An Equivalent Circuit for Analysis of Lightning-Induced Voltages on Multiconductor System Using an Analytical Expression

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Abstract--This paper proposes an equivalent circuit at a transition point of distribution lines for lightning-induced voltage analysis. The lightning-induced voltages caused by a piecewise linearly varying lightning current are expressed by analytical formulas. The equivalent circuit is represented by resistors and current sources known from past history and given by the lightning-induced voltage on an infinite-length line. The transient voltages and currents at the transition point are accurately obtained in less computation time. Thus, the proposed equivalent circuit is very convenient for analyzing the lightning induced effect in complicated electric power networks.

Keywords: lightning-induced voltage, equivalent circuit, analytical expression, lightning protection design

I. Introduction

LIGHTNING-INDUCED voltage due to indirect lightning stroke sometimes causes line outages. Accordingly, the insulation design of distribution lines should consider the lightning-induced effect. The lightning-induced voltage on the distribution line, therefore, has been investigated [1]. The distribution line has many power apparatuses and branches, and the span length between the transition points of the distribution line is extremely shorter than that of the transmission line. Accordingly, many transition points should be considered for lightning surge analysis. Equivalent circuits for the lightning-induced voltage analysis, therefore, have been studied to evaluate the overvoltge [2-5].

There are three famous coupling models for the evaluation of the lightning-induced voltage based on the transmission line approximation [1]. The Agrawal model [6] is rigorous and the most adequate. On the other hand, the Rusck model [2] has been widely used to analyze the lightning-induced effect. Sakakibara extended the Rusck model to analysis of the case of inclined lightning channel [7]. The Rusck model is identical to the Agrawal model even on a finite length line [8], when the ground and the line are perfectly conducting, and the return stroke model is assumed to be the transmission line model [9]. The Rusck model gives an analytical expression of the lightning-induced voltage due to the above assumptions. Thus, this paper adopts the Rusck model.

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Line equations for the lightning-induced voltages include distributed sources generated by electromagnetic fields due to return stroke current in the lightning channel. It is impossible to obtain general solution of the line equations in time domain with no approximation. The EMTP is a powerful tool to estimate transient voltages in large electrical circuit, but treats induced voltages on a parallel line. On the other hand, the lightning-induced voltages are caused by the return stroke currents on the channel which is perpendicular to the line. The finite difference method is frequently used to solve the line equations for lightning-induced effect [10]. The method, however, requires division of the line into many sections. As a result, it takes much computation time, and the calculated results sometimes show poor accuracy because the number of sections is often lacking. The frequency-domain method accurately considers frequency dependence and obtains the general solution in frequency domain, but has a difficulty of such the nonlinear phenomena as surge arresters duty. The finite difference method also shows numerical instability when many surge arresters operate. Accordingly, a more simplified and efficient simulation method is needed for lightning-induced voltage analysis. It is apparent that an analytical formula in time domain gives an exact solution, and does not include the above difficulties.

The authors derived an analytical formula for the lightning-induced surges on an infinite-length line generated by a piecewise linearly varying return stroke current based on the Rusck model [11]. The formula is very simple, and can be calculated by using a pocket calculator. Therefore, the formula is convenient for sensitive lightning surge analysis.

This paper proposes an equivalent circuit for the analysis of the lightning-induced voltage at a transition point using the analytical expression. The validity of the equivalent circuit is confirmed by comparing the calculated results of the proposed equivalent circuit with the finite difference method.

II. Analytical Formula of Lightning-Induced Surges

A. Assumptions

The following conditions are adopted in this paper.

(a) Conductivity of the line and the ground is infinite.
(b) The velocity of the return stroke current is constant.
(c) The return stroke current develops upward from the ground to a cloud with no wave deformation and no attenuation.
(d) The lightning channel is located perpendicularly to the ground plane.

(e) t=0 is defined as the time when the return stroke current starts developing.

Assumptions (b) and (c) correspond to the transmission line model of the return stroke.

The return stroke current $i(t)$ is approximated by piecewise linear characteristics given by:

$$i_R(t) = \sum_{k=1}^{N}(\alpha_{k+1} - \alpha_k)(t - T_k)u(t - T_k) \tag{1}$$

$$\alpha_k = \frac{I_{k+1} - I_k}{T_{k+1} - T_k} \tag{2}$$

where α_k: tangent of the approximate return stroke current on $[T_k, T_{k+1}]$, I_k: instantaneous current at $t=T_k$, $u(t)$: unit function.

B. Rusck model

The line equations on the basis of the Rusck model are given by:

$$\frac{\partial V_s}{\partial x} = -L\frac{\partial I_t}{\partial t} \tag{3}$$

$$\frac{\partial I_t}{\partial x} = -C\frac{\partial (V_s - e_s)}{\partial t} \tag{4}$$

$$U_x = V_s + e_m \tag{5}$$

$$e_s = \int_0^h \left(\frac{\partial V_i}{\partial z}\right)_{z=0} dz \tag{6}$$

$$e_m = \int_0^h \left(\frac{\partial A_i}{\partial t}\right)_{z=0} dz \tag{7}$$

where L and C: line inductance and line capacitance per unit length respectively, V_s: induced scalar potential, e_s: inducing scalar potential, e_m: inducing vector potential, U: lightning-induced voltage, I: line current h: height of conductor, V_i, A_i incident scalar and vector potentials generated by return stroke current.

C. Analytical expressions of lightning-induced surges

Fig. 1 illustrates a configuration of a line and a return stroke. The lightning-induced voltage $U(x, t)$ and the induced current $I(x, t)$ at an arbitrary point x, where the lightning channel located at x=0, on an infinite-length line due to a vertical lightning stroke is expressed by [11]:

$$U = U_1 + U_2 \tag{8}$$

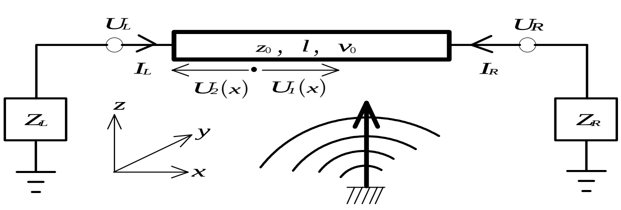


Fig. 1. Configuration of a line and a return stroke.

$$I = \frac{U_1 - U_2}{z_0} \tag{9}$$

$$U_n = V_{sn} + \frac{1}{2}e_m \tag{10}$$

where z_0: surge impedance of the line, n=1: forward component, n=2: backward component.

$$U_1, U_2 = \alpha\frac{30h}{v}\left\{\ln\left[\frac{\beta}{t_0 \mp t_x}\bullet\frac{(t\mp t_x)^2 + (t_y/\beta)^2}{\sqrt{t^2+\zeta t_0^2} + \beta(t\pm\zeta t_x)}\right] + \beta\ln\frac{t+\sqrt{t^2+\zeta t_0^2}}{t_0 + t_0/\beta}\right\} \tag{11}$$

$$U(x,t) = \alpha\frac{30h}{v}\left\{\ln\left[1+\left(\frac{\beta}{t_y}.\frac{t^2-t_0^2}{t+\beta\sqrt{t^2+\zeta t_0^2}}\right)^2\right] + 2\beta\ln\frac{t+\sqrt{t^2+\zeta t_0^2}}{t_0+t_0/\beta}\right\} \tag{12}$$

$$I(x,t) = \alpha\frac{30h}{vz_0}\ln\left[\frac{t_0+t_x}{t_0-t_x}\bullet\frac{(t-t_x)^2+(t_y/\beta)^2}{\left(\sqrt{t^2+\zeta t_0^2}+t_x/\beta\right)^2+t_y^2}\right] \tag{13}$$

where v: the return stroke velocity, v_0: light velocity in free space, h: the line height, y: the distance between the line and the return stroke, $\beta=v/v_0$, $\zeta=(1-\beta^2)/\beta^2$, $t_x=x/v_o$, $t_y=y/v_o$, $t_0 = \sqrt{t_x^2 + t_y^2}$.

Equations (11) to (13) indicate that the lightning-induced surges on an infinite-length line based on the Rusck model are expressed by logarithmic function, and are very simple.

III. Equivalent Circuit of Distribution Line for Lightning-Induced Voltage Analysis

Rusck proposed a calculation method of the lightning-induced voltage on a finite-length line by using the super-position theorem. The line is equivalent to injecting a current into the line with the same magnitude as, but with opposite direction to, the one flowing in the line before disconnection, namely infinite-length line, at the disconnection point [2]. According to the above method, an equivalent circuit of a line illuminated by the electromagnetic field generated by a return stroke current is represented by a distributed-parameter circuit neglecting the influence of the electromagnetic field, lightning-induced voltage $U_t(x, t)$ and current $I_t(x, t)$ on the infinite-length line at a transition point as illustrated in Fig. 2.

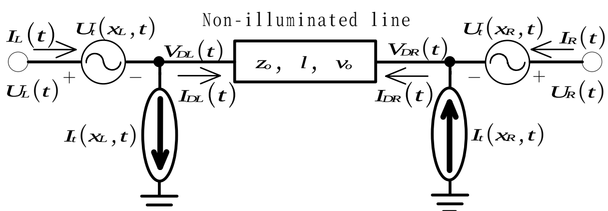


Fig. 2. An equivalent circuit of line illuminated by indirect lightning stroke.

The distributed-parameter circuit can be modeled by the equivalent current sources expressed by the node voltage and the current at the opposite terminal of the line, and the terminals are topologically independent. This line model is proposed by Dommel [12]. The circuit illustrated in Fig. 3 is obtained by replacing the distributed-parameter circuit of Fig. 2 with the Dommel's line model.

Then, applying Norton's theorem to the circuit of Fig. 3, another representation of the equivalent circuit is obtained as shown in Fig. 4. The equivalent circuit consists only of current sources and resistors, and can be directly installed into the EMTP. Thus, lightning-induced voltage analyses could be easily carried out in more complicated distribution line systems by using the proposed equivalent circuit.

The node voltage and the injected current at the resistor z_0 of Fig. 3 differ from those in Fig. 4. However, $J_R(t\text{-}T)$ and $J_L(t\text{-}T)$ of Fig. 4 must coincide with those of Fig. 3. Also, $U_L(t)$, $I_L(t)$, $U_R(t)$ and $I_R(t)$ of Fig. 3 must coincide with those of Fig. 4. From these relations, the current sources in Fig. 4 are given in the following equations.

$$
\begin{aligned}
&J_R(t\text{-}T)=\text{-}V_{PR}(t\text{-}T)/z_0\text{-}I_{PR}(t\text{-}T)+2U(x_R,\ t\text{-}T)/z_0\\
&J_L(t\text{-}T)=\text{-}V_{PL}(\ t\text{-}T)/z_0\text{-}I_{PL}(t\text{-}T)+2U(x_L,\ t\text{-}T)/z_0\\
&I_{NL}(t)=U(x_L,\ t)/z_0\text{-}I(x_L,\ t)=2U_2(x_L,\ t)/z_0\\
&I_{NR}(t)=U(x_R,\ t)/z_0+I(x_R,t)=2U_1(x_R,\ t)/z_0
\end{aligned}
\tag{14}
$$

where $T=l/v_0$, l: line length, x_R, x_L: x-coordinates at the right end and the left end

$J_R(t\text{-}T)$ and $J_L(t\text{-}T)$ are denoted using the node voltage and the injected current at the transition point as follows:

$$
\begin{aligned}
&J_R(t\text{-}T)=\text{-}U_R(t\text{-}T)/z_0\text{-}I_R(t\text{-}T)+2U_2(x_R,\ t\text{-}T)/z_0\\
&J_L(t\text{-}T)=\text{-}U_L(t\text{-}T)/z_0\text{-}I_L(t\text{-}T)+2U_1(x_L,\ t\text{-}T)/z_0
\end{aligned}
\tag{15}
$$

Appendix describes a circuit, which can consider an initial condition of the line in the EMTP.

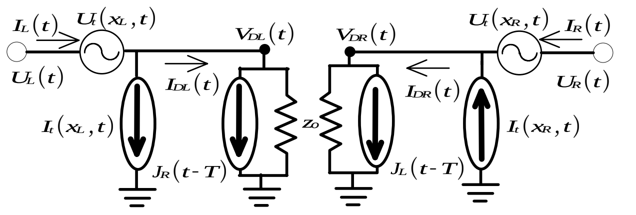


Fig. 3. An equivalent circuit including Dommel's line model of line illuminated by indirect lightning stroke.

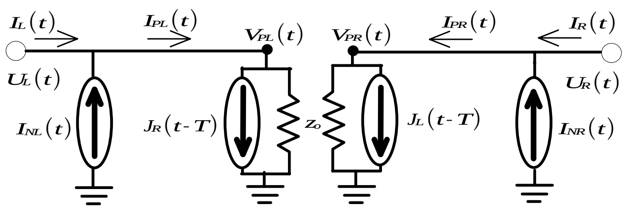


Fig. 4. A proposed equivalent circuit of line for lightning-induced voltage analysis.

IV. Equivalent Circuit for Multiconductor

Line equations for multiconductor are given by the same expression as (3) to (5) using matrix and vector variables. The modal theory [13] is useful to solve the line equations. Equations (3) and (4) in the modal domain are given by:

$$\frac{\partial \mathbf{V}_{sm}}{\partial x} = -\mathbf{L}_m \frac{\partial \mathbf{I}_{tm}}{\partial t} \tag{16}$$

$$\frac{\partial \mathbf{I}_{tm}}{\partial x} = -\mathbf{C}_m \frac{\partial (\mathbf{V}_{sm} - \mathbf{e}_{sm})}{\partial t} \tag{17}$$

where $\boldsymbol{U}=\boldsymbol{T}_V\boldsymbol{U}_m$, $\boldsymbol{I}=\boldsymbol{T}_I\boldsymbol{I}_m$, $\boldsymbol{T}_V$ $\boldsymbol{T}_I$: voltage and current transform matrices respectively, m: suffix for modal variable.

Equations (16) and (17) yields:

$$\frac{\partial^2 \mathbf{V}_{sm}}{\partial x^2} = \mathbf{L}_m \mathbf{C}_m \frac{\partial^2 (\mathbf{V}_{sm} - \mathbf{e}_{sm})}{\partial t^2} \tag{18}$$

$\mathbf{L}_m\mathbf{C}_m$ is a diagonal matrix, and variables in modal domain can be treated as single conductors. When the conductivity of the soil and the conductor is finite, the inductance matrix $\mathbf{L}$ is different from that for infinite conductivity, and there are some surge propagation velocities along the line. However, lightning surge analysis of distribution line usually treats short lines, and the soil resistivity affects propagation characteristics on overhead lines a little. The velocity can be assumed to be constant, and lightning induced voltage on a conductor is not affected by the other conductors considering $\mathbf{LC}$ is a diagonal matrix. Therefore, the analytical formulas and the method described in the last chapter are applicable to the multiconductor system. Figs. 5 and 6 illustrate the equivalent circuit of the multiconductor.

V. Validation of Equivalent Circuit

A. Comparison of the Proposed Method with Finite Difference Method

The finite difference method is frequently used to solve line equations. The time variable t and the position variable x are discredited as Δt and Δx respectively. The finite difference method transforms the line equations (3) to (5) into the

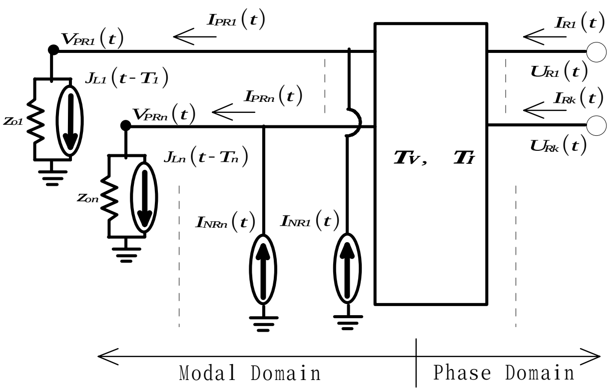


Fig. 5. An equivalent circuit including Dommel's line model of multi-conductor illuminated by indirect lightning stroke.

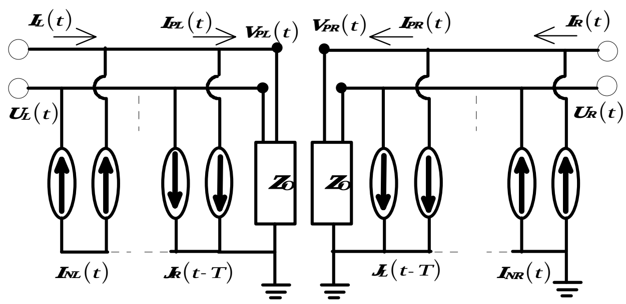


Fig. 6. A proposed equivalent circuit of multiconductor illuminated by indirect lightning stroke.

following algebraic equations:

$$\frac{V_s(x_{i+1}, t+\Delta t) - V_s(x_i, t+\Delta t)}{\Delta x} = -L\frac{I(x_i, t+\Delta t) - I(x_i, t)}{\Delta t} \tag{19}$$

$$\frac{I_s(x_{i+1}, t) - I(x_i, t)}{\Delta x} = -C\frac{V_s(x_i, t+\Delta t) - V_s(x_i, t)}{\Delta t} - C\frac{\partial e_s}{\partial t} \tag{20}$$

$$U(x_i, t+\Delta t) = Vs(x_i, t+\Delta t) + e_m \tag{21}$$

and solutions are obtained by the iterative procedure. The finite difference method is equivalent to the π-circuit representation of a distributed-parameter circuit. Therefore, a larger number of divided sections are necessary with a longer line and higher frequency [14].

To compare the proposed method with the finite difference method, the accuracy are examined for the variation of Δt and Δx in the circuit of Fig. 7 with $R_p=R_s=z_o$. Table 1 summarizes the accuracy of the crest voltage at node P. The accuracy is evaluated against theoretical solutions (Table (b)) as a percentage of calculated results in the crest voltage (upper figure in each block in Table (a)) and in the time to crest voltage (lower figure in each block). Symbol * in the table indicates that the condition of the finite difference method below [10]:

$$\frac{\Delta x}{\Delta t} > v_0 \tag{22}$$

is not satisfied, and numerical divergence occurs. The line length is adjusted to $(2n+1)\Delta x/2$ because the section length at the end of the line is $\Delta x/2$ in the finite difference method. The

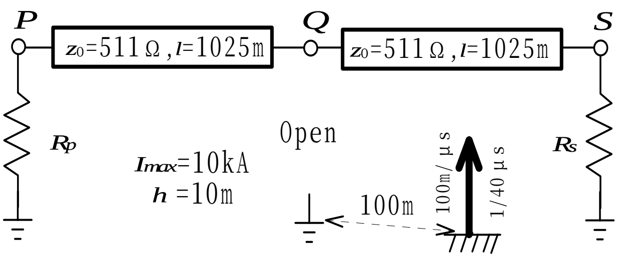


Fig. 7. Calculation circuit.

return stroke is located in front of the center of the line with y=100m, and the return stroke current with a triangle waveform of 1/40μs and a crest value of 10kA develops with 1/3 of the light velocity.

Table 1(a) shows that the accuracy of the finite difference method becomes poorer as Δx becomes larger. On the other hand, the proposed method shows good accuracy. The error of the proposed method is derived from the fact that the time to crest is not equal to $k\Delta t$.

Fig. 8 shows calculated waveforms in case of Δt=0.05μs. From Fig. 8, the calculated waveforms of the proposed method agree well with those of the finite difference method. The error of the crest voltage at node Q for both methods is less than 0.2%. An oscillation caused by the lack of high-frequency components due to the small number of divided sections, however, is observed in case of Δx=100m in the finite difference method after the reflection surges from the line ends arrived. Thus, it is clear that the proposed method gives a stable solution.

TABLE 1. COMPARISON OF FINITE DIFFERENCE AND PROPOSED METHOD.

(A) ACCURACY

		Δt [μs]			
		0.005	0.01	0.05	0.1
Finite difference method	Δx= 25m	0.39% 0.24%	0.39% 0.14%	0.29% 0.46%	*
	Δx=50m	2.13% 0.32%	2.13% 0.42%	2.04% 0.62%	1.57% 0.38%
	Δx=100m	6.93% 2.30%	6.92% 2.30%	6.80% 2.90%	6.51% 1.90%
Proposed method		0.00% 0.02%	0.00% 0.02%	0.02% 0.38%	0.02% 0.38%

(B) THEORETICAL SOLUTION

	Δx (l) [m]		
	25 (2025)	50 (2050)	100 (2100)
Crest value [kV]	28.6212	28.6243	28.6299
Time to crest [μs]	4.977	5.019	5.102

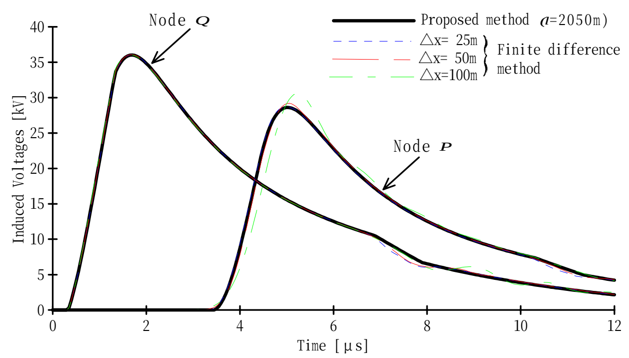


Fig. 8. Comparison of calculated results by the proposed and finite difference methods.

Fig. 9(b) shows calculated results of node voltage at nodes *P*, *Q* and *S* illustrated in Fig. 9(a) using the proposed and the finite difference methods in case of Δt=0.01μs, Δx=50m, z_0= 511Ω.

It is clear from Fig. 9 that the calculated results using the proposed method agree well with those using the finite difference method even in case of a non-matching finite-length multiconductor.

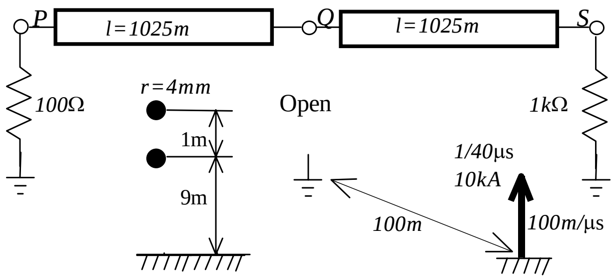
B. Experimental Results of Distribution line with Branched Lines

Figs. 10(a) and (b) illustrate an experimental circuit of a branched distribution line model and an approximated return stroke current model, respectively [15]. Experimental results and calculated results using the proposed method are shown in Figs. 10(c) and (d), respectively.

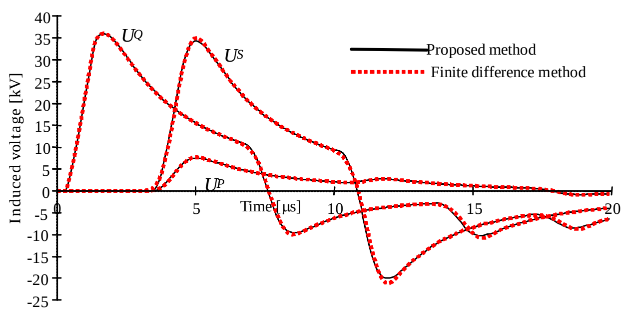
The calculated results by the proposed method agree comparatively with the experimental results. It is clear that the proposed method is applicable to a branched distribution line.

VI. Conclusions

An equivalent circuit of a distribution line for transition point transients calculation against indirect lightning stroke using an analytical expression of lightning-induced voltages on an infinite-length line based on the Rusck model has been proposed. The equivalent circuit consists of Dommel's line model and current sources expressed by the proposed analytical formula. Accordingly, it is easy to introduce the equivalent circuit into the EMTP. The equivalent circuit has advantages in computation time and accuracy in comparison to the finite difference method. Thus, the proposed equivalent circuit is applicable to and suitable for lightning-induced voltage analysis in a complicated distribution line system. Influence of finite soil conductivity can be considered by introducing additional current source, which depends on the conductivity [16].



(a)

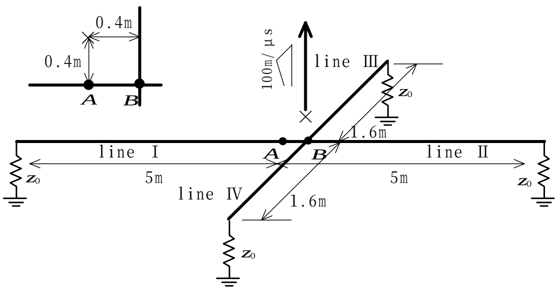


(b)

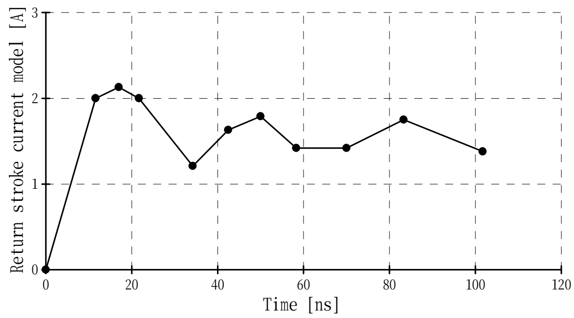
Fig. 9. Comparison of calculated results by the proposed method with the finite difference method. (a) Calculation circuit. (b) Calculated waveforms.

VII. Appendix

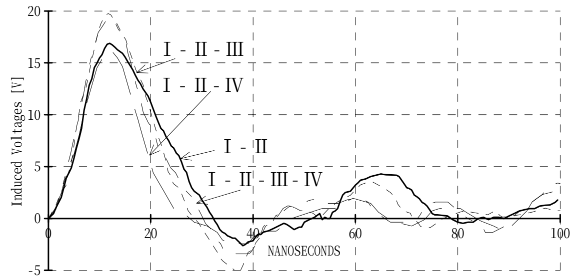
Distributed-parameter circuit is replaced by a π circuit to calculate initial voltage and current distribution in the EMTP.



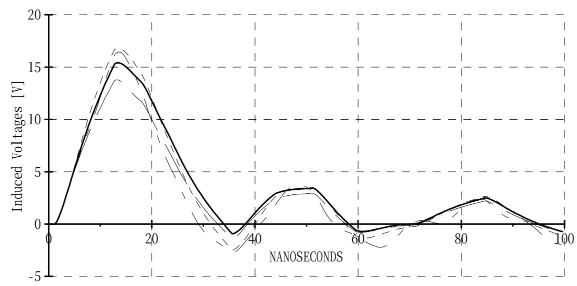
(a)



(b)



(c)



(d)

Fig. 10. Comparison between experimental results and proposed method for branched line. (a) Experimental circuit. (b) Return stroke current model. (c) Experimental results. (d) Calculated results.

The use of line models in the EMTP has advantage in the consideration of the initial conditions. A voltage source between nodes can be replaced by two current sources with opposite polarity and a resistor. Fig. 11 shows the equivalent circuit, which corresponds to Fig. 2, using the EMTP line model and current sources. The resistor R is chosen so that the value is sufficient smaller than the line surge impedance and the external circuit impedance.

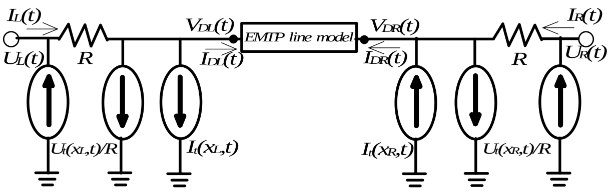


Fig. 11. An equivalent circuit for analysis of lightning-induced voltage using the Dommel line model prepared in the EMTP.

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IX. Biographies

Shozo Sekioka was born in Osaka, Japan on December 30, 1963. He received the B. Sc., and D. Eng. degrees from Doshisha University in 1986 and 1997, respectively. He joined Kansai Tech Corp. in 1987, and is an assistant professor of Shonan Institute of Technology from 2005. He has been engaged in the lightning surge analysis in electric power systems.