s-Domain Analysis of Lightning Surge Response of a Transmission Tower with Phase Conductors

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***Abstract* – Lightning surges on a transmission tower with a shield wire and phase conductors are analyzed using an *s*-domain method. The voltages induced on the phase conductors and surges at the position of the tower crossarms are computed to obtain the insulator voltages. Response of the system determined in the *s*-domain is transformed into the time domain using Fast Inverse Laplace Transform (FILT). It has been shown that lightning surge response of a transmission tower can be determined easily using Coupling Coefficient Matrix (CCM) defined in this paper. The solution procedure is programmed with MATLAB. The results obtained using the proposed method are compared with those obtained using Electromagnetic Transients Program (EMTP).**

Keywords – Transmission Lines, Lightning Surge, *s*-Domain Analysis, Fast Inverse Laplace Transform, EMTP.

I. INTRODUCTION

Analysis of lightning surges is important for power systems, since lightning strokes on transmission towers, shield wires and phase conductors may cause insulation flashover which may lead interruption of continuous power transmission. When a lightning stroke occurs on a tower or on a shield wire near the tower, potential across the insulation strings becomes excessive and a fault between the phase conductor and a tower crossarm may occur. One end of the insulation chain follow the potential at tower crossarm and other follow the potential on the phase conductor, which is the sum of nominal system operating voltage and induced potential due to lightning. For calculation of the potential across the insulator strings, the surges both on the tower and induced voltages on the phase conductors need to be determined.

There are many works that have been devoted to the analysis of lightning surges on transmission towers. Experimental studies on this subject have usually been carried out in Japan [1-4] and measured results have become an important base for computer simulations. In the experimental work done by Kawai surge response of transmission tower has been measured [1]. Later Ishii performed experiments on a full scale tower with phase conductors and ground wires [2, 3] and as a result of these experiments a multistory tower model has been proposed for multiconductor analysis in EMTP. Series of experiments later have been continued on a different tower configuration [4] to generalize the results obtained in previous works. On the other hand, in the area of numerical developments, Almeida and Correia De Barros [5] used finite element method for the EMTP simulation. Later Ishii and Baba [6] analyzed a large scale UHV transmission tower using moment method by solving electric field equations directly, and in their study the effect of slant elements, horizontal elements and crossarm has been investigated. Using the same method, they also investigated lightning surge characteristics of a transmission line comprising a tower, a shield wire and phase conductors with the help of computer program NEC-2 [7]. Next, in one of the works tower body and crossarms are modeled in details using short lines sections [8]. Recently, Mozumi et al. analyzed archorn voltages of a simulated 500 kV twin-circuit line [9].

In this paper, lightning surges on a transmission tower are analyzed using *s*-domain formulation by taking the effect of the shield wire and phase conductors into account. Potential on the insulator strings is determined from the difference of the voltages at position of tower crossarms and induced voltages on the phase conductors. The study in this paper proceeds the work done to obtain the tower surge response using nonuniform single phase line modeling [10-12]. In [13] the lightning surges on the tower with a ground wire has been computed but the effect of phase conductors has not been considered. This technique is extended here to involve the effect of the phase conductors. The lightning surges are computed by means of CCM defined in the next section. For the frequency to time domain conversion Fast Inverse Laplace Transform (FILT) is used. Computed results show good agreement with the results obtained using EMTP. Main advantage of the *s*-domain analysis is that frequency dependent effects can be included directly into the analysis. Although variation in system topology as well as the inclusion of nonlinear effects such as the back-flashover events require major numerical/theoretical efforts, for specific applications, the frequency domain approach can be used as a standard against which to compare time domain solutions.

II. *S*-DOMAIN ANALYSIS OF THREE-PHASE LINES

In this study, a three phase transmission system with a shield wire shown in Fig. 1 is considered. Current and voltage relations on a multiphase transmission line in the phase domain can be described by the following differential equations:

$$-\frac{dV}{dx} = ZI \tag{1a}$$

$$-\frac{dI}{dx}=YV \qquad (1b)$$

where ***V*** and ***I*** are the vector of voltage and current phasors, ***Z*** and ***Y*** are series impedance and shunt admittance matrices of the transmission line, respectively. Using the theory given in the Appendix, matrix ***Z*** and ***Y*** for a four conductor system can be constructed as

$$Z=\begin{bmatrix} z_{11} & z_{12} & z_{13} & z_{14} \\ z_{21} & z_{22} & z_{23} & z_{24} \\ z_{31} & z_{32} & z_{33} & z_{34} \\ z_{41} & z_{42} & z_{43} & z_{44} \end{bmatrix} \qquad (2)$$

$$Y=\begin{bmatrix} y_{11} & y_{12} & y_{13} & y_{14} \\ y_{21} & y_{22} & y_{23} & y_{24} \\ y_{31} & y_{32} & y_{33} & y_{34} \\ y_{41} & y_{42} & y_{43} & y_{44} \end{bmatrix} \qquad (3)$$

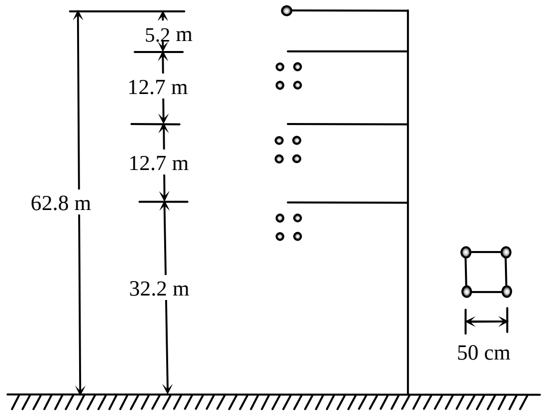


Fig. 1 Conductor positions of the vertical system.

Frequency domain solution of Eq. (1) gives the terminal equations of a multiphase transmission line as [11]

$$\begin{bmatrix}[V_S]\\ [I_S]\end{bmatrix}=\begin{bmatrix}[\cosh(\gamma l)] & [Z_0][\sinh(\gamma l)]\\ [Z_0^{-1}][\sinh(\gamma l)] & [\cosh(\gamma l)]\end{bmatrix}\begin{bmatrix}[V_R]\\ [I_R]\end{bmatrix} \qquad (4)$$

Since Z_0 and γ in the above equation are 4x4 matrices, determination of matrix functions are required for calculation of $Z_0=\sqrt{Z/Y}$, $\gamma=\sqrt{ZY}$, $\cosh(\gamma l)$ and $\sinh(\gamma l)$. One way for determination of matrix functions is to use the modal decomposition techniques [17, 18]. Defining modal variables $V_m=TV$ and $I_m=TI$, Eq. (1) is rewritten as

$$\frac{dV_m}{dx}=T^{-1}ZTI_m \qquad (5a)$$

$$\frac{dI_m}{dx}=T^{-1}YTV_m \qquad (5b)$$

In the above equations $Z_m=T^{-1}ZT$ and $Y_m=T^{-1}YT$ are diagonal matrices. Since function of a diagonal matrix can be evaluated by calculating function of its diagonal elements algebraically, the matrix of propagation constants for the independent modes (mode 1,2,3,4) can be obtained as

$$\gamma_m=\begin{bmatrix}\gamma_1 & 0 & 0 & 0\\ 0 & \gamma_2 & 0 & 0\\ 0 & 0 & \gamma_3 & 0\\ 0 & 0 & 0 & \gamma_4\end{bmatrix}=\begin{bmatrix}\sqrt{Z_1Y_1} & 0 & 0 & 0\\ 0 & \sqrt{Z_2Y_2} & 0 & 0\\ 0 & 0 & \sqrt{Z_3Y_3} & 0\\ 0 & 0 & 0 & \sqrt{Z_4Y_4}\end{bmatrix} \qquad (6)$$

Similarly, modal characteristic impedance matrix can be written as

$$Z_{0m}=\begin{bmatrix}Z_{01} & 0 & 0 & 0\\ 0 & Z_{02} & 0 & 0\\ 0 & 0 & Z_{03} & 0\\ 0 & 0 & 0 & Z_{04}\end{bmatrix}=\begin{bmatrix}\sqrt{Z_1/Y_1} & 0 & 0 & 0\\ 0 & \sqrt{Z_2/Y_2} & 0 & 0\\ 0 & 0 & \sqrt{Z_3/Y_3} & 0\\ 0 & 0 & 0 & \sqrt{Z_4/Y_4}\end{bmatrix} \qquad (7)$$

where Z_i and Y_i *(i=1,2,3,4)* are the diagonal element of matrix Z_m and Y_m respectively. Matrix $\cosh(\gamma_m)$ is then calculated as

$$\cosh(\gamma_m)=\begin{bmatrix}\cosh(\gamma_1) & 0 & 0 & 0\\ 0 & \cosh(\gamma_2) & 0 & 0\\ 0 & 0 & \cosh(\gamma_3) & 0\\ 0 & 0 & 0 & \cosh(\gamma_4)\end{bmatrix} \qquad (8)$$

and $\sinh(\gamma_m)$ can be found in the same manner. The matrices Z_{0m} , γ_m , $\cosh(\gamma_m l)$ and $\sinh(\gamma_m l)$ are transformed into the phase domain as

$$Z_0=TZ_{0m}T^{-1} \qquad (9)$$

$$\cosh(\gamma l)=T\cosh(\gamma_m l)T^{-1} \qquad (10)$$

$$\sinh(\gamma l)=T\sinh(\gamma_m l)T^{-1} \qquad (11)$$

Using the symmetry of the lines at the tower position, terminal equations of transmission lines (phase conductors and ground wire) can be obtained as [8],

$$\begin{bmatrix}[V_S]\\ [I_S]\end{bmatrix}=\begin{bmatrix}[\cosh(\gamma l)] & [Z_0/2][\sinh(\gamma l)]\\ [2/Z_0^{-1}][\sinh(\gamma l)] & [\cosh(\gamma l)]\end{bmatrix}\begin{bmatrix}[V_R]\\ [I_R]\end{bmatrix} \qquad (12)$$

The above equation is in the form

$$\begin{bmatrix}V_{S1}\\ V_{S2}\\ V_{S3}\\ V_{S4}\\ \hline I_{S1}\\ I_{S2}\\ I_{S3}\\ I_{S4}\end{bmatrix}=\left[\begin{array}{cccc:cccc} a_{11} & a_{12} & a_{13} & a_{14} & b_{11} & b_{12} & b_{13} & b_{14}\\ a_{21} & a_{22} & a_{23} & a_{24} & b_{21} & b_{22} & b_{23} & b_{24}\\ a_{31} & a_{32} & a_{33} & a_{34} & b_{31} & b_{32} & b_{33} & b_{34}\\ a_{41} & a_{42} & a_{43} & a_{44} & b_{41} & b_{42} & b_{43} & b_{44}\\ \hdashline c_{11} & c_{12} & c_{13} & c_{14} & d_{11} & d_{12} & d_{13} & d_{14}\\ c_{21} & c_{22} & c_{23} & c_{24} & d_{21} & d_{22} & d_{23} & d_{24}\\ c_{31} & c_{32} & c_{33} & c_{34} & d_{31} & d_{32} & d_{33} & d_{34}\\ c_{41} & c_{42} & c_{43} & c_{44} & d_{41} & d_{42} & d_{43} & d_{44}\end{array}\right]\begin{bmatrix}V_{R1}\\ V_{R2}\\ V_{R3}\\ V_{R4}\\ \hline I_{R1}\\ I_{R2}\\ I_{R3}\\ I_{R4}\end{bmatrix} \qquad (13)$$

This equation can also be written in compact form as

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \tag{14}$$

In the obvious that the type of termination at line ends will not effect the results if the maximum observation time of the analysis is selected to be less than twice the travel time of the lines. Therefore, the receiving-end of the transmission line can be assumed to be open circuit and $(I_R = 0)$; hence V_S is calculated as

$$[V_S] = [A][C]^{-1}[I_S] \tag{15a}$$

or

$$[V_S] = [E][I_S] \tag{15b}$$

where $[E] = [A][C]^{-1}$ is defined as the Coupling Coefficient Matrix (CCM). As the sending-end currents are zero for the phase conductors ($I_{S1} = I_{S2} = I_{S3} = 0$), Eq. (15b) can be written as

$$\begin{bmatrix} V_{S1} \\ V_{S2} \\ V_{S3} \\ V_{S4} \end{bmatrix} = \begin{bmatrix} e_{11} & e_{12} & e_{13} & e_{14} \\ e_{21} & e_{22} & e_{23} & e_{24} \\ e_{31} & e_{32} & e_{33} & e_{34} \\ e_{41} & e_{42} & e_{43} & e_{44} \end{bmatrix} \begin{bmatrix} I_{S1} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{16}$$

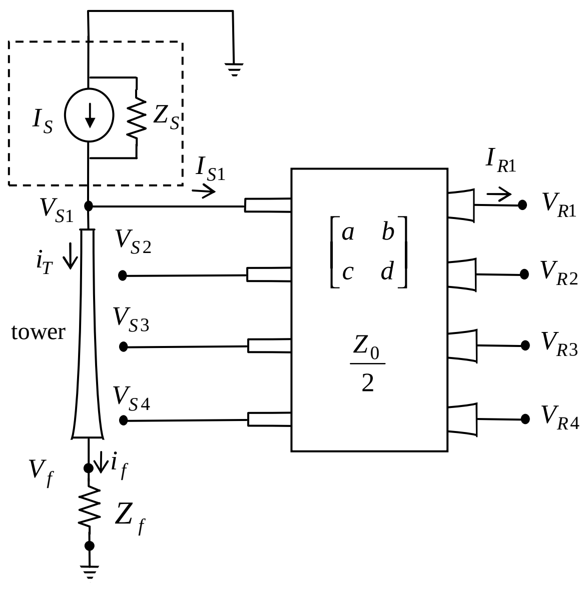


Fig. 2 Equivalent circuit model of the system.

From the above equation, sending-end voltages of the lines are obtained as

$$V_{S1} = e_{11} I_{S1} \tag{17a}$$
$$V_{S2} = e_{21} I_{S1} \tag{17b}$$
$$V_{S3} = e_{31} I_{S1} \tag{17c}$$
$$V_{S4} = e_{41} I_{S1} \tag{17d}$$

For determination of these voltages, I_{S1} should be known. To obtain this, first tower top voltage (voltage of ground wire at the position of tower top) is determined considering Fig. 2. Using the theory given in [12] we obtain tower top voltage as

$$V_{S1}(s) = \frac{I_S(s)}{Y(s)} \tag{18}$$

where

$$Y(s) = \frac{1}{Z_S} + \frac{1}{e_{11}} + \frac{Z_f c_T + d_T}{Z_f a_T + b_T} \tag{19}$$

In the above equation a_T, b_T, c_T and d_T indicates {ABCD} parameters of the tower. We substitute V_{S1} into Eq. (17a) to obtain I_{S1}, and then we get *s*-domain induced voltages of the phase conductors at tower position from Eqs. (17b, c, d) as

$$V_{S2} = \frac{e_{21}}{e_{11} Y(s)} I_S(s) \tag{20a}$$

$$V_{S3} = \frac{e_{31}}{e_{11} Y(s)} I_S(s) \tag{20b}$$

$$V_{S4} = \frac{e_{41}}{e_{11} Y(s)} I_S(s) \tag{20c}$$

The above equations are in *s*-domain and for frequency to time domain conversion FILT given in the next section is used.

III. Frequency to Time Domain Conversion

Frequency to time domain conversion formula for any function *F(s)* in *s*-domain is

$$f(t) = \frac{1}{2\pi j} \int_{\gamma - j\infty}^{\gamma + j\infty} F(s) \exp(st) ds \tag{21}$$

The above closed form formula can be solved analytically for only some particular form of *F(s)*. Therefore, usually numerical methods are used. In this study, the response of the system obtained in the *s*-domain is converted into time domain by using numerical Fast Inverse Laplace Transform (FILT) [15]. This technique has been applied to several transmission line transient problems and its efficiency has been proved [12, 16-17].

In [15] it as been show that, the following formula can be used to obtain *f(t)* numerically

$$f_{ec}^{kp}(t,a) = (e^a / t) \left[\sum_{n=1}^{k-1} F_n + (1/2^{p+1}) \sum_{n=0}^{p} A_{pn} E_{k+n} \right] \tag{22}$$

where

$$F_n = (-1)^n \operatorname{Im} F\{[a + j(n - 0.5)\pi]/t\} \tag{23}$$

and

$$A_{pp} = 1, \quad A_{pn-1} = A_{pn} + \binom{p+1}{n} \tag{24}$$

In order to compute *f(t)* from Eq. (22); first, proper values are chosen for *a*, *k* and *p* appearing in Eq. (22). Satisfactory results are obtained in the applications by FILT with $a=5$, $k=15$ and $p=10$ [10-12].

IV. Simulation Results

The system shown in Fig. 1 is considered for application. Reference to the measurements made by Ishii et al. [2], the characteristic impedance of the tower is taken to be varying from 220 Ω at tower top to 150 Ω at the tower base. This variation is defined by the exponential function $Z_0(x) = 150\exp(0.00609x)$, where x is the distance measured from the ground level. Nonuniform single-phase transmission line model is used to represent the tower. The velocity of surge propagation in the tower is taken to be the velocity of light (300 m/μs), tower footing impedance is taken to be 17 Ω, and ground resistivity is 500 Ωm. A current source with ramp function and rise time of 20 ns is used to represent the lightning source and an impedance of 400 Ω is considered for the lightning path. Based on the developed theory, a computer program in MATLAB [20] is prepared. Tower top voltage computed using *s*-domain method is shown in Fig. 3. Peak value of the tower top voltage is about 188.4 p.u. This value is agreement with the theoretical one which is calculated simply by multiplying peak value of the lightning current (2 p.u.) with the approximate value of the impedance seen from the tower top Z= 95 Ω. The voltages at tower crossarm positions, the voltages induced on the phase conductors and the insulator voltages computed using the proposed method are shown in Figs. 4(a), (b) and (c) for upper, middle and lower phase, respectively. The insulator voltages are also obtained using EMTP by constant parameter modeling. The results obtained by the method presented in this paper and those obtained using EMTP for the voltages between insulator strings are shown in Figs. 5(a), (b) and (c) for comparison. Nonuniform variation of tower surge impedance is simulated by cascaded connection of 5 uniform line sections each of which having different characteristic impedance. A good agreement can be observed between the curves. The obtained results are also in agreement with the numerical results obtained in[5]. However, there are some differences

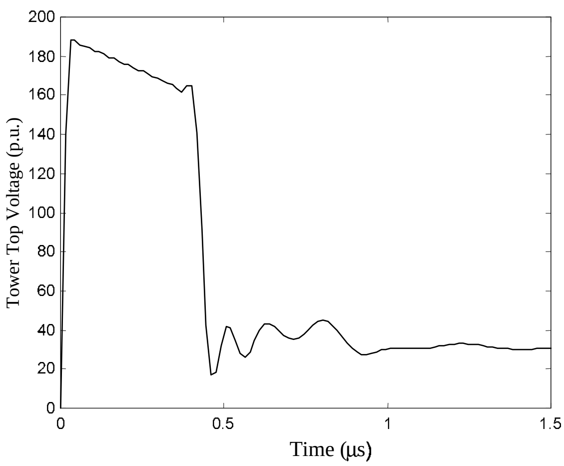
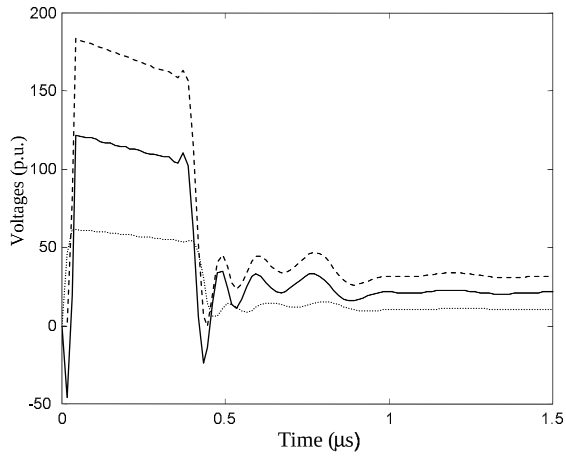
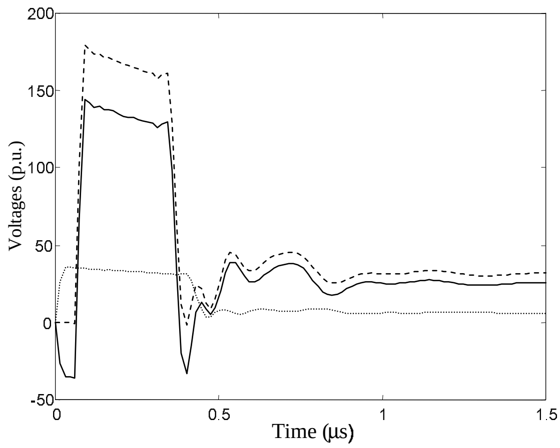


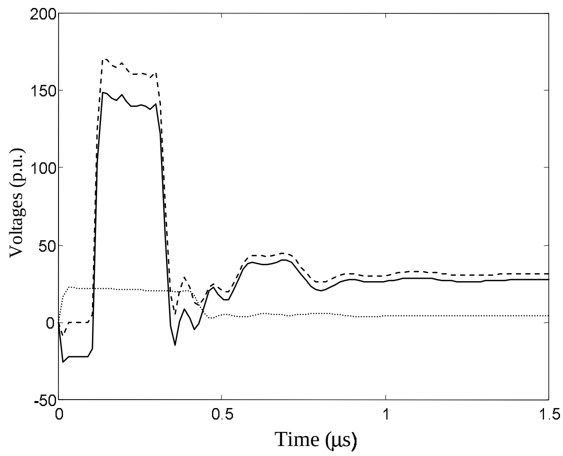
Fig. 3 Tower top voltage computed by *s*-domain method.



(a)



(b)



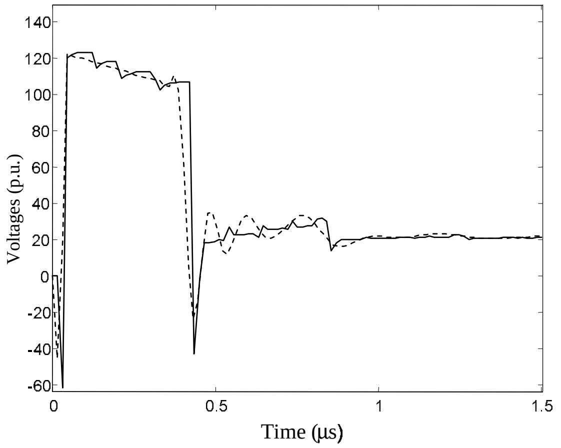
(c)

Fig. 4 Computed (a) Upper, (b) middle and (c) lower phase voltages.

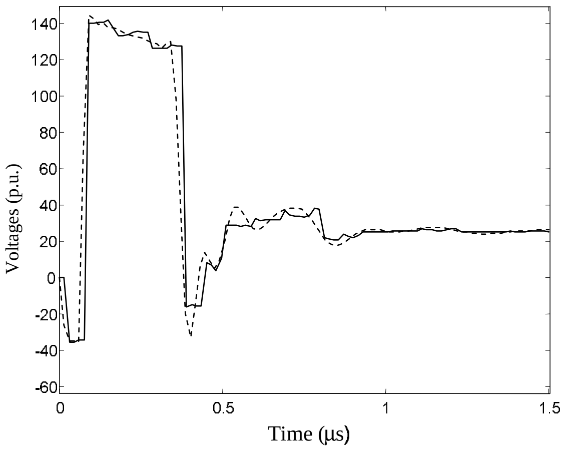
--- tower phase position

— insulator string

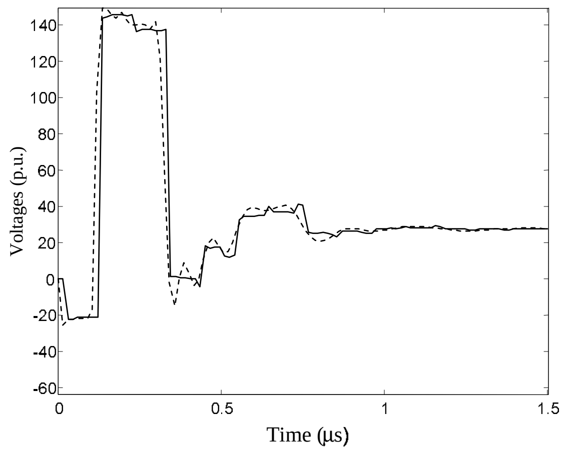
.... phase conductor



(a)



(b)



(c)

Fig. 5 Comparison of insulator voltages for (a) Upper, (b) middle and (c) lower phase.
— EMTP results, --- *s*-domain results

between the computed and measured results. Contrary to the measured results [2,7] the peak value of the computed lower-phase insulator voltage is the highest and that of the upper-phase is the lowest. This is because the induced voltages on the conductor located far from the ground wire is being lower due to weak mutual effects while the voltages on the tower at position of lower conductor is approximately equal to the tower top voltage. Which is probably because the tower is represented by a lossless line in *s*-domain approach. Comparisons with EMTP results show that the effect of frequency dependent of the line parameters is not effective for tower surge response.

V. CONCLUSIONS

Tower surge response and voltages on the insulator strings in a multiphase system are obtained using *s*-domain analysis including the effect of the shield wire and the phase conductors. The Coupling Coefficient Matrix (CCM) described in this paper facilitates determination of tower surge response in the s-domain. Obtained results show good agreement with those obtained using EMTP. Advantage of the *s*-domain analysis are that frequency dependent effects can be included directly and nonuniform variation of the tower parameters are represented accurately. The method will be further improved to include frequency dependence and resistive losses of tower elements.

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APPENDIX

To form impedance matrix ***Z*** for a multiphase transmission line, self impedance of conductor *i* and mutual impedance between conductor *i* and *j* can be calculated as [14]

$$Z_{ii} = R_i + \frac{\mu_0}{2\pi} s \ln \frac{2(H_i + p)}{D_{si}} \quad \Omega/m \qquad \text{(A.1)}$$

$$Z_{ij} = \frac{\mu_0}{2\pi} s \ln \frac{\sqrt{(H_i + H_j + 2p)^2 + D_{ij}^2}}{\sqrt{(H_i - H_j)^2 + D_{ij}^2}} \quad \Omega/m, \text{i} \neq \text{j} \qquad \text{(A.2)}$$

where

$$p = \frac{1}{\sqrt{s\mu_0\sigma}} \qquad \text{(A.3)}$$

is defined as the complex depth, which is used to include frequency dependent characteristics of earth, and other variables are

- s : complex frequency ($s = j\omega$).
- σ : earth conductivity, ℧ m-1.
- H_i : average height above ground of conductor i, m.
- D_{ij} : horizontal distance between conductors i and j, m.
- D_{si} : self GMR of conductor i, m.
- R_i : resistance of conductor i, Ω/m .

For determination of matrix ***Y***, first potential matrix ***P*** should be determined. Elements of this matrix are calculated as

$$P_{ii} = \frac{1}{2\pi\varepsilon_0} \ln \frac{2H_i}{r_i} \qquad \text{(A.4)}$$

$$P_{ij} = \frac{1}{2\pi\varepsilon_0} \ln \frac{\sqrt{(H_i + H_j)^2 + D_{ij}^2}}{\sqrt{(H_i - H_j)^2 + D_{ij}^2}} \qquad \text{(A.5)}$$

where *r* is the radius of conductor *i* and other variables are defined as in Eqs. (2) and (3). Once matrix ***P*** is determined, capacitance matrix can be found

$$C = P^{-1} \qquad \text{(A.6)}$$

and therefore shunt admittance matrix is determined as

$$Y = j\omega C \qquad \text{(A.7)}$$