Calculation of Lightning-induced Voltages in MODELS Including Lossy Ground Effects

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***Abstract* – The paper outlines how lightning-induced voltages (LIV) in multi-conductor overhead lines can be calculated in MODELS in the ATP-EMTP. Lossy ground effects on the radial electrical field is taken into account by the Cooray-Rubinstein approximation, but the overhead line itself is assumed to be lossless. This is a reasonable assumption for line lengths up to 1-3 km, depending on the ground resistivity. The simple transmission line model is used for the lightning channel. This is an "engineering model" and a good compromise between accuracy and complexity, based on measurable parameters. The model is valid for the first few microseconds where the peak voltage mostly occurs.**

Keywords – Lightning induced voltages, MODELS, lossy ground

I. INTRODUCTION

Lightning-induced voltage (LIV) in overhead electrical power systems is an important source of failure. The problem increases with reduced voltage level since classically, the LIV is proportional to the overhead line height. Calculation of LIV on overhead lines is addressed by many authors as summarized in [1, 2]. Implementation of LIV in EMTP software is reported in [3-6]. In [3] the source code of M39 is modified, while in [4] an induced voltage term is calculated externally and included as an empirical source [7]. In [5,6] the induced voltage calculation is included in EMTP (and MatLab) using a finite difference approach. The present paper proposes an efficient and analytical method to include LIV calculations via MODELS [8].

Two critical points in the calculation process are the lightning channel model and the handling of lossy ground effects. Among all the lightning channel models the transmission line model (TL) is used in this paper since it is reported to give a reasonable agreement with measurements [9]. It also results in analytical expressions for the electrical fields when integrating the contributions along the lightning channel [10]. The propagation effect on the electromagnetic field is modeled by a simple surface impedance method called the Cooray-Rubinstein approximated which has shown to be a suitable method for LIV calculations over reasonable well conducting ground (σ_g>0.001 S/m) [11, 12]. Combined with a linear lightning current shape at ground this results in a final solution containing only a single convolution integral.

The electromagnetic field from the developing leader is assumed to be semi-static compared to the one from the return stroke. As a consequence only the return stroke field is considered in this paper. Too close to the lightning location this is not a reasonable assumption, however as seen in [13].

II. MODELING

The configuration of the system is shown in Fig. 1. The main assumptions in the model are:

- The lightning channel is straight and vertical.
- The return stroke current propagates unattenuated upward with a constant velocity, v (TL model).
- The shape of the lightning current is linear in time.
- Only the field from the return stroke is considered.
- The earth is homogenous with constant conductivity and permittivity.
- The overhead line is considered to be lossless. The lossy ground is assumed to affect the horizontal electrical field only.

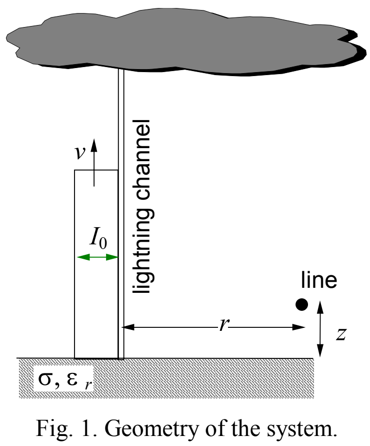


Fig. 1. Geometry of the system.

A. Electrical fields

If a constant current I_0 is assumed the electromagnetic fields over a lossless ground can be calculated analytically, following the approach from Rusck [10].

$$E_{x0}(x,y,z,t) \approx \frac{\mu_0 \cdot c \cdot I_0}{2\pi \cdot \beta} \cdot x \cdot z \cdot \left[-\frac{1-\beta^2}{\xi^3} + \frac{1}{r^3} \right] \quad (1)$$

$$E_{z0}(x,y,z=0,t) = \frac{\mu_0 \cdot c \cdot I_0}{2\pi \cdot \beta} \cdot \left[\frac{1-\beta^2}{\xi} - \frac{1}{r} \right] \quad (2)$$

$$B_{y0}(x,y,z=0,t) = \frac{\mu_0 \cdot I_0}{2\pi \cdot r^2} \cdot \frac{v \cdot t \cdot x}{\xi} \quad (3)$$

with $\xi = \sqrt{(v \cdot t)^2 + (1-\beta^2) \cdot r^2}$ and $r = \sqrt{x^2 + y^2}$ (4)

E_{x0} is directed along the overhead line from terminal B to A. E_{z0} is directed upward from ground and B_{y0} is directed perpendicular to the overhead line. A positive lightning current propagates upward.

According to the Cooray-Rubinstein approximation the horizontal electric field over a lossy ground is the sum of the lossless field and a surface impedance contribution depending on the horizontal magnetic field:

$$E_x^\sigma(x, y, z, t) = E_{x0}(x, y, z, t) - g_0(t) * B_{y0}(x, y, 0, t) \tag{5}$$

where the sign '*' denotes a convolution and $g_0(t)$ is the surface function in the time domain. The surface function is in the frequency domain [11]

$$g_0(\omega) = c \cdot \sqrt{\frac{\varepsilon_0 \cdot j\omega}{\sigma + \varepsilon_r \cdot \varepsilon_0 \cdot j\omega}} \tag{6}$$

where σ is the conductivity and ε_r is the relative permittivity of the ground. The lossy ground effect on the vertical electrical field and the magnetic field is further ignored. This is a good approximation within 10 km distance.

B. Coupling to overhead line

Fig. 2 shows the orientation of the overhead line. The end A of the overhead line is oriented so that $x_A > x_B$. The overhead line has length L and height $z = h$ above ground.

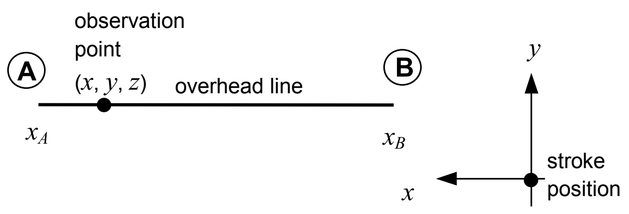


Fig. 2. The co-ordinate system and the line orientation.

The Agrawal coupling model [14] is further used to calculate the LIV in an overhead line. The total line voltage is a sum of the scattered voltage U^S and the incident voltage U^i

$$U(x,t) = U^S(x,t) + U^i(x,t) \tag{7}$$

where

$$U^i(x,t) = -\int_0^h E_z(x, y, z, t)\, dz \approx -h \cdot E_z(x, y, 0, t) \tag{8}$$

and

$$\begin{aligned} &\frac{\partial U^S(x,t)}{\partial x} + \left(R' + L' \cdot \frac{\partial}{\partial t} \right) \cdot i(x,t) = E_x^\sigma(x, y, h, t) \\ &\frac{\partial i(x,t)}{\partial x} + C' \cdot \frac{\partial}{\partial t} U^S(x,t) = 0 \end{aligned} \tag{9}$$

The general solution in the frequency domain to these equations is

$$\begin{aligned} U(x,\omega) = &-h \cdot E_{z0}(x, y, 0, \omega) + K_1 \cdot e^{\gamma \cdot x} + K_2 \cdot e^{-\gamma \cdot x} \\ &+ \frac{1}{2} \int_{-\infty}^{x} E_x^\sigma(\lambda, y, h, \omega) \cdot e^{-(x-\lambda)\cdot\gamma} d\lambda \\ &- \frac{1}{2} \int_{x}^{\infty} E_x^\sigma(\lambda, y, h, \omega) \cdot e^{(x-\lambda)\cdot\gamma} d\lambda \end{aligned} \tag{10}$$

where $\gamma = \sqrt{j\omega C' \cdot (R' + j\omega L')}$. K_1 and K_2 are constants determined by the boundary conditions.

At the terminal A, the last integral in (10) vanishes. A term called the inducing voltage $U_{ind0}^\sigma(x_A, \omega)$ is now defined which is composed of both the integrals of the horizontal field and the vertical field at both terminals. The remaining integral is split in two terms based on (5).

$$\begin{aligned} U_{ind0}^\sigma(x_A, \omega) = &\int_{x_B}^{x_A} E_x^\sigma(\lambda, y, h, \omega) \cdot e^{-(x-\lambda)\cdot\gamma} d\lambda \\ &- h \cdot E_{z0}(x_A, y, 0, \omega) + h \cdot E_{z0}(x_B, y, 0, \omega) \cdot e^{-\gamma \cdot L} \\ = &\, U_0(x_A, \omega) - g_0(j\omega) \cdot U_\Delta(x_A, \omega) \end{aligned} \tag{11}$$

where

$$\begin{aligned} U_0(x_A, \omega) = &\int_{x_B}^{x_A} E_{x0}(\lambda, y, h, \omega) \cdot e^{-(x-\lambda)\cdot\gamma} d\lambda \\ &- h \cdot E_{z0}(x_A, y, 0, \omega) + h \cdot E_{z0}(x_B, y, 0, \omega) \cdot e^{-\gamma \cdot L} \end{aligned} \tag{12a}$$

is the lossless part and

$$U_\Delta(x_A, \omega) = \int_{x_B}^{x_A} B_{y0}(\lambda, y, 0, \omega) \cdot e^{-(x-\lambda)\cdot\gamma} d\lambda \tag{12b}$$

is the lossy ground contribution.

C. Implementation

To be able to implement the calculation of induced voltages in MODELS a fundamental assumption is that the overhead line itself is lossless which is reasonable for short lines (<2 km) [15]. With this lossless line assumption the integrals in (12) can actually be solved analytically in the time domain as shown in [16], since $\gamma = j\omega/c$.

The overhead line can then further be modeled with the classical Bergeron model as shown in fig. 3.

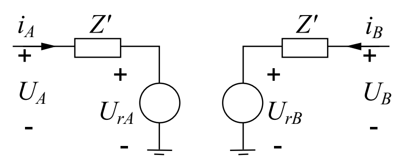


Fig. 3. Electric model of overhead line.

Where the voltage sources are formulated [17]:

$$\begin{aligned} U_{rA}(t) &= U_{ind0}^\sigma(x_A, t) + U_B(t - \tau) + Z' \cdot i_B(t - \tau) \\ U_{rB}(t) &= U_{ind0}^\sigma(-x_B, t) + U_A(t - \tau) + Z' \cdot i_A(t - \tau) \end{aligned} \tag{13}$$

The source U_{rA} at terminal A consists of a resultant inducing voltage term called $U^{\sigma}_{ind0}(x_A,t)$ and the reflection from terminal B delayed the traveling time τ.

So far a step current is assumed. An arbitrary current shape can be taken into account by a final convolution integral

$$U^{\sigma}_{ind}(x,t) = g_1(t) * U^{\sigma}_{ind0}(x,t) \qquad (14)$$

where

$$g_1(t) = \frac{1}{I_0}\frac{\partial i(t)}{\partial t} = \frac{I_m}{I_0 \cdot t_c} \cdot \left(1 - H(t-t_c)\cdot b\right) \qquad (15)$$

The last part of (15) applies to a linear rising lightning current. t_c is the time to crest while b is the current tail factor, $b = (t_h - t_c/2)/(t_h - t_c)$. H is the unit step function.

Based on (11) the total inducing voltages becomes

$$U^{\sigma}_{ind}(x,t) = U^{0}_{ind}(x,t) + U^{\Delta}_{ind}(x,t) \qquad (16a)$$

where

$$U^{0}_{ind}(x,t) = g_1(t) * U_0(x,t) \qquad (16b)$$

$$U^{\Delta}_{ind}(x,t) = -g_1(t) * g_0(t) * U_{\Delta}(x,t) \qquad (16c)$$

Assuming a linear lightning current the convolution $g_1(t)*g_0(t)$ can be solved analytically. This results in a Bessel function which is further simplified with its large argument approximation as shown in [16].

Equation (13) is now efficiently reformulated into a recursive expression without the overhead line current and the characteristic impedance matrix [16]. The lossless contribution $U^{0}_{ind}(x_A,t)$ is scaled with the height of the conductor j, h_j (the lossy ground contribution $U^{\Delta}_{ind}(x_A,t)$ is height independent). It is assumed that all conductors have the same distance y to the lightning stroke.

$$U^{j}_{rA}(t) = \left(\frac{h_j}{h}\cdot U^{0}_{ind}(x_A,t) + U^{\Delta}_{ind}(x_A,t)\right) + 2\cdot U^{j}_{B}(t-\tau) - U^{j}_{rB}(t-\tau) \qquad (17a)$$

$$U^{j}_{rB}(t) = \left(\frac{h_j}{h}\cdot U^{0}_{ind}(-x_B,t) + U^{\Delta}_{ind}(-x_B,t)\right) + 2\cdot U^{j}_{A}(t-\tau) - U^{j}_{rA}(t-\tau) \qquad (17b)$$

where j is the phase number (1..n).

When the overhead line is matched with its characteristic impedance the total induced line voltage will be half of the inducing voltage formulated in (16).

D. Practical considerations

The calculation of the voltages sources U_{rA} and U_{rB} is implemented in the MODELS language [8] as shown in appendix A. This results in an ATPDraw circuit as shown in fig. 4, where two phases are supported.

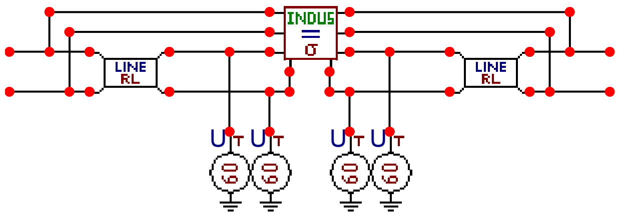


Fig. 4 Implementation in ATPDraw.

The overhead line is further modeled with type 51..52 branches with the resistances equal to the characteristic impedances. Type 60 sources are used to represent the U_{rA} and U_{rB} voltages [7]. Only the terminal voltages can be calculated and to obtain voltages along the line it must be sectioned.

The model also handles the orientation of the overhead line. If the positions of the overhead line terminals A, B is given by (x_A', y_A') and (x_B', y_B') and the lightning position by (x_0', y_0') in a global system, the co-ordinates in the specific induced voltage co-ordinate system in fig. 2 will be:

$$\begin{aligned} x_A &= (x_A' - x_0')\cdot\cos(\alpha) + (y_A' - y_0')\cdot\sin(\alpha) \\ x_B &= (x_B' - x_0')\cdot\cos(\alpha) + (y_B' - y_0')\cdot\sin(\alpha) \\ y &= \left|(y_A' - y_0')\cdot\cos(\alpha) - (x_A' - x_0')\cdot\sin(\alpha)\right| \end{aligned} \qquad (18)$$

with

$$\tan(\alpha) = \frac{y_A' - y_B'}{x_A' - x_B'} \quad \text{(full 4-quadrant solutions)}$$

The x co-ordinates x_A and x_B now satisfy the requirement $x_A > x_B$ and the line length is $L = x_A - x_B$.

E. Verification

Fig. 5 shows calculations of LIV in a matched overhead line similar to fig. 6b compared with a calculation by Rachidie et.al. [15] where the modified transmission line model is used and the calculations are performed numerically. The agreement is very good for the first few microseconds.

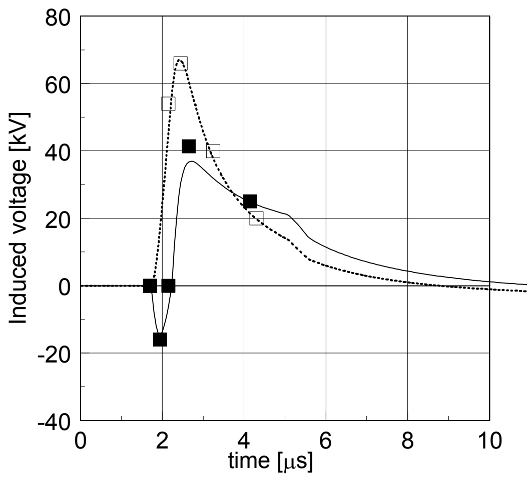


Fig. 5. Induced voltage at the terminals A and B in overhead line terminated with its characteristic impedance. Dotted line: lossless. Solid line: lossy ground (σ=0.001 S/m, ε_r=10). □, ■: values from [Rachidie et. al., 1996 [15]].

III. CALCULATIONS

The induced voltage from a return stroke is very dependent on the orientation and terminations of the overhead line. If the lightning strike is located near one of the terminal the inducing voltage at this terminal is a sum of a large positive vertical field contribution and a negative horizontal field contribution, while the situation at the other terminal is the opposite. When the ground becomes lossy the radial electrical field is reduced. This will result in a higher voltage at the near end and a lower voltage at the far end, that in fact can change sign. When the overhead line is matched with it's characteristic impedance the resulting induced voltage will be half the value of the inducing voltage, and when the line terminal is open the induced voltage will be initially doubled.

The lightning current has in all cases amplitude 30 kA, a front time of 2 µs, a time to half value 50 µs and a velocity $v=c/2=1.5\cdot10^8$ m/s. The height of the overhead line is h=10 m and it's characteristic impedance is 400 Ω.

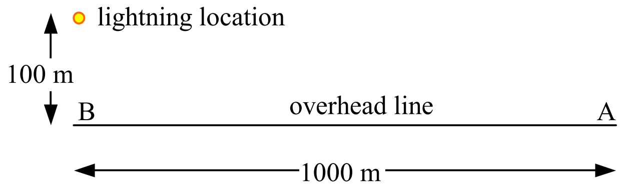


Fig. 6a) Configuration 1.

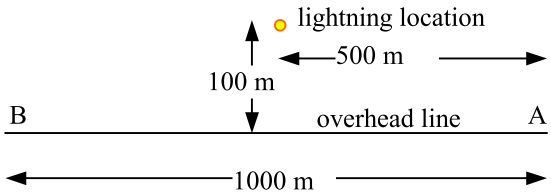


Fig. 6b) Configuration 2.

Fig. 7 shows the inducing voltage from (16) at the near and far end of configuration 1 in fig. 6a.

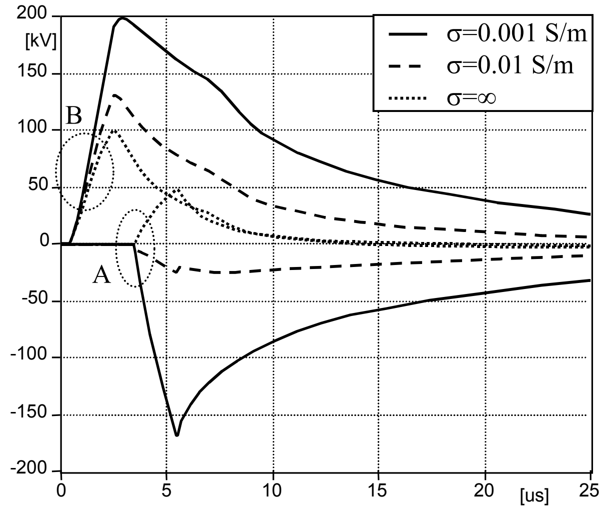


Fig. 7. Inducing voltage at the near and far end according to configuration 1.

Fig. 8 shows the inducing voltage at the end- and mid-point of configuration 2. The inducing voltage at the mid-point in the lossless situation will be almost the voltage calculated by Rusck [10]. The reduction is caused by the 2 µs front time. The amplitude according to Rusck would result in $U_{max} \approx 60\cdot I_m\cdot\frac{h}{y}\cdot\left(1+\frac{\beta}{\sqrt{2-\beta^2}}\right)=248\,\mathrm{kV}$.

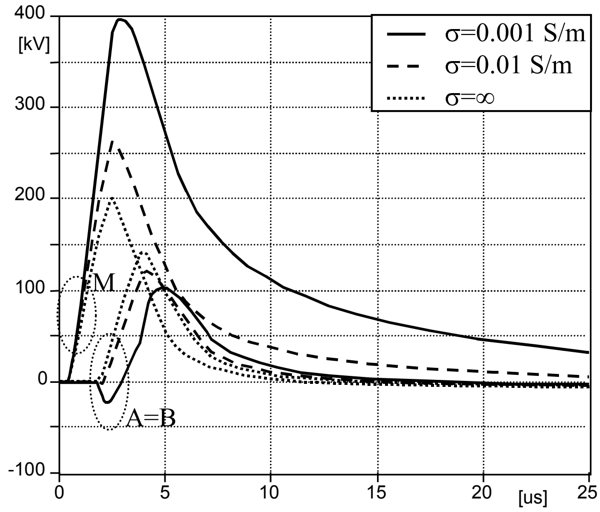


Fig. 8 Inducing voltage at the end- (A=B) and mid-point (M) of configuration 2.

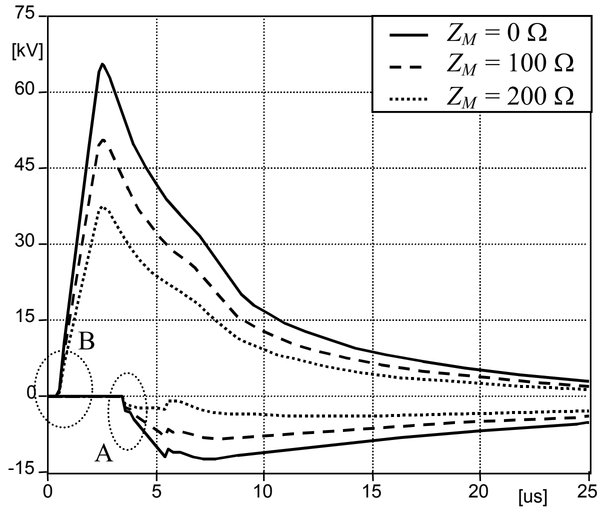


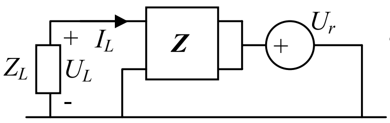
Fig. 9 Induced voltage at the phase wire, config. 1 including a grounded conductor. Ground conductivity is 0.01 S/m.

The effect of the ground conductivity is clearly seen in fig. 7 and 8. For a lightning stroke near the line's terminal the effect is large; the voltage at the near end is increased and at the far end reduced to actually change sign. For a lightning stroke near the mid-point of the line the lossy ground effect is less pronounced, resulting in a minor reduction of the induced voltage.

The developed model can handle multiple phases. Usually two phases is sufficient since the zero sequence system is of greatest importance. The phase conductors can thus be merged into one and the ground or neutral conductor taken as the other. The effect of a grounded second wire is demonstrated in fig. 9, using configuration 1. The phase wire is matched at both ends with it's characteristic impedance Z_L of 400 Ω. The neutral wire also has impedance Z_N = 400 Ω and it is ideally grounded at both ends. The mutual impedance Z_M varies between 0 and 200 Ω. Without the neutral wire (Z_M = 0) the induced voltage is half the values in fig. 7. A ground conductivity of σ=0.01 S/m is assumed. At the near end the maximum induced voltage is reached before any reflections from the far side. The maximum voltage is therefor reduced by a factor χ due to the grounded conductor. At the far end the reduction is larger and more complicated since the voltage reaches it's maximum after reflections from the near end.

$$\chi = \frac{U_L}{U_r / 2} = \frac{2 \cdot Z_L \cdot (1 - Z_M / Z_N)}{2 \cdot Z_L - Z_M^2 / Z_N} \qquad (19)$$

is based on the terminal model



$$\boldsymbol{U} - \boldsymbol{Z} \cdot \boldsymbol{I} = U_r \quad \wedge \quad \boldsymbol{U} = \begin{bmatrix} -Z_L \cdot I_L \\ 0 \end{bmatrix}$$

and the assumption that incoming voltage waves to the line and neutral terminals are initially equal (same line height and no reflections).

IV. Discussion

The developed model is based on some fundamental assumptions.

- The lightning channel is modelled as a transmission line. This is an engineering approach that gives reasonable agreement with measurements [9]. An extension called the modified transmission line model is used in for example [15]. This model seems to give better results for the fields, but the effect on the induced voltage is limited and the peak value is almost unaffected. It is important to realise that the lightning channel in reality is very complicated with a stochastic nature, so the level of accuracy for these calculations is anyhow limited.
- The overhead line is lossless. This is a fundamental assumption which is reasonable for shorter lines (1-3 km) [15]. It seems very complicated at this moment to solve the full integral in (12) in MODELS. Longer lines can as a first approximation be modelled with line sections interconnected by concentrated and constant resistances. A modification of the *Vance approximation* (large frequency approximation) [15] is suggested in [16] with the frequency replaced by $1/4t_c$):

$$R' \approx \frac{1}{2\pi \cdot h} \cdot \sqrt{\frac{\mu_0}{\pi \cdot \sigma \cdot 4 \cdot t_c}} \quad [\Omega/\mathrm{m}].$$

- Only the induced voltage from the return stroke is considered. This is doubtful for nearby lightning since the field from the leader becomes important as seen in [13].

V. Conclusion

The presented model is a reasonable engineering model that enables calculation of lightning induced voltages in overhead line systems in the ATP-EMTP. The analytical formulation results in a fast and stable calculation. The model is reasonable for distances of 100 m -10 km and line lengths up to 2 km.

Appendix A. MODELS code

In the model `Indloss` the inducing voltages are calculated in the `INIT` section. The variables `UindA0[i]` and `UindAD[i]` represent the lossless and lossy ground contributions respectively, as a function of time. In order to allocate sufficient memory the number of time steps in the simulation is here declared as a constant `Tmax`. Due to the special handling of the $g_1(t)*g_0(t)$ convolution function at small times [16] the following restriction applies to the time step: $\Delta t > \varepsilon_r \cdot \varepsilon_0 / (\pi \cdot \sigma)$, which is reasonable in most cases.

```
MODEL indloss
CONST Tmax   {VAL:500}      --num. time steps
      n      {VAL:2}        --num. phases
      c      {VAL:3.e8}     --speed of light
      I0     {VAL:1}        --step current ampl.
      eps0   {val:8.85e-12}--permittivity
INPUT UA[1..n],UB[1..n]     --terminal voltages
DATA XA0,YA0,XB0,YB0        --line positions
     z[1..n] {DFLT:10}      --phase heights
     Im      {DFLT:3e4}     --current amplitude
     tc      {DFLT:5e-5}    --time to half value
     th      {DFLT:2e-6}    --time to crest
     v       {DFLT:1.5e8}   --current velocity
     X0      {DFLT:0.0}     --lightning x-pos
     Y0      {DFLT:0.0}     --lightning y-pos
     sigma   {DFLT:0.01}    --ground conductivity
     epsr    {DFLT:10}      --rel. permittivity
OUTPUT UrA[1..n],UrB[1..n] --type 60 sources
VAR UindA0[0..Tmax],UindB0[0..Tmax],g1,g2,g2s,
    UindAD[0..Tmax],UindBD[0..Tmax],Ui0,UiD,
    UrA[1..n],UrB[1..n],Tr,Te,i,AB,dt,uk[1..4],
    b,L,x,tau,a1,b1,c0,c1,c2,c3,Y,XA,XB,a
FUNCTION F0(x,tr):= (c*tr-x)/
         (y*y+(b*(c*tr-x))**2)
FUNCTION F1(x,tr):= (x+b*b*(c*tr-x))/
         sqrt((v*tr)**2+(1-b*b)*(x*x+y*y))
FUNCTION F2(x,tr):= x+b*b*(c*tr-x)+
         sqrt((v*tr)**2+(1-b*b)*(x*x+y*y))
FUNCTION F3(x,tr):= (v*tr+sqrt((v*tr)**2+(1-
         b*b)*(x*x+y*y)))/sqrt(x*x+y*y)
FUNCTION t0(x):= sqrt(x*x+y*y)/c
HISTORY
  UrA[1..n] {dflt:0}, UrB[1..n] {dflt:0}
  UA[1..n]  {dflt:0}, UB[1..n]  {dflt:0}
DELAY   CELLS   DFLT:   (sqrt((XA0-XB0)**2+(YA0-
YB0)**2))/(c*timestep)+1
INIT
  a:=atan2(YA0-YB0,XA0-XB0)
  XA:=(XA0-X0)*cos(a)+(YA0-Y0)*sin(a)
  XB:=(XB0-X0)*cos(a)+(YB0-Y0)*sin(a)
  Y :=(YA0-Y0)*cos(a)-(XA0-X0)*sin(a)
  dt:= timestep b:=v/c L:=XA-XB tau:=L/c
  a1:=Im/(I0*tc)
  b1:=0.5*tc/(th-tc)+1
  c0:=PI*sigma/(eps0*epsr)
  c1:=a1/sqrt(epsr)
  c2:=-1.0736/c0+0.2153/(dt*c0**2)+
      4/3*sqrt(dt/c0)
  c3:=-0.2153/(dt*c0**2)+1/6*sqrt(dt/c0)
```

```
  FOR AB:=1 TO 2 DO
    if AB=1 then
      x:=XA else x:=-XB
    endif
    FOR i:=0 TO Tmax DO
      Tr:=i*dt
      if Tr>t0(x)
      then
        if Tr>t0(x-L)+tau
        then
          Ui0:=f0(x,Tr)*
               (f1(x,Tr)-f1(x-L,Tr-tau))
          UiD:=ln( f2(x,Tr)/f2(x-L,Tr-tau) )
              -1/b*ln( f3(x,Tr)/f3(x-L,Tr-tau) )
        else
          Ui0:=f0(x,Tr)*(f1(x,Tr)+1)
          UiD:=ln( f2(x,Tr)*f0(x,Tr) )
              -1/b*ln( f3(x,Tr)/(1+b) )
        endif
      else
        Ui0:=0, UiD:=0
      endif
      if AB=1 then
        UindA0[i]:=Ui0
        UindAD[i]:=UiD
      else
        UindB0[i]:=Ui0
        UindBD[i]:=UiD
      endif
    ENDFOR
  ENDFOR
  g1:=a1*dt
  g2:=g1*sqrt(eps0/(PI*sigma*dt))
  i:=Tmax
  WHILE i>1 DO
    uk[1..4]:=0
    FOR Tr:=1 TO i-1 DO
      g2s:=sqrt(i-Tr)
      uk[1]:=uk[1]+UindA0[Tr]
      uk[2]:=uk[2]+UindB0[Tr]
      uk[3]:=uk[3]+UindAD[Tr]/g2s
      uk[4]:=uk[4]+UindBD[Tr]/g2s
    ENDFOR
    UindA0[i]:=(uk[1]+0.5*UindA0[i])*g1
    UindB0[i]:=(uk[2]+0.5*UindB0[i])*g1
    UindAD[i]:=uk[3]*g2+
      (UindAD[i]*c2+UindAD[i-1]*c3)*c1
    UindBD[i]:=uk[4]*g2+
      (UindBD[i]*c2+UindBD[i-1]*c3)*c1
    i:=i-1
  ENDWHILE
  Tr:=trunc(tc/dt)
  Te:=trunc(2*th/dt)
  FOR i:=Tmax TO Tr BY -1 DO
    if i>Te
    then
      UindA0[i]:=0, UindB0[i]:=0
      UindAD[i]:=0, UindBD[i]:=0
    else
      UindA0[i]:=UindA0[i]-b1*UindA0[i-Tr]
      UindB0[i]:=UindB0[i]-b1*UindB0[i-Tr]
      UindAD[i]:=UindAD[i]-b1*UindAD[i-Tr]
      UindBD[i]:=UindBD[i]-b1*UindBD[i-Tr]
    endif
  ENDFOR
ENDINIT
EXEC
  FOR i:=1 to n DO
    UrA[i]:=60*I0*b*
     (z[i]*UindA0[t/dt] - UindAD[t/dt]) +
     2*delay(UB[i],tau-dt,1)-delay(UrB[i],tau,1)
    UrB[i]:=60*I0*b*
    (z[i]*UindB0[t/dt] - UindBD[t/dt]) +
     2*delay(UA[i],tau-dt,1)-delay(UrA[i],tau,1)
  ENDFOR
ENDEXEC
ENDMODEL
```

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