

Negative Impedances as Power System and Control Elements in EMTP-Type Programs

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Abstract – Negative impedances have long been used in power system analysis for the representation of three-winding transformers as star circuits. They do not cause problems as long as precautions are observed about the correct placement of magnetizing branches. With the addition of ideal operational amplifiers, negative resistances and capacitances have recently been used to represent first-order transfer functions of control circuits, with only one (“non-inverting”) ideal operational amplifier. While not physically realizable, it can be shown that the equations are identical to those of a physically based model with positive impedances and an extra inverting ideal operational amplifier, if the system of equations of the latter is reduced. In the past, some attempts have been made to represent transfer functions of control circuits with R,L,C-elements directly connected to the measuring point of the power system. This R,L,C-circuit does load the power system with an impedance, however, which is unacceptable unless its value is very high. A simple approach for avoiding the load on the power system measuring point is the connection of an equal impedance, but with negative value, to the same point. Examples show that these negative impedances do not cause problems in EMTP simulations.

Keywords – electromagnetic transients simulation, control systems, transformer modelling, negative impedance.

I. INTRODUCTION

Negative impedances are usually regarded with suspicion in EMTP-type programs, and rightly so. For example, if an ac voltage source were connected to an R - L branch to ground, with a positive resistance of $R = 1\Omega$ and a negative reactance of $\omega L = -10\Omega$, the steady-state phasor solution would simply be $I = V/(1 - j10)$. This might indeed be a reasonable answer if the negative reactance represents the impedance of a capacitor. In EMTP-type time-domain solutions, solving the differential equation

$$Ri + v_L = v, \text{ where } v_L = L \frac{di}{dt}, \quad (1)$$

numerically, with whatever technique, would make the current grow to infinity with L being negative. In this simple case, assuming the source is now a dc voltage connected at $t = 0$, the exact solution is

$$i = \frac{V_{dc}}{R} \left(1 - \exp\left(-\frac{t}{T}\right) \right), \quad (2)$$

where the time constant $T=L/R$ becomes negative for a negative value of L , and the exponent therefore positive. If the impedance of -10Ω is that of a capacitor, the term v_L in Eq. (1) is of course incorrect, and must be replaced by

$$v_c = \frac{1}{C} \int i \cdot dt. \quad (3)$$

II. NEGATIVE INDUCTANCES IN TRANSFORMER EQUIVALENT CIRCUITS

A. Two-winding transformers

Negative inductances occur in two-winding transformers when its two coupled windings (branches) are represented by equivalent circuits with uncoupled branches. In general, the two coupled branches are best described by branch matrix equations, with the form

$$\begin{bmatrix} Y & -tY \\ -tY & t^2Y \end{bmatrix} \begin{bmatrix} V_H \\ V_L \end{bmatrix} = \begin{bmatrix} I_H \\ I_L \end{bmatrix} \quad (4)$$

for steady-state solutions, where the branch voltages are defined as $V_H = V_{H1} - V_{H2}$; $V_L = V_{L1} - V_{L2}$, assuming the branches go from nodes H_1 to H_2 and L_1 to L_2 . For transient simulations, starting from the equation

$$[R][i] + [L] \left[\frac{di}{dt} \right] = [v], \quad (5)$$

and recognizing that $[L]$ does not exist because $[Y]$ in Eq. (4) is singular, we obtain with the inverse inductance matrix

$$[L]^{-1} = \begin{bmatrix} \frac{1}{L} & \frac{-t}{L} \\ \frac{-t}{L} & \frac{t^2}{L} \end{bmatrix}, \quad (6)$$

the equation for the time domain

$$[L]^{-1} [v] - [L]^{-1} [R][i] = \left[\frac{di}{dt} \right]. \quad (7)$$

For simplicity, assume that $[R]$ in Eq. (7) is zero, and

that both windings go from node to ground. It can then be seen that both branch equations of (4) and (7) can be represented as a nodal Π -circuit (Fig. 1).

This π -circuit is well known from power flow and short-circuit studies done with per unit quantities, whenever “off-nominal turns ratios” appear in cases where the transformer ratio differs from the ratio of the base voltages. This Π -circuit has a series admittance element tY between nodes

H_1 and L_1 (or a series inductance $\frac{L}{t}$), a shunt admittance

element in node H_1 of $(1-t)Y$ (or a shunt inductance $\frac{L}{1-t}$), and another shunt admittance element in node L_1 of

$(t^2 - t)Y$ (or a shunt inductance $\frac{L}{t^2 - t}$).

Assuming $t > 1$, the shunt element in H_1 is negative. Equations (4) and (7) can also be obtained from a cascade connection of a branch with admittance Y (or inductance L), with an ideal transformer of ratio t . The latter representation indicates that the negative shunt inductance in node H_1 of Fig. 1 should not create numerical problems. Both representations, as a Π -circuit and as a cascade connection, are equivalent.

If the transformer is energized from node H_1 , but open-circuited in node L_1 , then Fig. 1 shows that the series impedance $Z_{series} = 1/(tY)$ from H_1 to L_1 with the shunt impedance $Z_{shunt} = 1/[(t^2 - t)Y]$ from L_1 to ground is in effect a voltage divider, which gives us a ratio $V_H/V_L = (Z_{series} + Z_{shunt})/Z_{shunt} = t$. If resistances are ignored, it would be an inductive voltage divider.

What is furthermore interesting is the fact that the shunt impedance of $1/[(1-t)Y]$ in node H_1 is the negative value of the impedance $(Z_{series} + Z_{shunt})$ of the voltage divider. It therefore compensates for the loading on the network that $(Z_{series} + Z_{shunt})$ creates. This addition of a negative impedance to prevent loading of the network will be discussed further in Section V. One can also notice that the sum of the three impedances in the Π -circuit is zero; they form a resonance circuit.

A more general equivalent circuit that does not require that the windings go from node to ground is shown in Fig. 2. In that more general case, the inductances of the two diagonal branches are always negative, even in the case of $t = 1.0$.

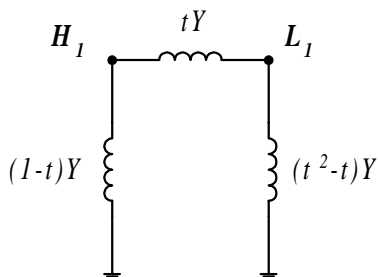


Fig. 1 Π -circuit with uncoupled branches.

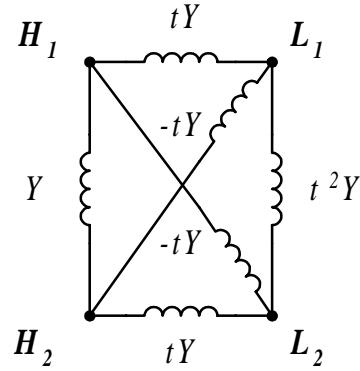


Fig. 2 Equivalent circuit with uncoupled branches for arbitrary connection.

The correctness of Fig. 2 can easily be verified by looking at the contribution of a branch current with Eq. (4) to the nodal equation of a node. For example, the current $I_{H1 \text{ to } H2}$, which is the same as $I_H = Y(V_{H1} - V_{H2}) - tY(V_{L1} - V_{L2})$, is the same current as the sum of the currents through the three branch admittances connected to node H_1 in Fig. 2.

B. Three-winding transformers

Negative inductances have long been used in power system analysis in the representation of three-winding transformers as star circuits, as shown in Fig. 3, with impedances in per unit quantities or referred to one side. For three-winding transformers with a high, medium and low voltage winding, it is usually the medium voltage branch that has a negative inductance value. Negative values do not cause problems for steady-state as well as time-domain solutions as long as precautions are observed about the correct placement of magnetizing branches.

Without a magnetizing branch, the sum of the two inductances between any two windings will always be positive (it is, in fact, the short-circuit inductance between those 2 windings). Note that v_L of Eq. (1) must be used for all three branches, including the one with the negative value.

If there are shunt branches at the star point for the representation of saturation and iron core loss effects, then there are situations where the time-domain simulation may go unstable. A simple case would be an infinite magnetizing inductance in parallel with a resistance for iron core losses.

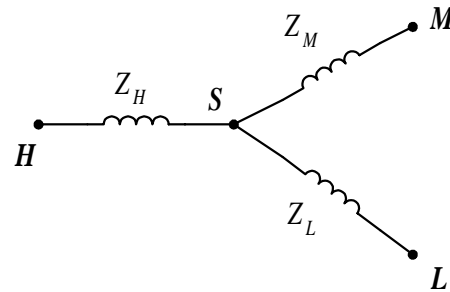


Fig. 3 Star circuit for three-winding transformer with p.u. quantities, or quantities referred to one side.

If the transformer is unloaded and only energized from the medium voltage side, assuming a negative inductance in that branch, then we have the situation of Eq. (2), with a negative time constant $T=L/R$, and consequently a growing exponential term. The problem arises from the fact that placing the branches for saturation and iron core loss effects at the star point is not correct, as pointed out previously [1-4].

In power flow and short-circuit studies, the values of the star circuit are usually obtained from the short-circuit impedances between pairs of windings. For example, the impedance Z_M can be obtained from

$$Z_M = \frac{1}{2}(Z_{HM} + Z_{ML} - Z_{HL}).$$

If this calculation is done with complex quantities, whereby $Re\{Z_{HM}\}$, etc. represents the per unit losses in the short-circuit test between H and M etc., then $Re\{Z_M\}$ may become negative. For steady-state solutions, that may be acceptable. For transient simulations with a magnetizing inductance in the star point, this would lead to a solution with a growing exponential if the transformer is only energized from side M , unless the sum of the resistance of the network connected to M and of $Re\{Z_M\}$ becomes positive. The safest way to avoid this possible instability is to separate $[R]$ from $[L]^{-1}$ according to Eq. (7), and to check for negative values in $[R]$ when these matrices are created with support routines such as BCTAN. For example, Microtran's version of the transformer support routine warns the user with the message

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Resistance of winding xxx was calculated as x.xxxx p.u. based on 1 MVA (3-phase). A negative resistance is not acceptable if the exciting current is taken into account. If you want to accept the negative resistance, run the case again with an extra line after the title card with "$IGNORE" in columns 1 - 7. If you want to set the resistance to zero, use "$ZERO" instead in columns 1 - 5.
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In more complicated network situations, the reason for a growing exponential term may not be that easy to see. If the network is linear, or linearized around an operating point, an eigenvalue analysis would be necessary to see whether there are any eigenvalues with real positive values.

III. NEGATIVE CAPACITANCE CREATED BY CONVERSION TO P.U. QUANTITIES

Assume, for example, a stray capacitance C between the high voltage terminal H_I and the low voltage terminal L_I of a 230/115 kV transformer bank in wye/wye connection, with the neutral solidly grounded. In equations with actual quantities, this is a "normal" branch with an admittance $Y = j\omega C$ between nodes H_I and L_I . In building the nodal admittance matrix $[Y_{actual}]$ for steady-state solutions, $+j\omega C$ will contribute to the diagonal elements $H_I - H_I$ and $L_I - L_I$, and $-j\omega C$ will contribute to the off-diagonal elements $H_I - L_I$ and $L_I - H_I$.

For conversion to per unit quantities, the base values are needed. Let's assume base values of $V_{HI} = 230/\sqrt{3}$ kV and $V_{LI} = 115/\sqrt{3}$ kV as single-phase voltage bases, and $S = 100/3$ MVA as the single-phase apparent power base. The conversion to the per unit admittance matrix $[Y_{p.u.}]$ is achieved by multiplying row H_I and column H_I of $[Y_{actual}]$ with V_{HI} , and row L_I and column L_I with V_{LI} , and then dividing all elements by S (Equation (IV.12) of Appendix IV in [4]). What shows up in the diagonal element $H_I - H_I$ will then be $j\omega C * V_{HI}^2/S$, in the diagonal element $L_I - L_I$ it will be $j\omega C * V_{LI}^2/S$, and in the off-diagonal elements $H_I - L_I$ and $L_I - H_I$ it will be $-j\omega C * V_{HI}V_{LI}/S$.

This is no longer a single branch between nodes H_I and L_I . Instead, it has become a Π -circuit, with a series admittance between nodes H_I and L_I of $j\omega C * V_{HI}V_{LI}/S$, with a shunt admittance in node H_I of $j\omega C * (V_{HI}^2 - V_{HI}V_{LI})/S$, and in node L_I of $j\omega C * (V_{LI}^2 - V_{HI}V_{LI})/S$. This Π -circuit is very similar to the Π -circuit of two-winding transformers discussed earlier. With $V_{LI} < V_{HI}$ in this example, the shunt capacitance in node L_I will be negative.

IV. TRANSFER FUNCTIONS WITH IDEAL OPERATIONAL AMPLIFIERS AND NEGATIVE RESISTANCES, INDUCTANCES AND CAPACITANCES

After the modelling of current and voltage dependent sources in EMTP-based programs in [5], [6], reference [7] has presented a technique which uses circuit components, such as resistances, capacitances, and ideal operational amplifiers, for the computer modelling of control transfer functions. This novel approach can be used by any EMTP-type electromagnetic transients program or by similar simulation programs, independently of the method used for the time-domain integration, because of its generality and flexibility.

For an efficient digital computer implementation, it is assumed that resistances and capacitances can be assigned negative values. Ideal operational amplifiers can be represented with the Modified Nodal Analysis (MNA) method, which would result in an unsymmetric nodal conductance matrix.

The compensation method with an iterative Newton-Raphson algorithm is used, because nonlinear effects, such as hard and soft limits, or saturation, can then easily be handled as well as showed in [7].

With the compensation method, a simultaneous solution of control and electric power system equations is obtained at every time step of the digital computer simulation.

Following is the proof that positive impedances with 2 amplifiers can be converted to a circuit with one amplifier with negative impedances, exactly. Let's assume that we use two ideal operational amplifiers as in Fig. 4, with a physically based network, for a transfer function. The first one is between nodes 2 and 3, and the second one is between nodes 4 and 5. For the second inverting operational amplifier, let's assume that the two admittances are equal to Y , because it should just invert.

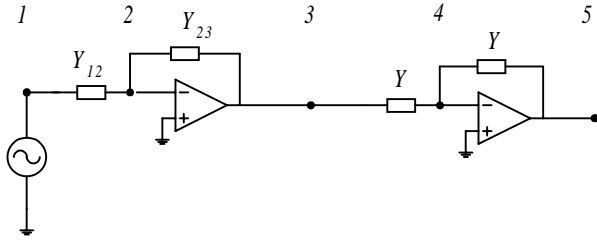


Fig. 4 Two ideal operational amplifiers.

With one non-inverting circuit, the diagram would be as shown in Fig. 5:

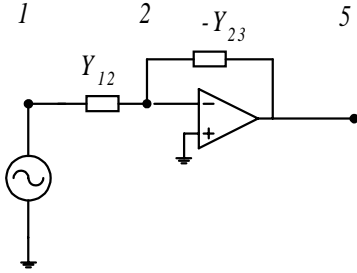


Fig. 5 One non-inverting ideal operational amplifier.

If we write the equations in the frequency domain, and use the transfer function approach, we know from Fig. 4 that $V_3/V_1 = -Y_{12}/Y_{23}$ and $V_5/V_3 = -Y/Y$ or $V_5/V_3 = -1$. This leads to the result $V_5/V_1 = Y_{12}/Y_{23}$ for the single non-inverting circuit model of Fig. 5.

What many people, including in academia, probably do not believe immediately is the fact that the EMTP network equations can also be converted from the first to the second circuit. Start with equations for nodes 2,3,4,5, using the fact that

$$\begin{aligned} I_2 &= 0, \\ I_4 &= 0, \end{aligned}$$

$$\text{for node 2: } -Y_{12}V_1 + (Y_{12} + Y_{23})V_2 - Y_{23}V_3 = 0 \quad (8)$$

$$\text{for node 3: } -Y_{23}V_2 + (Y_{23} + Y)V_3 - YV_4 - I_{30} = 0 \quad (9)$$

$$\text{for node 4: } -YV_3 + 2YV_4 - YV_5 = 0 \quad (10)$$

$$\text{for node 5: } -YV_4 + YV_5 - I_{50} = 0 \quad (11)$$

We also have the 2 extra equations

$$V_2 = 0 \quad (12)$$

$$V_4 = 0 \quad (13)$$

Therefore, we have now 6 equations to solve for $V_2, V_3, V_4, V_5, I_{30}, I_{50}$. The latter 2 currents are the currents going into the amplifiers on the output side. These equations would partly be solved in the main program, and partly in CONNEX, or if we solve all network equations together with those of the ideal operational amplifiers, they would

be solved as one system of equations.

By inserting equations (12) and (13) into equations (8) to (11), we obtain:

$$-Y_{12}V_1 - Y_{23}V_3 = 0, \text{ from Eq. (8)} \quad (14)$$

$$-YV_3 - YV_5 = 0, \text{ from Eq. (10)} \quad (15)$$

The second equation (15) simply says $V_3 = -V_5$, which inserted into equation (14) leads to

$$-Y_{12}V_1 + Y_{23}V_5 = 0 \quad (16)$$

This is exactly what we get from the circuit of Fig. 5, when we write the equation for node 2 and set $V_2 = 0$.

I_{30} and I_{50} in equations (9) and (11) seem to be dependent variables, and can be calculated once the voltages are known (they are needed, of course, if CONNEX is used). Only equation (12) would be needed for Fig. 5.

To include limits in the transfer function representation, one can either include the limiting functions into the equations of the ideal operational amplifier directly, or use nonlinear resistances as models for Zener diodes. For the non-inverting circuits with negative resistances discussed above, the nonlinear resistances may have to be negative as well. In most EMTP versions, nonlinear resistances can be solved with the compensation method, with Newton's method for the iterations. Assume that the program is written in such a way that a subroutine is called to find the current i and its derivative di/dv for a given approximate solution of v . Then it becomes easy to treat the case of negative nonlinear resistances by simply changing the signs on i and di/dv , just before return from the subroutine. Little else must then be changed in the code. This has been tested successfully with UBC's version of the EMTP on a limited number of cases.

V. BUILDING TRANSFER FUNCTIONS WITH R,L,C-CIRCUITS

As described in the preceding section, transfer functions can easily be modelled with ideal operational amplifiers and positive or negative impedances. Before operational amplifiers became available in EMTP-type programs, some users have attempted to represent transfer functions of control circuits with R,L,C-elements. In general, these elements do load the power system at the measuring point, unless their impedance is very high.

The discussion of the unloaded two-winding transformer in Section II, Subsection A, with the series element tY and shunt element $(t^2-t)Y$ forming a voltage divider, and the shunt element $(1-t)Y$ in node H_1 providing a negative compensating impedance that unloads the network at node H_1 , suggests an approach for building transfer functions directly from network elements alone.

Let us assume that we want to create a first-order transfer function $\frac{K}{1+sT}$, with the input being the voltage between nodes i and k , and with $K = 10$ and $T = 1$ ms (Fig. 6).

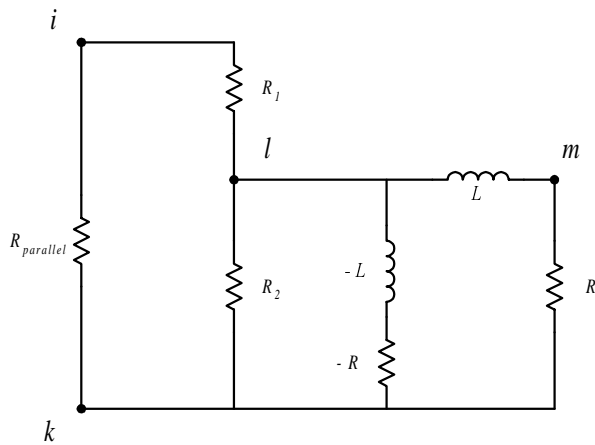


Fig. 6 Transfer function built from R,L,C-circuits.

By connecting a resistive voltage divider with $R_1 = -9\Omega$ and $R_2 = 10\Omega$ between nodes i and k , we achieve the correct amplification for the voltage between nodes l and k of $\frac{R_2}{R_1 + R_2} = 10$. To avoid loading the network at the measuring point with the resistance $(R_1 + R_2) = 1\Omega$, we now connect a negative resistance of $R_{parallel} = -1\Omega$ to compensate for the load. If the voltages in nodes i and k could be solved before connecting these elements, they can still be solved because nothing has been changed in their diagonal elements of the matrix. The extra node equation for node l is simply the voltage divider equation.

The dynamics of $(1.0 + sT)$ can be modelled by adding a branch with $L = 1$ mH and $R = 1\Omega$ between nodes l and k . The voltage across the resistance between nodes m and k will then have the correct transfer function output. With a resistance of 1Ω , the current would be the correct output as well. Again, this R - L branch would load the network, which can be avoided by connecting another R - L branch with negative values in parallel.

Fig. 7 shows an EMTP simulation, where node k was ground, and where a dc voltage source with an internal resistance of 1Ω was connected to node i . The diagonal elements in the triangularized matrix, without pivoting, were

$$G_{ii} = -0.111111, G_{ll} = 0.102381, G_{mm} = 1.025581.$$

As can be seen, there is no sign of ill-conditioning in the matrix. If the amplification were increased from 10.0 to 1000.0, the diagonal elements of the triangularized matrix would become

$$G_{ii} = -0.0010010, G_{ll} = 0.0033810, G_{mm} = 0.3105634,$$

and there is still no ill-conditioning.

If the output were now connected to a network with an unknown impedance as seen between nodes m and k , then it becomes impossible to add a compensating negative impedance to avoid loading because of its unknown value, and this simple approach would eventually become unworkable.

On the other hand, if the output voltage is just used as a variable, e.g. as the field voltage of a synchronous generator, the method would work. Since field windings have typical time constants of a few seconds, using it as a field voltage with a delay of one time step would probably be acceptable.

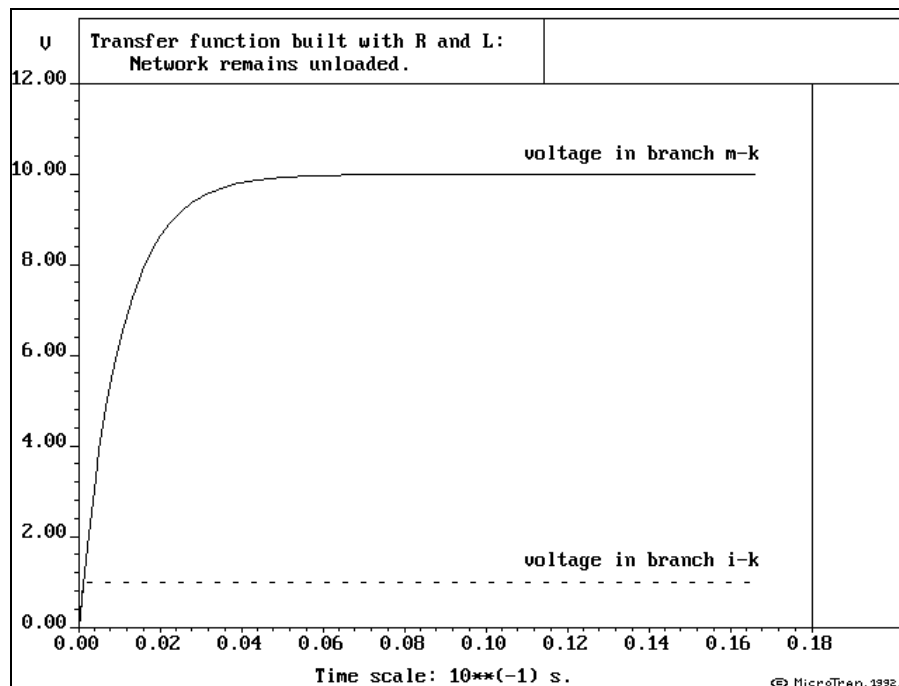


Fig. 7 Transfer function simulation results.

V. CONCLUSIONS

This paper has discussed the issue of negative impedances as power system and control elements in EMTP-type programs. Negative impedances have long been used in power system analysis for the representation of three-winding transformers as star circuits. They do not cause problems as long as precautions are observed about the correct placement of magnetizing branches.

With the addition of ideal operational amplifiers, negative resistances and capacitances have recently been used to represent first-order transfer functions of control circuits. Examples have shown that the use of negative impedances to avoid loading measuring points in power systems networks does not cause problems in EMTP simulations.

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