

## Indices For Ferroresonance Performance Assessment in Power Distribution Network

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**Abstract--** In a power distribution system having long radial cable fed transformers provided with fuse protection at the tap off point, ferroresonance is a common phenomenon. During expansion and operational planning of a distribution system, to shortlist critical operating conditions exhibiting ferroresonance for detailed temporal simulation, the paper proposes a system parameter based index named characteristic ratio CR and determines its relationship with the geometrical ratio GR, a descriptor of the phase plane trajectory. To identify existence of ferroresonance during actual system operation, a frequency jump index FJI, which can be obtained through real time data analysis, is also proposed and its relationship with GR is determined. A table showing CR and corresponding GR values based on initial detailed studies can be used during subsequent planning process, for identifying only the critical cases for detailed studies and determining the limiting parameters, to avoid ferroresonance. Similarly, the online calculated FJI can be compared to the FJI values in a lookup table based on initial detailed studies and the corresponding GR value can be used to indicate severity of the ferroresonance and prompt the need of initiating corrective action during system operation.

*Index Terms*—Bifurcation, Ferroresonance, Distribution System, Computer aided analysis, Contingency Analysis

### I. INTRODUCTION

Disturbance due to ferroresonance is a common phenomenon in electric power distribution system operation. Depending on circuit conditions, its effect may be a random over voltage that could be either a short transient for few cycle, a continuous over voltage or even a jump resonance. It causes both phase-to-phase and phase-to-ground high sustained oscillating over voltages and over currents with sustained levels of distortion to the current and voltage waveforms, leading to transformer heating together with excessively loud noise due to magnetostriction, electrical equipment damage, thermal or insulation breakdown and mal-operation of the protective devices.

A distribution system may have characteristic features such as a long overhead line or a cable feeding a delta or floating wye connected primary of a transformer serving a varying load, a low short circuit level power supply point interfacing the transformer feeder, a three phase isolator with protective fuse on its each phase at the tap-off point feeding the pad

mounted transformer primary. To minimize the distribution system losses the current trend has been to provide low loss power transformers. A phase-to-earth fault is usual in a distribution system, which often leads to a blown fuse on the faulty phase only. This results in a long radial feeder connected to a delta or a ungrounded star connected transformer with blown fuse on one of the phases at the feeding point [1],[2],[3].

The distribution system also has circuit capacitances in the form of phase-to-phase and phase-to-ground capacitances of the cable feeding the transformer. A distribution transformer's ferromagnetic core saturates and exhibits hysteresis. The saturation point is characterized by a dramatic change in the slope of the core's current-flux density curve. The slope of the curve is proportional to the inductance of the winding. As the current in a ferromagnetic coil increases beyond the saturation point, its inductance changes rather suddenly, achieving minimum inductance value in the saturated region of the core. The hysteresis in a ferromagnetic material is due to two current-flux density characteristics. The magnitude of excitation causing the transformer core to go into saturation and the magnitude of excitation at which the core comes out of saturation are different. As the iron core goes into saturation due to increased excitation current in a lightly loaded transformer with its feeding circuit's one of the phases open due to single phase switching, the sudden change in its inductance triggers a sudden change in the frequency at which it resonates with circuit capacitance. Sudden and unpredictable changes in inductance means, an interaction with a wide range of circuit capacitances and existence of several stable steady state responses to any given change of parameters.

To predict existence of multiple stable states for the same operating voltage in a distribution system and to determine generalised values of parameters such as maximum permissible feeder length and minimum permissible load limit on a transformer that will ensure good operating safety margin, many variants and combinations of parameters, initial conditions and switching instants for a given circuit configuration need to be simulated. During system expansion and operational planning exercises, to reduce number of cases for detailed study, it is desirable to shortlist the critical operating conditions exhibiting ferroresonance. Also, to fine

tune the operation, detection of existence of ferroresonance through data analysis during actual system operation is of preliminary interest.

## II. HYSTERESIS & BIFURCATION FRAMEWORK

The hysteresis in a transformer and the resulting multiple stable resonant states due to different inductance values, interacting with wide range of circuit capacitances can be explained in a bifurcation theory framework. A three phase cable feeding a transformer with delta connected primary and a fuse blown on one of the phases can be viewed as a Thevenin's equivalent sinusoidal source feeding a capacitance in series with a nonlinear inductance  $f(I)$ . The series capacitance  $C$  is given by  $C=2C_m+C_g$ , where  $C_m$  is phase-to-phase and  $C_g$  is phase-to-ground capacitance of the feeder cable [4]. The loop equation

$$V = \omega f(I) - I/\omega C \quad (1)$$

has two right hand terms viz.  $V_L = \omega f(I)$  a smooth curve and  $V_C = -I/\omega C$  a straight line plot with a slope  $\tan \alpha = 1/\omega C$ , which reduces as  $C$  or the cable length is increased. Each straight line corresponds to a specific cable length. The applicable inductance  $L$  on the smooth curve depends on the initial circuit operation together with state of saturation of the transformer core. Depending on the operating condition and circuit configuration i.e.  $L$  &  $C$  parameters, operating point voltages for a current will be given by the intersection of the smooth curve and the relevant straight line. The intersections on the linear region of the curve yield stable equilibrium or operating points while intersections on the saturated region result in unstable operation or jumps. The simultaneous intersections of the straight line with the inductance curve at two or more than two points yields multiple stable operation. The series circuit yields a second order differential equation given by

$$Ae^{-I/B} dI/dt + (1/C) \int I dt = V \sin(\omega t + \phi) \quad (2)$$

and the associated phase portrait shown in figure 1 gives the stable orbit structure [5] of the differential equation corresponding to stable equilibrium points, sufficiently small variations of the parameters around which do not cause alteration in qualitative structure of the flow. The parameter values in the saturated region of the curve  $V_L = \omega f(I)$  are bifurcation values.

At the point of intersections  $V_L$  equals  $V_C$  implying

$$I\omega L = I/\omega C \quad (3)$$

since at this point  $\omega f(I) = \omega L$ . This yields a resonant frequency  $1/(2\pi\sqrt{LC})$ . The resonance is described by

$$Ld^2I/dt^2 = (I/C) \quad (4)$$

indicating multiple solutions. The inductance and circuit capacitance being both energy storage elements, complete to and fro energy transaction between a range of circuit capacitance and the transformer coil during resonance, is responsible for multiple stable operating state. A loaded transformer will result in bleeding of some of the stored energy in the coil during each transfer cycle and will introduce damping. Considering no damping, at resonance

$$\frac{1}{2} LI^2 = \frac{1}{2} CV^2 \quad (5)$$

yields a characteristic ratio

$$\sqrt{\frac{I}{I(2Cm + Cg)}} \quad (6)$$

implying severity of ferroresonance with reduction in the ratio, which could be either due to inductance approaching its saturated value or due to increase in feeder length. At minimum value of the ratio, jump or branching may be witnessed. The proposed characteristic ratio, a system parameter based index, is linked to geometrical descriptors of a phase plane portrait, to facilitate short listing of only critical cases for detailed study and subsequent determination of safe parameters free from ferroresonance risk during operational planning of a power distribution system.

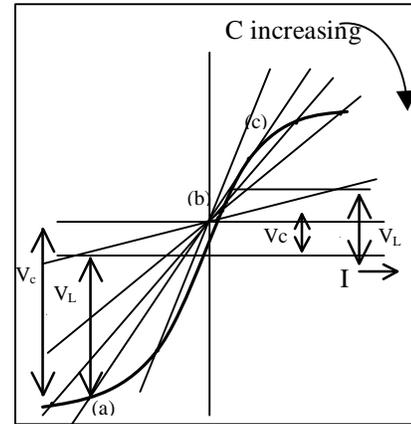


Figure 1: The effect of increasing C

The inductance assuming minimum value in the saturated region of the core results in, lower characteristic ratio and wider spread out of the phase plane trajectory, implying increase in severity of the operating condition for a given primary feeder length.

The resonant frequency being  $1/(2\pi\sqrt{LC})$ , core inductance assuming minimum value also results in increased frequency of oscillation. This exhibits a relatively higher jump in resonant frequency, when the characteristic ratio (CR) crosses over from a higher value in the linear region to a lower value

during operation in saturated region of the current-flux density curve.

A wavelet analysis of the time series of primary voltages acquired during actual system operation yielding, highest pseudo frequency component present in the time series during level-1 decomposition and lowest pseudo frequency component present in the time series during level-5 decomposition, can reveal the extent of jump in oscillation through the ratio of the two pseudo frequencies. Thus the ratio termed frequency jump index (FJI) can be used to identify existence of ferroresonance for initiating remedial action.

### III. SYSTEM STUDY

Effectiveness of the proposed characteristic ratio (CR) and the frequency jump index (FJI) and their relationship with the geometrical descriptors of the phase plane trajectory is verified using the 22kV system shown in figure 2. The EMTDC simulation for generating time samples of source voltage, transformer primary voltage, transformer primary delta winding flux and its derivative and also the phase current, is carried out for 0.56 seconds with phase-A fuse opening at 0.2 secs. Simulation is repeated for source voltage variation from 0.1 p.u. to 1.5 p.u. The delta-star transformer is kept at close to no load.

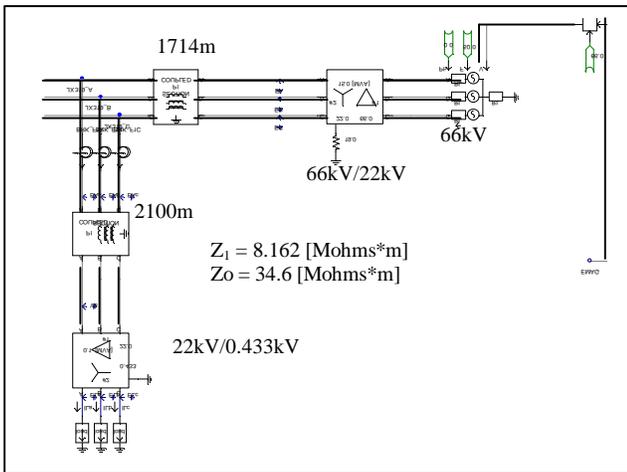


Figure 2: Case study network

### IV. RELATIONSHIP OF CR & NONLINEAR PERFORMANCE

Branching or bifurcation diagrams together with Phase plane trajectories, serve as unique form of identifiers for detecting existence of ferroresonance. For the network studied, bifurcation diagram in Figure 3 is plotted for source voltage varying from 0.1 to 1.5p.u. Here circuit capacitance is not changed. However, due to change in source voltage and hence the change in state of core saturation, transition without jump takes place implying soft loss of stability or periodic

behaviour up to 0.85 p.u. excitation. As the source voltage is increased beyond 0.9 p.u., aperiodic period doubling scenario is witnessed. Phase plane portraits of phases AB and CA with flux in the winding plotted on X-axis and its derivative plotted on Y-axis, while source voltage clamped at 1.0 p.u. in Figure 4 & Figure 5, clearly show the period doubling phenomenon. Periodic solution during initial period shown by closed phase plane trajectories or periodic attractors can also be seen.

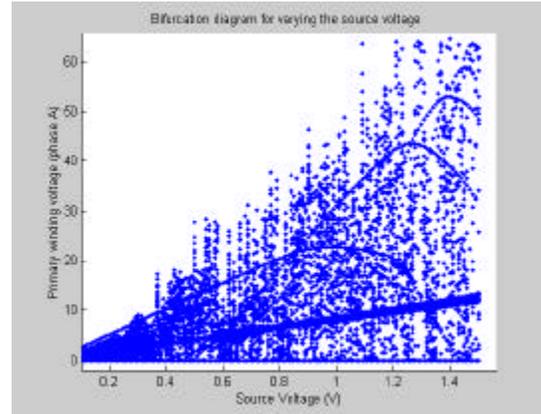


Figure 3: Bifurcation diagram with varying E

The trajectories during the later part do not close back on themselves and exhibit period doubling [6]. These are strange attractors indicating a nonperiodic or chaotic waveform since they are attracted to a constrained region and seem to wander randomly within this region. These attractors exhibit sensitive dependence of the nonperiodic ferroresonance on initial conditions. Initial condition here implies circuit condition before fuse blowing.

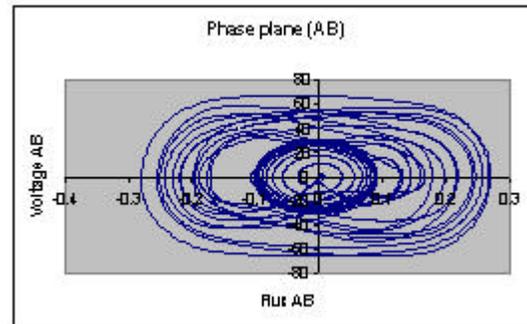


Figure 4: Phase plane portrait of phase AB

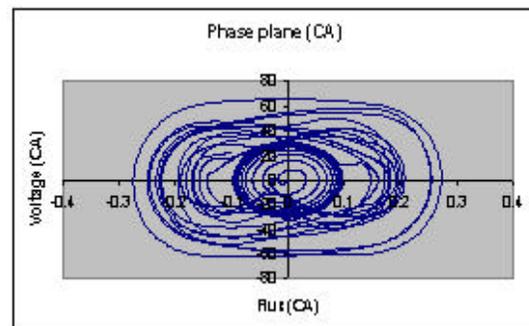


Figure 5: Phase plane portrait of phase CA

To shortlist only the critical cases out of the large number of possible operating configurations for detailed temporal simulation and to identify the limiting parameters so as to avoid occurrence of ferroresonance, a characteristic ratio is proposed and the same is linked to geometrical descriptors of the likely phase plane trajectory. Figure 6 depicts the characteristic ratio as seen by phase AB as the inductance changes over time corresponding to a source voltage of 1.0 p.u.

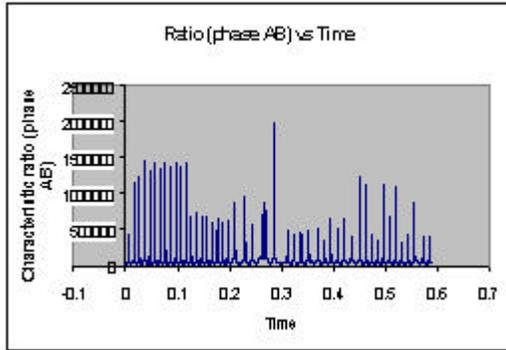


Figure 6: Characteristic ratio of phase AB

The regimes of high and low characteristic ratios are identified and the corresponding phase plane trajectories drawn are shown in figure 7 & 8 for phase AB.

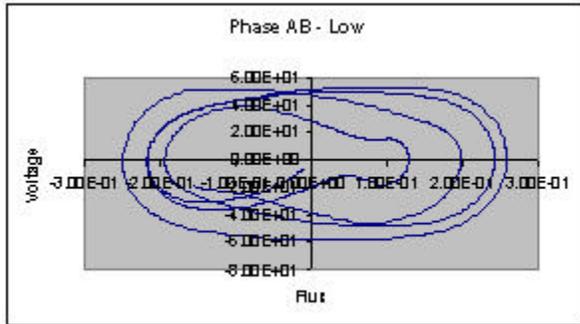


Figure 7: Phase plane corresponding to Low CR

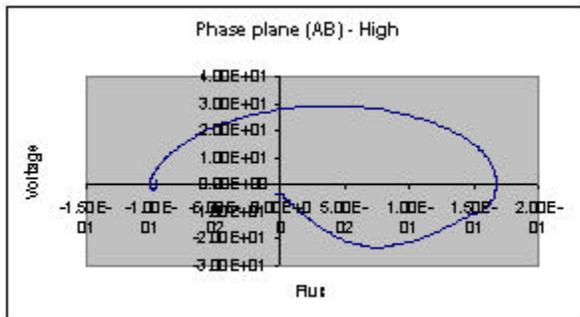


Figure 8: Phase plane corresponding to High CR

The corresponding geometrical descriptors of the phase plane trajectories together with their respective CR are summarized in table 1.

Phase		Characteristic Ratio (CR)	Geometrical Ratio (GR)	Oscillations Counts		
				D1	D2	FJI
A	Low	500,000	2.197	48	2	24
B	High	2,000,000	2.02	48	3	16
C	Low	500,000	2.43	56	3	18.6
A	High	6,000,000	2.27	50	3	16.7

Table 1: Characteristic ratio, Geometrical ratio and FJI

Lower the characteristic ratio, more severe the operating condition becomes. The geometrical ratio of the maximum spreads of an elliptical trajectory along X and Y-axes is also higher for the phase plane trajectory corresponding to the regime of low characteristic ratio. The elliptical phase plane trajectories depict the existence of ferroresonance with increase in severity being indicated by increasing degeneration in elliptical shapes. The proposed system parameter based characteristic ratio (CR) correlates well with the geometrical descriptors (GR) of the phase plane trajectories. A table indicating values of these parameter based indices CR with their likely phase plane trajectory patterns encrypted in terms of their geometrical descriptors GR can be framed for a system, on the basis of detailed simulations carried out initially. The table 1 clearly brings out that lower the value of CR, higher the value of GR implying a more degenerated elliptical phase plane trajectory. During subsequent planning studies, only cases requiring detailed investigation can be short-listed using the table 1.

## V. WAVELET ANALYSIS & FREQUENCY JUMP INDEX

Wavelets analysis of the voltage time series acquired during system operation enables determination of pseudo frequency or oscillation counts and hence oscillation activity at a given time represented in terms of sample number in the plots in fig 9 and fig10.

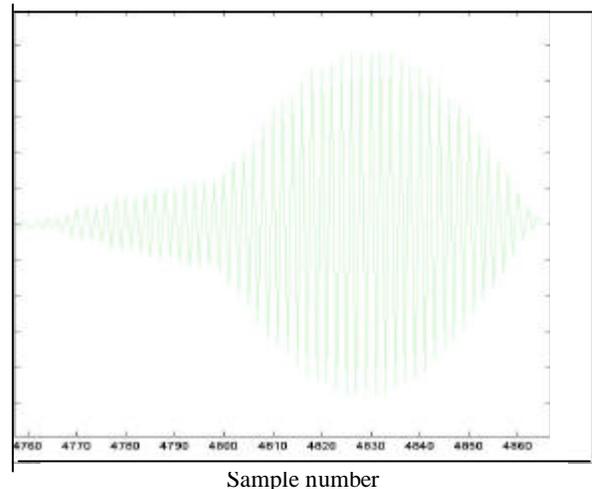


Figure 9: Detail 1 oscillation counts phase CA corresponding LOW CR

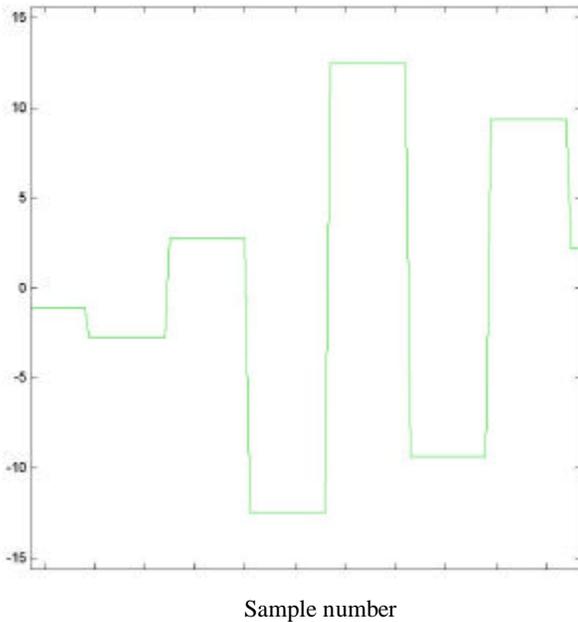


Figure 10: Detail 5 oscillation counts phase  
CA corresponding to LOW CR

Daub-4 is used as the mother wavelet and the decomposition of the time series into approximations (a1 to a5) and details (d1 to d5) is carried out till level-5. This implies that the acquired voltage time series during existence of a specific CR is processed through 5 stages of successive filtering. The most rapid oscillation present in the signal is shown by D1 available at the end of first level of filtering and the slowest oscillation present in the voltage time series is given by D5 available at the end of fifth stage of filtering. Their instant of occurrence is shown in terms of sample numbers. Each frequency represents a stable solution. The ratio of oscillation counts in D1 and D5 in a period gives the frequency jump index FJI corresponding to a CR and GR and indicates the extent of jump from one extreme solution to the other. With the available similar lookup table based on initial studies and the FJI based on the wavelet analysis of real time voltage time series, severity of ongoing ferroresonance can be assessed.

## VI. CONCLUSIONS

During expansion and operational planning of a distribution system, to shortlist only the critical cases out of the large number of possible operating configurations for detailed temporal simulation, the paper proposes a system parameter based index termed characteristic ratio CR and determines its relationship with the geometrical descriptor (GR) of the phase plane trajectory. A table showing CR and corresponding GR values based on initial detailed studies can be used during subsequent planning process, to select only the critical cases for detailed studies to determine the limiting parameters, and avoid ferroresonance.

To identify existence of ferroresonance during actual system operation in order to initiate corrective action, a frequency jump index termed FJI obtainable from real time data analysis, is also proposed and its relationship with GR is determined. A lookup table showing GR and FJI values based on initial detailed studies can then be used to indicate severity of the ferroresonance and prompt the need of initiating corrective action during system operation.

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