Application of Vector Fitting to High Frequency Transformer Modeling

Bjørn Gustavsen

SINTEF Energy Research, N-7465 Trondheim, Norway (e-mail: bjorn.gustavsen@sintef.no)

***Abstract* – This paper describes a procedure for frequency dependent modeling of power transformers from measured terminal characteristics. The admittance matrix is measured in the frequency domain using a network analyzer and a dedicated measurement setup. Subsequent approximation of the admittance matrix with rational functions using Vector Fitting gives an EMTP-type compatible model suitable for transient studies. The approach is demonstrated for a 3-winding rectifier transformer and for the calculation of internal voltages in a winding. It is shown that that highly accurate results can be obtained**

Keywords – Power system transients, transformer overvoltages, frequency dependence, modeling, rational functions, system identification.

I. INTRODUCTION

The calculation of transferred overvoltages through transformers requires that the frequency dependent behavior of the transformer be taken into account. The modeling can in principle be carried out by calculating an electrical network based on the transformer geometry [1],[2] but this approach requires detailed information about the transformer construction, which is usually proprietary to the manufacturer. Another procedure is to assume a certain circuit topology and then identify the circuit elements using measurements [3],[4].

An alternative procedure is to measure the admittance matrix $Y(s)$ with respect to the transformer terminals. Subsequent rational approximation of Y gives a realization which can be included in EMTP-type programs by a network equivalent or by convolutions [5]-[10]. This approach gives a terminal equivalent only, meaning that internal overvoltages cannot be computed. The voltages on taps can still be calculated as long as they are available for measurements. It is noted that the resulting model is linear and so it cannot simulate non-linear effects in the transformer. However, above a few kHz the transformer essentially behaves as a linear component.

This paper outlines a procedure for terminal modeling of transformers, based on measurements in the frequency domain. The procedure is based on a special measurement setup [9] and rational approximation by the Vector Fitting approach [11]. The approach is demonstrated for a 3 winding rectifier transformer, and for internal overvoltage calculation of a winding assembly. All measurements were performed during a 1-year visit at the University of Stuttgart, Germany.

II. RATIONAL APPROXIMATION OF LINEAR MULTI-TERMINAL COMPONENTS

The terminal behavior of a linear component can be characterized by its voltage/current relationship defined by the admittance matrix, Y

$$I(s) = Y(s)\cdot V(s) \tag{1}$$

A model suitable for EMTP-type simulation programs can be obtained by approximating Y with rational functions. Several procedures have been applied for solving this non-linear least squares problem, including polynomial fitting in [5], Levenberg-Marquardt in [6], and Vector Fitting [11] in [7]-[10]. In this paper results produced by a matrix application of Vector Fitting are presented. The matrix application [12] produces an approximation in the form of residue matrices and a common set of guaranteed stable poles which are real or come in complex conjugate pairs. The realization is on the form

$$Y(s) = \sum_{m=1}^{N} R_m \cdot \frac{1}{s-a_m} + D + sE \tag{2}$$

In addition is produced the alternative form

$$Y(s) = C(sI-A)^{-1}B + D + sE \tag{3}$$

In order to guarantee a stable simulation, passivity needs to be enforced. A procedure for passivity enforcement is shown in [13].

III. MEASUREMENT PROCEDURE

The elements of Y can be measured one-by-one using a network analyzer and a current sensor. It follows from (1) that element Y_{jj} can be measured by applying a 1 p.u. voltage to terminal j with the remaining terminals grounded. The current flowing from the voltage source into terminal j then equals Y_{jj}. Element Y_{ij} equals the current flowing from ground into terminal i.

A very careful measurement setup is required in order to obtain a sufficiently accurate result. Fig. 1 shows the measurement setup which comprises a network analyzer, a connection box with built-in current sensor, and measurement cables between the connection box and the transformer. The results shown in this paper were obtained using a preliminary version of this setup, which is described in full detail in [9]. All measurement were done on the connection box, thus producing a model of the transformer *and* the measurement cables. Reference [9] shows how to mitigate the effect of the measurement cables, but this was not done for the results shown in the current paper.

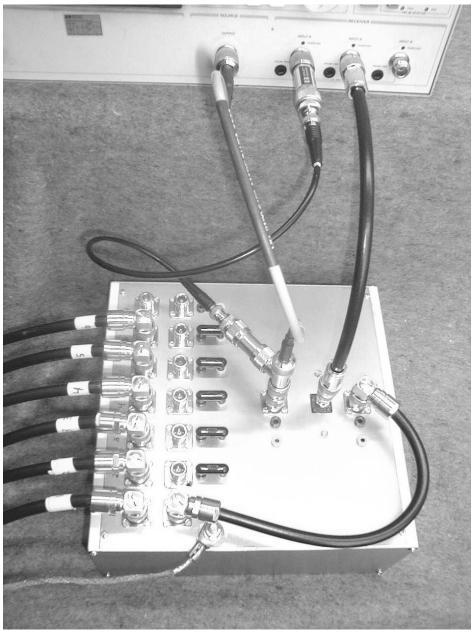

Fig. 1 Measurement setup

IV. MEASUREMENT VALIDATION

It is recommended to make additional measurements so as to validate the accuracy of the measured Y-matrix. A first test is to verify that Y is symmetric. In addition, one should make measurements with different terminal conditions. E.g., one can measure the open circuit voltage ratio between windings, from high to low, and from low to high. For a 2-winding transformer this gives 3x3 matrices V_{LH} and V_{HL}, respectively. Next, one can calculate V_{LH} and V_{HL} from Y and compare with the directly measured quantities.

The calculation is done as follows. Consider a 2-winding transformer partitioned into 3x3 blocks:

$$\begin{bmatrix} I_H \\ I_L \end{bmatrix} = \begin{bmatrix} Y_{HH} & Y_{HL} \\ Y_{LH} & Y_{LL} \end{bmatrix} \cdot \begin{bmatrix} V_H \\ V_L \end{bmatrix} \tag{4}$$

From (4) we get

$$V_{HL} = -Y_{HH}^{-1} \cdot Y_{HL} \tag{5}$$

$$V_{LH} = -Y_{LL}^{-1} \cdot Y_{LH} \tag{6}$$

V. APPLICATION TO 3-WINDING TRANSFORMER

A. Measurements

As an example we consider a 2 MVA 3-winding rectifier transformer, see fig. 2. Because the transformer was available for only a very short time, it was decided to create a model with respect to only two windings, with the terminals of the third winding (3 kV) being open. The neutral of the 2 kV winding was grounded during the measurements.

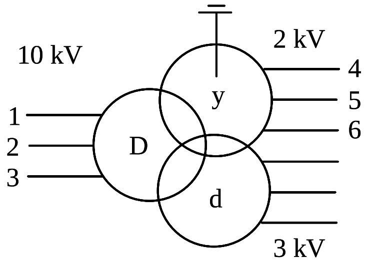

Fig. 2 3-winding transformer

The 6x6 admittance matrix was measured with respect to the 10 kV and 2 kV winding, giving 401 linearly spaced samples between 1 kHz and 1 MHz. The 36 measured elements are shown in fig. 3.

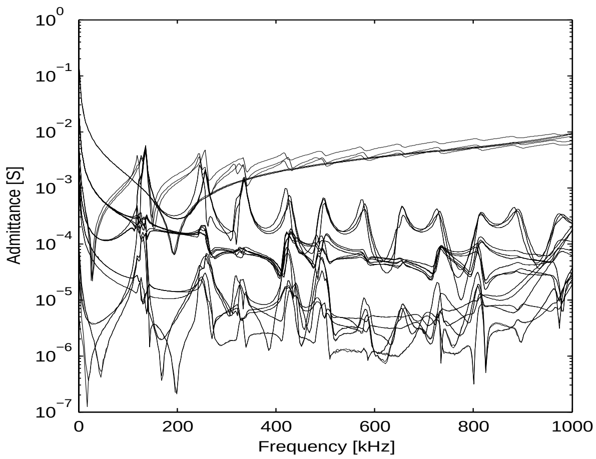

Fig. 3 Measured elements of Y

Symmetry was enforced for Y, and the resulting accuracy was assessed by calculating the voltage ratio between the 10 kV winding and the 2 kV winding by (5) and (6). The voltage ratio was next measured and a direct comparison was made.

Fig. 4 shows the calculated and the measured voltage ratio from high to low (9 elements). It is seen that a very good agreement has been obtained, including the small elements.

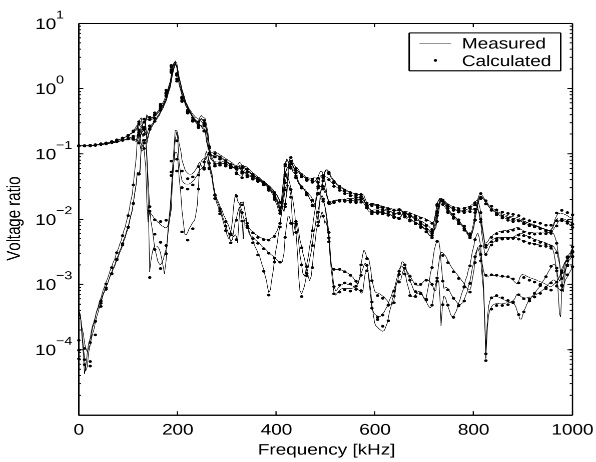

Fig. 4 Voltage ratio from high to low (V_{LH})

Fig. 5 shows the corresponding result for the voltage ratio from low to high. Again, a good agreement has been achieved.

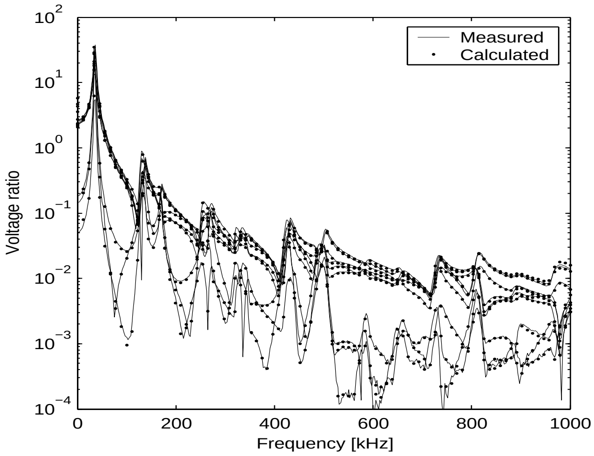

Fig. 5 Voltage ratio from low to high (V_{HL})

Fig. 6 shows an expanded view of the low frequency area in fig. 5. A significant discrepancy can be observed below 10 kHz. This deviation is caused by the 10 kV winding being ungrounded. This causes the submatrix Y_{HH} in (5) to get a high condition number (ratio between largest and smallest singular value) due to a very small zero sequence current at low frequencies, thus causing the measurement errors to be magnified by the matrix inversion. Fig. 7 shows the condition number of submatrices Y_{HH} and Y_{LL} as function of frequency. Reference [10] shows how to overcome this accuracy problem by separate measurement and modeling of the zero-sequence system. That method was however not available at the time when the measurements on the 3-winding transformer were carried out.

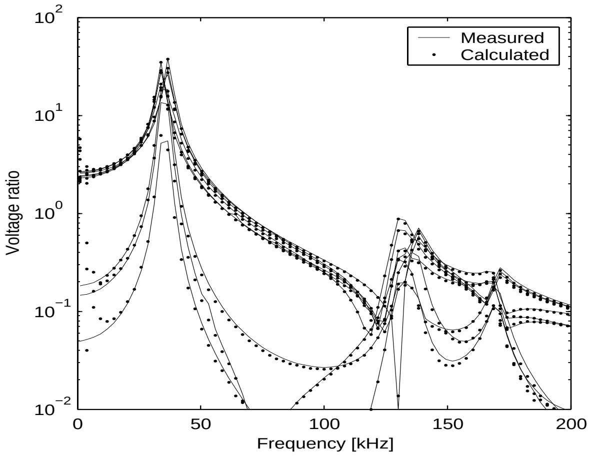

Fig. 6 Voltage ratio from low to high (V_{HL})

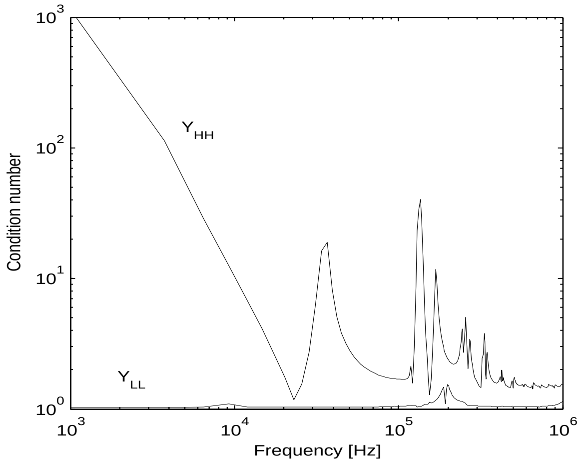

Fig. 7 Condition number for Y_{HH} and Y_{LL}

B. Rational approximation

The Matrix Fitter [12] was applied to Y using 60 poles per column, with 3 iterations and enforcement of symmetry. Each element in Y was in the fitting process weighted with the inverse of its magnitude. The resulting approximation is shown in fig. 8. It can be seen that the rational approximation gives a very good agreement with the measurements.

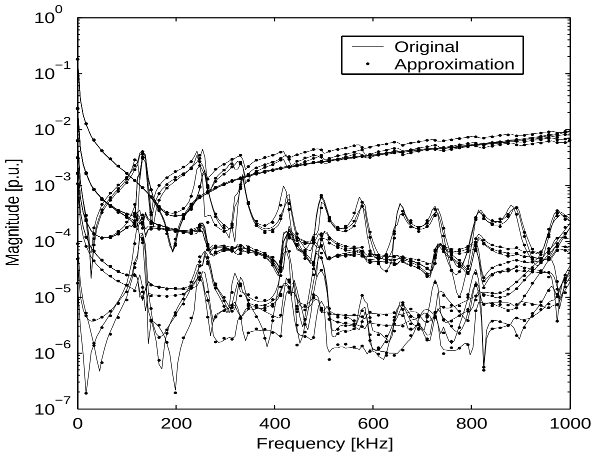

Fig. 8 Rational approximation of Y

C. Time domain validation

The rational approximation was included in an EMTP-type program as a Norton equivalent with history sources updated using convolution with trapezoidal integration.

Fig. 9 shows a case where terminal 1 of the HV-winding is energized by a step voltage with the other terminals of that winding grounded. The applied voltage was measured and taken as input in the simulation by an ideal voltage source. Fig. 10 compares the measured and simulated voltages on terminals 4,5,6 on the low-voltage winding. A very good agreement can be observed, including the small voltage on terminal 5.

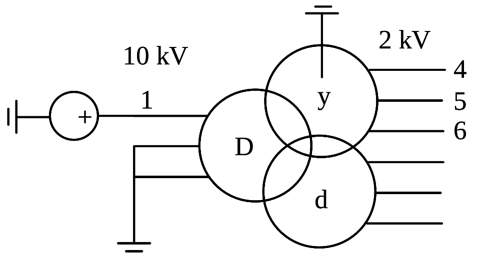

Fig. 9 Excitation of high voltage winding

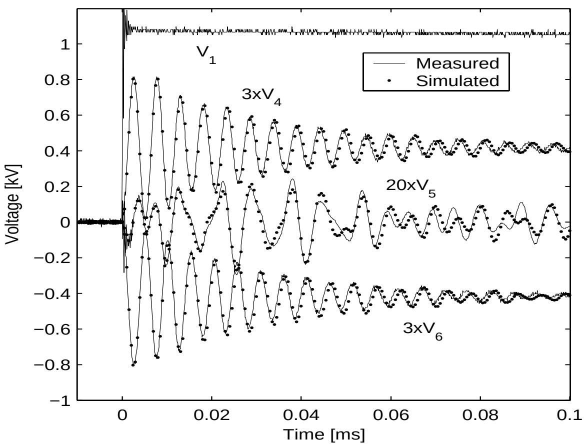

Fig. 10 Measured and simulated responses

Fig. 11 shows a case where a step voltage is applied to terminal 4 of the low voltage winding with terminal 5 open and terminal 6 grounded. All terminals of the high-voltage winding are open. The applied voltage was measured and taken as input in the simulation by an ideal voltage source. Fig. 12 compares the measured and simulated voltages on terminals 1,2,3 of the high-voltage winding. Although a significant deviation emerges over time, the simulation captures the major characteristic in terms of peak value, attenuation, and fundamental frequency components.

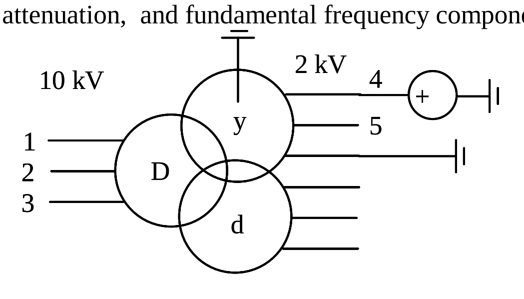

Fig. 11 Excitation of low voltage winding

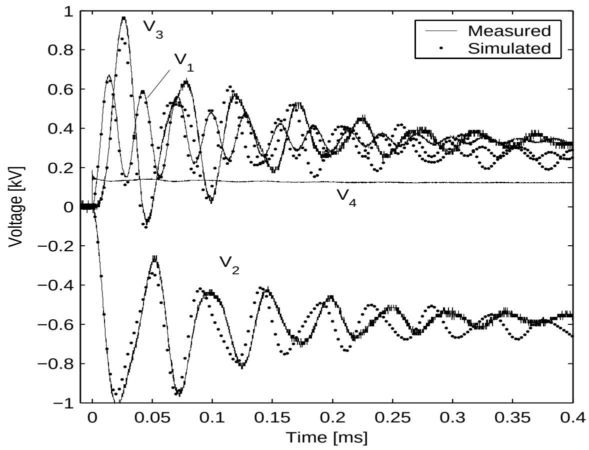

Fig. 12 Measured and simulated responses

VI. Application to winding Assembly

In principle, the voltage at any point in a transformer can be included in the model by introducing an additional row and column in Y, provided that the point is available for measurements. Some points, e.g. taps, will never be connected to an external network. In such situations it is more efficient to calculate the voltage on that point using the voltage transfer function between the transformer terminals and the additional point [14]. For a two-winding transformer, the voltage V_x is calculated as

$$V_x = H \cdot V \tag{7}$$

where H is a 1x6 row matrix and V is a 6x1 column vector containing the terminal voltages.

As an example a free-standing winding was considered which contained an internal cylinder to represent the capacitive coupling to the core, see fig. 13. Measurement taps were available so that the voltage at internal points could be measured.

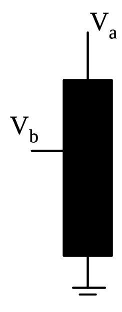

Fig. 13 Internal point (b) in winding

Fig. 14 shows the measured transfer function V_b/V_a from the top to the internal point as obtained by a network analyzer with logarithmic spaced frequency samples. Also is shown a 30th order rational function approximation calculated by Vector Fitting. A very good agreement can be observed.

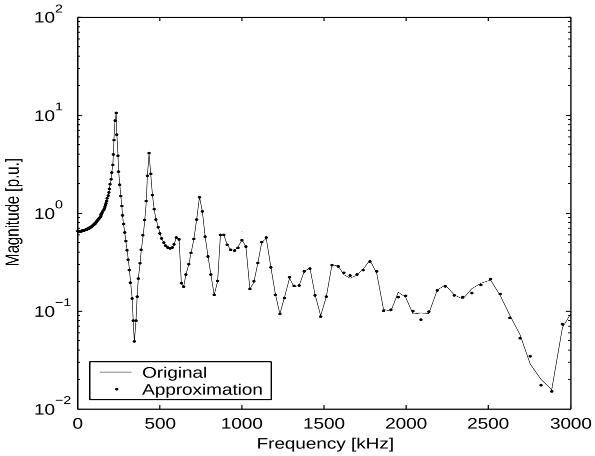

Fig.14 Rational approximation of transfer function V_b/V_a

Fig. 15 shows with solid trace the measured voltage on terminal b when applying a near step voltage to terminal a. The measured voltage on terminal a was taken as a known quantity and the voltage on terminal b was simulated, shown with dotted line. A very good agreement can be observed. Fig. 16 shows the initial part of the response. (All measurements were done using an active, high impedance voltage probe. The setup in fig. 1 was not used).

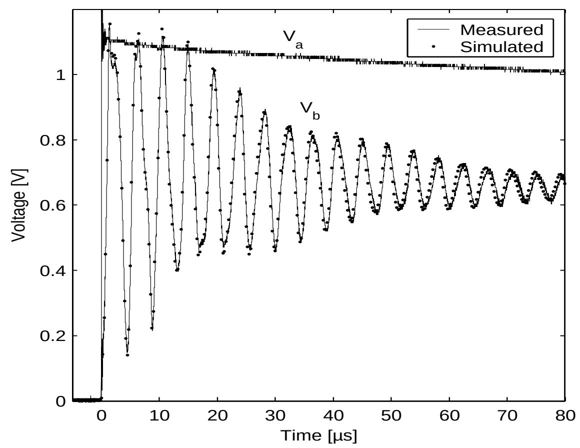

Fig.15 Measured and simulated response

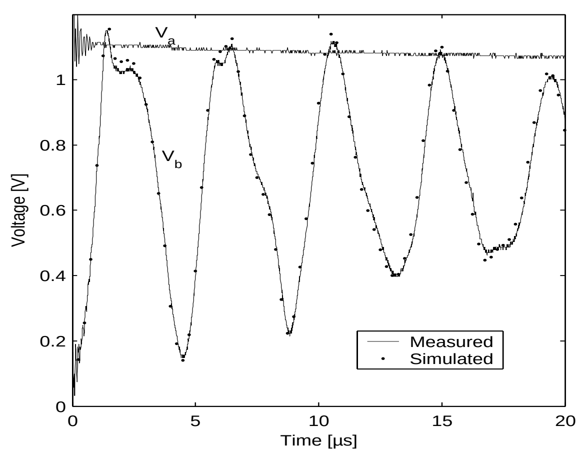

Fig.16 Expanded view of initial response

VII. Conclusions

This paper has demonstrated the suitability of Vector Fitting (VF) for high frequency modeling of power transformers. A terminal model was first obtained by measuring the admittance matrix *Y* in the frequency domain using a network analyzer. *Y* was next subjected to rational approximation using VF, thereby producing a realization which is compatible with EMTP-type simulation programs. Application to a 3-winding transformer demonstrated the accuracy of the procedure. It was also shown how to apply VF for the calculation of transferred overvoltages at open points/terminals. Both the VF routine and the matrix fitter routine are freely available from the author in Matlab code.

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