \alpha_1 = \frac{-Z_s + \sqrt{Z_s^{\,2} + 4Z_s Z_p}}{2}, \alpha_2 = -(Z_s + \alpha_1).$$

This type of error is expressed here as the % absolute difference between |Z| calculated from (1) and $|Z_c \coth(\gamma\ell)|$. It depends basically on frequency and also on electrode length, number of segments, and soil characteristics. Division to shorter elementary segments when calculating at higher frequencies results in smaller numerical error. In [10] an elementary segment length of at most 1/10 of the wavelength in the soil at a particular energization frequency is suggested. Values of this type of error versus frequency are shown in figure 2 for practical application cases. For longer electrode lengths, the error is higher at low frequencies, but in high frequencies its increment slope reduces. Higher soil resistivity leads to smaller numerical error, as well as the increment of the total number of division segments. It can be observed that division into segments of 1m results in error lower or equal to 5% at the MHz frequency range. Division into segments of 0.71m results in error less or approximately equal to 1% at the MHz frequency range.

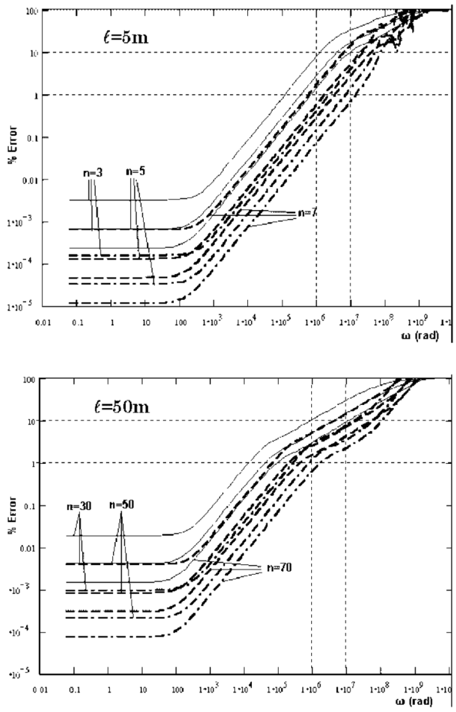


Figure 2 : Error due to segmentation expressed as percentage % on the impedance of the open ended transmission line Z=coth( $\gamma\ell$), where (__ $\rho$ =100 $\Omega$ m),(- - - - $\rho$ =500 $\Omega$ m),( -.-.-.- $\rho$ =2000 $\Omega$ m)

2. Application of a numerical integration method to solve the equations of the n $\pi$-circuits network. The numerical error of this type cannot be expressed by a closed-form mathematical expression. It depends on the length of each elementary segment and the time step used by the numerical procedure. It reduces with the reduction of the integration step, increasing this way the computational time.

The trapezoidal rule implemented in EMTP requires that the time step remains constant during the whole simulation time. This is a disadvantage of application of the lumped elements $\pi$-circuits model, because the time step at the "tail" of the injection current, is possible to increase without seriously affecting accuracy. Use of specialized calculation software enables the use of variable time step although requiring more complicated programming code. In general, when lumped elements circuit models are used, the recommended integration time step should be less or equal to the time needed for the voltage or current wave to run across the elementary segment length. This way the response "sees" artificially the end of the segment before or just when the real wave reaches the end of the segment. This way the integration time step can be related to excitation frequency, and the double exponential waveform of the injection current. Shorter time step should be used in the analysis of the response under a faster impulse current, with shorter time to its maximum value.

Variations of this model for specific applications can be found in literature. These models are derived simplifying or altering the model shown in figure 1, which however can be applied in a wider range of cases met in practice.

### *B. Time Domain Models – Modified Pi Circuits Model*

To construct the modified $\pi$-circuits model, the electrode is divided into elementary segments of equal length. Each segment is represented by a modified $\pi$-circuit where inductance and capacitance elements are considered distributed in the middle of each segment and resistance and conductance are considered lumped. The modified $\pi$-circuits model of the grounding electrode is shown in figure 3. This model is described and used in SGSYS grounding software [3],[22].

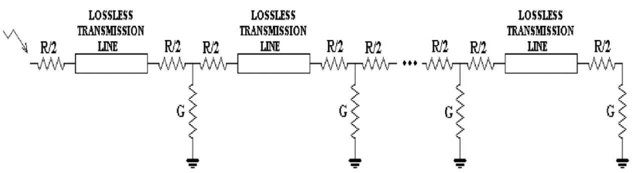


Figure 3 : Modified $\pi$ -circuits model of the grounding electrode

In the middle of each segment, the Bergeron's accurate expressions for the lossless transmission line are written. These expressions are extended using Kirchoff 's laws to obtain values for voltages and currents at the ends of each segment. All the end nodes of elementary segments will be n+1 leading to a system of 2n+2 equations when solving for the unknown currents and voltages at these nodes. In general the system of equations is written in the form:

$$x(t) = M^{-1}Ex(t-\tau) - M^{-1}x_0$$

where $\tau$ is the traveling time for a wave along the elementary segment's transmission line. Matrices M, and E have their non-zero elements very close to the diagonal, allowing the application of sparsity techniques.

Numerical error when this model is used, is caused considering $R$ and $G$ as lumped. This is opposed to the distributed nature of $R$ and $G$. The error of this model is smaller compared to that when the lumped elements $\pi$-circuits model is used, because the solution expressions for the lossless transmission line part of each segment, are mathematically accurate. Increasing the number of the elementary segments of the electrode, the response of the network tends to the response of the open-ended transmission line. Furthermore, the input impedance $Z$ of the model obtains very close values to $Z_c \coth(\gamma\ell)$ as shown in figure 4, when the electrode is divided in segments less but close to 1/10 of the wavelength in the soil. In figure 4 it can also be observed that the effect of segmentation when $d\ell$ is below 1m, is eliminated up to 10 MHz frequency.

Elementary segments length, and solution time step $\tau$ are closely related and remain constant during calculation. Solution of the network of figure 3 requires a fixed time step equal to the travelling time of the lossless transmission line part at the center of each segment. Considering division to 1m long segments, the solution time step will be $\tau$ = 3.33x10$^{-9}$ and it is much smaller than the allowable value when the lumped elements $\pi$-circuits model is used.

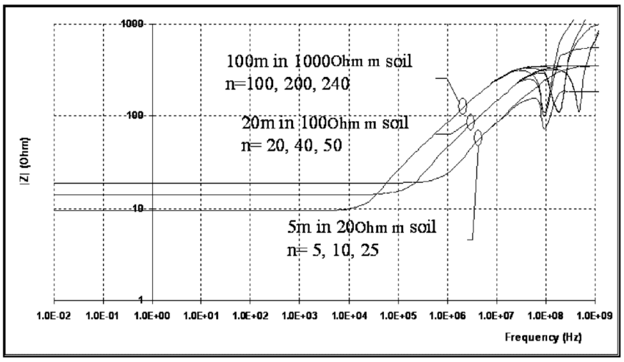


Figure 4 : Input |Z| vs. frequency, contrasted to open-ended T.L.

### *C. Time Domain Models – Modified Lumped Elements Pi-Circuits Model*

This model has been proposed by the authors [20],[21] in order to overcome the problems in calculation of the response of long electrodes or extended grounding systems. This can be the case of windfarm grounding systems.

The modified lumped elements-circuits model is obtained from the lumped elements-circuits model. It is better suited for calculation of the transient response because it is based on the effective length[23] of the electrode. The term "effective length" denotes the electrode length, above which, no serious reduction of impulse impedance and GPR of the grounding conductor are observed. Values of the effective length are provided in literature. They are obtained either graphically [21] or from mathematical interpolation formulae [24]. An example is shown in figure 5 where the max. potentials at the energization point are plotted for a 50Hz $I(t)=I_o \cdot \sin(\omega t)$ kA and for a 9kA 1.4/17 μs source current. The resistivity of the soil is equal to 1000Ωm, $I_o$=9kA and the electrode is placed 0.75m below the surface. In the case of the electrode of figure 5, effective length is approximately equal to 70m.

Effective length depends on soil type and excitation front which is closely related to the range of the maximum injected frequency. The effect of soil type can be distinguished in the effect of the soil resistivity and the effect of the relative permittivity. Those two are in some cases related to each other and to frequency, as suggested in[25]. In general, higher relative permittivity denotes soil with a higher percentage of moisture, having a lower resistivity. Consequently the soil relative permittivity can make the behavior of the electrode more reactive or capacitive when it obtains lower(wet soil) or higher(dry soil) values.

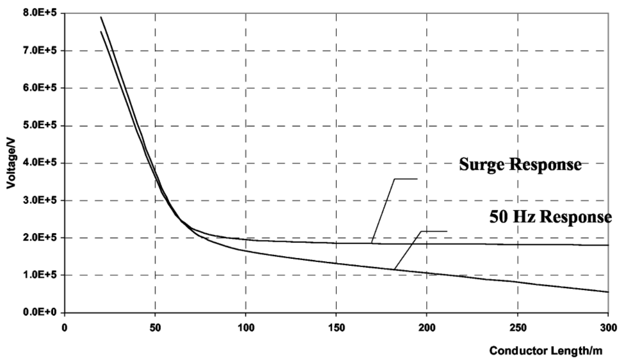


Figure 5 : Maximum Voltage at the injection point, vs. electrode length

The proposed model is constructed by division of the electrode into elementary segments of unequal length and representation of each one by a $\pi$-circuit with lumped parameters. Only a few meters of the electrode close to the source that correspond to the effective length need to be accurately modeled by short segments. Longer segments without seriously affecting accuracy can model the rest of the electrode. The proposed model is shown in figure 6.

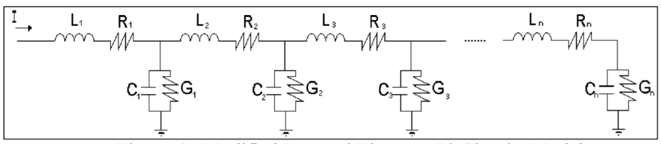


Figure 6 : Modified Lumped Elements Pi-Circuits Model

A proposed relationship for calculation of the length that corresponds to the $\mu$-th $\pi$-circuit versus its distance from the energization point is:

$$length_{\mu} = C_1 e^{d \cdot C_2} + C_3 \tag{2}$$

$C_1$, $C_2$ and $C_3$ are constants which depend on the desired accuracy. Expression (2) is obtained from interpolation of values for various cases of successive construction of the model. A first estimation was that the length of the segments comprising the effective length should not be greater than 1m. An exponential increment of the length of the segments vs. distance from 0, can be considered analogous to the exponential variation of an elementary pulse travelling along the electrode in one direction.

The fact that the effective length does not generally exceed 70m allows an estimation of $C_1$, $C_2$ and $C_3$ :

$C_1$=0.1353, $C_2$=0.1298, $C_3$= 0.4176

Using these values a model is constructed which has not a minimum number of $\pi$-circuits in all cases, but leads to results of acceptable accuracy.

Using the above values, the number of n elementary $\pi$-circuits when modeling conductors of 100m and above is shown in table 1. The total number of elementary circuits compared to the number required when the $\pi$–circuits model of section A.1 is used, has been dramatically reduced.

Table 1 : Reduced Number of Elementary Circuits

| 100m | 500m | 1000m | 5000m |
|---|---|---|---|
| n=34 | n=47 | n=53 | n=65 |

It should be noted that the integration time step used when the lumped parameters modified $\pi$-circuits model is implemented should be less or equal to the travelling time that takes the voltage or current wave to cross the shorter electrode segment.

*D. Time Domain Models – Distributed Parameters Transmission Line Model*

This technique uses the Bergeron's travelling wave technique extended for calculation of the response of grounding electrodes. According to this approach, the inherent frequency dependence of the transmission line characteristic impedance Zc(ω) and the corresponding propagation constant A(ω), due to the existence of resistive elements, is taken into account. The functions Zc(ω) and A(ω), the values of which depend on the line configuration, are calculated using the supporting calculation subroutine LINE CONSTANTS. These are expressed in the frequency domain, by rational functions of the form [1]:

$$Q\frac{\prod_{i=1}^{n}(s+z_i)}{\prod_{j=1}^{m}(s+p_j)} = k_0 + \sum_{j=1}^{m}\frac{k_j}{s+p_j} \tag{3}$$

where the zeros, poles and residues are denoted by $z_i$, $p_j$ and $k_j$ respectively. This approach is known as JMARTI approach [12]. The advantage of this approximation is that the left-hand side of the above equation is transformed in the time domain as quickly damped exponential functions. This facilitates and accelerates the simulation calculations involving convolutions of Zc and A.

The impedance Z' and the susceptance Y' per unit length of a horizontally buried or a vertical bare electrode are obtained by formulae proposed in [17] and [5]. These formulae can however be replaced in case the coating of the conductors is not taken into account in calculations. This is permissible in most practical cases where grounding conductors are usually made of bare copper.

Frequency dependent transmission line model has the advantage of being suitable, for a wide range of frequencies. The error introduced in this case is due to the fact that approximation of Zc(ω) and A(ω), is based on magnitude values while phases are ignored. Phase angle of impedance given by is different than that of grounding electrode's characteristic impedance. An example of this difference can be seen in figure 7 for a 1m long electrode. Line (b) on fig.7 is the Zc phase angle obtained by the approximation of Zc by the rational function (3). Curve (a) on fig.7 indicates the original value.

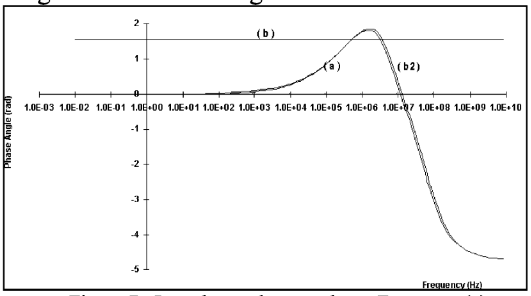


Figure 7 : Impedance phase angle vs. Frequency(a) Impedance approximation phase angle vs. Frequency (b)

A method is discussed in [26], for reducing the angle difference between the two approximations by adding a zero-pole pair at specific frequency. Furthermore, use of the formulae that give the transmission line characteristic impedance and propagation constant, and R, L, C, G values those calculated for the grounding electrode, improves seriously the accuracy of the results (Curve (b2) in figure 7).

### *E. Frequency Domain Models – Electromagnetic Model*

Fundamental principles of electromagnetism can be used to solve the propagation and the equations that govern the dispersion of current from the surface of a grounding electrode into the soil. Calculations follow the stages:

1. The electrode is divided into elementary segments. The length of elementary segments depends on the calculation frequency and their distance from the energization source.
2. Each elementary segment is modeled by an electrical dipole because the resultant magnetic field has negligible value.
3. Given the intensity of the current source, current distributions at the elementary segments are calculated :

- Every elementary segment is substituted by an elementary current source with known current distribution, and described by suitably selected interpolation functions.
- The electromagnetic field produced by a horizontal or vertical electric dipole is calculated.
- Electromagnetic interactions between the dipoles are calculated from the intensity of the current in the electric field surrounding the energized dipole, using the Method of Moments. Mutual interactions between the grounding system elements depend on their geometry, topology of the system, and soil characteristics.

4. Conductance matrix is inverted and multiplied by the vector of the injection sources. Unknown current values in the middles of dipoles are calculated.
5. Surface potentials and touch and step voltages are calculated from current distribution at the system conductors.
6. On the last stage the Inverse Fast Fourier Transformation is applied, translating results into time domain values.

This electrode model is constructed with the least possible neglects and solved with no assumptions. For this reason, it provides results of high accuracy. Results of this model are contrasted to $\pi$ -circuits model results when a 10m (figure 8) and a 500m grounding electrode (figure 9) is considered. Soil resistivity is 1000Ωm and Energization current is 31kA · sin(ωt). Results agree in all cases examined.

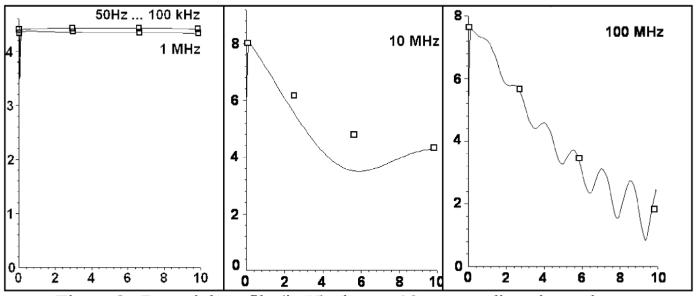


Figure 8 : Potential profile (in V) along a 10m grounding electrode, vs. electrode length(m), EMTP results(□□□), Electromagnetic model results(___)

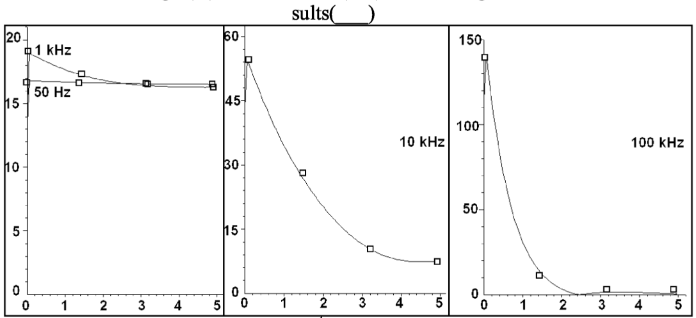


Figure 9: Potential profile (in $10^4$ V) along a 500m grounding electrode, vs. electrode length (x10m), EMTP results (□□□), Electromagnetic model results (___)

### *F. Hybrid Models*

Hybrid models imply the use of electromagnetic theory in time domain calculations. Solution is provided by EMTP. The electromagnetic field equations are used to calculate self and mutual impedances between elementary grounding electrode segments at every single frequency. The J.Marti transmission line model is constructed next to represent each of the impedances using a set of poles zeros and residues. Mutual interactions between the dipoles are considered in calculation using a specialized subroutine written in ATP-MODELS language. Time step must equal to $\tau = 3.33x10^{-9}$ in all cases, because the J.Marti model is used. This fact, and the necessary frequency domain precalculations, increase dramatically the computational time. However hybrid models of grounding electrodes are highly accurate.

## III. Comparative Evaluation of the Models

Regarding their computational characteristics, the following comments can be summarized for the models described in this paper

1. The lumped elements $\pi$ -circuits model is easily constructed and implemented in EMTP. Accuracy of results is comparable to the accuracy achieved when using specialized grounding software. A disadvantage of this model is the limit in lengths that can be handled by EMTP or other

software. More $\pi$ -circuits results in higher accuracy, but increase computational time.

2. The modified $\pi$ -circuits model is easy to construct and implement in EMTP. The accuracy of the model is good. Application range and computational speed are limited due to the division to segments of equal length.

3. The lumped elements modified $\pi$ -circuits model remains simple while its applicability range is extended in calculations of the response of very long electrodes. The time needed for calculations is seriously reduced, since the number of circuits that need to be analyzed is reduced. There is practically no limit in the electrode lengths that can be handled by EMTP, extending its capabilities. This model has been used in the practical case of the calculation of long interconnection electrodes running between windturbines in a wind farm.

4. The distributed parameters transmission line model is more complicated and difficult to implement in EMTP. Time step is also restricted to be less or equal to $\tau = 3.33\mathrm{x}10^{-9}$. However this model can be highly accurate in cases of calculation of the transient response of grounding electrodes.

5. The electromagnetic model of grounding electrode provides highly accurate results because no neglects are made in the construction stage. However it requires high computational cost, which makes it inappropriate for calculations of long grounding electrodes.

6. Hybrid models of grounding electrodes are highly accurate. However, they inherent the disadvantages of the models of categories E and F

The above comments can be summarized in Table 2 where IV characterizes the best model and I the worst:

Table 2 : Comparative Evaluation of Grounding Electrode Models

| | Lumped elements $\pi$ -circuits model | Modified $\pi$ -circuits model | Modified lumped elements $\pi$ -circuits model | Distributed parameters T.L. model | Electromagnetic model | Hybrid model |
|---|---|---|---|---|---|---|
| (1) | IV | II | I | I | I | I |
| (2) | III | II | IV | II | I | I |
| (3) | III | III | IV | III | IV | III |
| (4) | III | II | IV | III | IV | III |

(1) : Easiness of pre-processing data and construction of the model
(2) : Computational speed
(3) : Accuracy
(4) : Application Range

## VI. CONCLUSIONS

An important effort has been made in the last years, to solve the equations of propagation along a grounding electrode. This paper summarises the models that appear in the main works related to this topic, with emphasis on those approaches which have been implemented into an EMTP-like tool. It can be concluded that, although some refinements are still needed, the solutions developed during the last years have provided efficient codes and extended the capabilities of EMTP and similar programs to analyse the transient performance of grounding electrodes.

## VI. REFERENCES

[1] S.Cattaneo, A.Geri, F.Mocci, G.Veca: "Transient Behavior of Grounding Systems Simulation: Remarks on the EMTP's and special code's use" Proceedings of the 21st EMTP Users Group Meeting, 5-7 June 1992 Kolympari Crete,Greece

[2] N.D. Hatziargyriou, M.Lorentzou: 'Grounding Systems Design Using EMTP', 23rdEuropean EMTP Users Group Meeting, Barcelona, 9-11 November 1997

[3] P. Meliopoulos, M.G.Moharam: "Transient Analysis of Grounding Systems" Systems" IEEE Transactions on Power Apparatus and Systems.Vol.PAS-102, no.2, February 1983.

[4] F.Menter,' Accurate Modelling of conductors imbedded in Earth with Frequency Dependent Distributed Parameter Lines' (Proceeding of the 1st EMTP User Group Meeting, 1992)

[5] F.Menter : "EMTP Based Model for Grounding System Analysis" IEEE Trans on PWRD, Vol.9, no.4, October 1994.

[6] L.Grcev F.Dawalibi: "An electromagnetic model for transients in grounding systems" IEEE Trans., on Power Delivery, vol.5, 1990, pp.1773-1781

[7] L.Grcev, V.Arnautovski: "Frequency Dependent and Transient Impedance of Grounding Systems: Comparison between Simulation and Measurement" Lightning & Mountains, Chamonix, France 1997

[8] M.Heimbach, L.Grcev: «Grounding System Analysis in Transients Programs Applying Electromagnetic Fields Approach», IEEE, 96 SM 392-1 PWRD.

[9] L.Grcev, M.Heimbach : «Frequency Dependent and Transient Characteristics of Substation Grounding Systems», IEEE, 96 SM 390-5, PWRD.

[10] H.W. Dommel 'EMTP Theory Book', BPA, Oregon, 1986

[11] R.G.Olsen, M.C.Willis : "A comparison of exact and quasi-static methods for evaluating grounding systems at high frequencies" IEEE Trans., on Power Delivery, vol. 11, 1996, pp.1071-1081.

[12] J.Marti:'Accurate Modelling of frequency Dependent Transmission Lines in Electromagnetic Transient simulations' IEEE Transactions on Power Apparatus and Systems.Vol.PAS-101, no.1, January 1982

[13] M.I. Lorentzou, N.D. Hatziargyriou, B.C.Papadias, S.Sekioka : «Modeling of Soil Ionization for Calculation of the Transient Response of Grounding Electrodes», Proc. of the IPST 2001 Conference, June 24-28, Rio de Janeiro, BRAZIL.

[14] R.Kosztaluk, M.Loboda, D.Mukhedkar : "Experimental Study of Transient Ground Impedances", IEEE Trans. 1981, PAS-100, pp.4653-4660.

[15] A.Geri, G.M.Veca, E.Garbagnati, G.Sartorio : "Non-Linear Behavior of Ground Electrodes under Lightning Surge Currents : Computer Modelling and Comparison with Experimental Results", IEEE transactions on Magnetics, vol.28, no.2, March 1992.

[16] S.Yamaguchi, M.Inoue, S.Sekioka, T.Sonoda, Y.Kato, N.Nagaoka, A.Ametani:"A Frequency Dependent Counterpoise Model for a Transient Analysis" Proceedings of the International Conference on Electrical Engineering (ICEE'98) Vol.1

[17] E.D. Sunde, 'Earth conduction effects in transmission systems', 2nd ed., New York: Dover Publications, 1968

[18] W. Dwight "Calculation of Resistances to Ground" Electrical Engineering, pp.1319-1328, 1936.

[19] G.F.Tagg :"Earth Resistances" Pittman Publishing Corporation, London 1964.

[20] M.Lorentzou, N.Hatziargyriou: " Modeling of Long Grounding Conductors Using EMTP", IPST '99 - International Conference on Power Systems Transients, Budapest, 20-24 June 1999.

[21] M.I. Lorentzou "Contribution to Modeling and Analysis of the Transient Behavior of Grounding Systems", PhD Thesis, November 2001.

[22] A.P.Meliopoulos, M.G. Moharam : «An advanced computer model for grounding system analysis», IEEE Trans. on Power Delivery, vol. 8, No.1, January 1993

[23] M.I. Lorentzou, N.D.Hatziargyriou : "Effective Dimensioning of Extended Grounding Systems for Lightning Protection", Proc. of the 25th ICLP Conference, 18-22 Sept.2000, Rhodes, Greece.

[24] L.Grcev : "Improved Earthing System Design Practices for Reduction of Transient Voltages", CIGRE WorkGroup 1998.

[25] S.Devgan, E.R.Whitehead : «Analytical models for distributed grounding Systems» IEEE Trans. on Power Apparatus and Systems, Vol.PAS-92, Part III, pp.1763-1770, Sep./Oct. 1973.

[26] R.Popa, I.Baran, J.Trecat : «Aspects concerning the analysis of Grounding systems Transient Behavior using EMTP», Proc. of the IPST'95 Conference, Lisbon 3-7 September, 1995.

[27] Yaqing Liu; Zitnik, M.; Thottappillil, R : " An improved transmission-line model of grounding system ", IEEE Trans. on Electromagnetic Compatibility, Volume: 43 Issue: 3 , Aug. 2001

[28] H.W. Dommel, "Digital computer solution of electromagnetic transients in single- and multi-phase networks," IEEE Trans., Power App. and Syst., Vol. PAS-88 (4), pp. 388-398, 1969.