A Comparison of Transient Simulation with EMTDC and State Space Diakoptical Segregation Methodology

K. K.C. Yu, *Student Member, IEEE* and N. R. Watson, *Senior Member, IEEE*

Abstract—The use of diakoptical technique can considerably reduces the computational burden when formulating network equations. Nodal analysis with diakoptical segregation of the plant component can efficiently modifies the necessary equations when the system undergoes topological changes. It avoids changes of the whole network equations and only the relevant parts of the network equations are considered. This particular benefits simulations that undergo frequent switching. In this paper, a state-space diakoptical segregation method is applied to transient simulation of a.c systems under different fault conditions.

For comparison, the transient response of the proposed technique is compared with the EMTDC simulation. Two test scenarios based on the Lower South Island of the New Zealand system were set up. The scenarios looked at the performance of the proposed method under symmetrical and asymmetrical line-to-ground faults.

Keywords: Electromagnetic transients, Transient simulation, State variable equations

I. Introduction

DIGITAL simulations in the time domain via electromagnetic transient program (EMTP, EMTDC or ATP) has become the industrial standard. They play a critical part in design and operation of modern power systems where analytical solution is prohibitive. These simulations provide the basis for; system control testing, equipment protection design, performance testing under disturbance/fault conditions to name but a few. Transient simulation based on state space theory [1]-[3] were once popular, however the high computation cost has caused interest to diminish even though it has many advantages [4]-[5]. One such methodology specifically developed to analyse the dynamic behavior of HVDC systems is the Transient Converter Simulation (TCS) program [6]. A diakoptically based nodal approach provides an ideal environment for the analysis of system with frequently switching components such as converters. It avoids involving the whole network in unnecessary topological changes and only modifies the relevant network equations to represent the new system state. Furthermore, the nodal approach becomes more efficient if sparsity of coefficient matrices is used to improve computational efficiency.

The general state variable approach requires the identification of *independent* state variables and formulation of the appropriate equations [7]. Arbitrary use of inductor current and capacitor voltage is not sufficient due to the possible presence of inductor cut-sets or loops of capacitors and voltage sources. Use of graph theory or linear matrix methods can find the appropriate state variables. However, when the system is undergoing topological changes due to frequent switchings, this identifying of state variables and formation of state equations is not practical due to the computational effort to identify independent state variables and form the state equations. The diakoptically segregation of the sub-network presented avoids the need to identify *independent* state variables. In the present approach a diakoptical technique is used as an efficient method of forming the state equations for systems which may have frequently switching components in them. There is a slight loss of generality, for example every capacitive sub-network must have a connection to ground, but this is minor to the enormous computational benefits.

In the TCS algorithm, the power system is represented by an equivalent network of lumped inductive, resistive and capacitive. The set of network equations are formulated from diakoptical segregation of the sub-network where the nodes are partitioned into three possible groups depending on what type of branches are connected to them. This results in a set of first order differential equations being developed and the system matrices in compact form containing their indexing information, assigns node numbers for branch elements and interconnection information. The state variable solution can then be obtained using numerical integration technique, such as implicit trapezoidal integration. The state equations are solved at each time-step iteratively, with no restriction imposed on the selection of the integration time step length. Thus, when changes in the system topology occur, the network equations, the connection matrix and the step length can be modified to fall exactly on the time of the changes to represent the new conditions.

In this paper, a new implementation of the TCS algorithm is applied to an a.c. system for prediction of transient response under symmetric and asymmetric fault conditions. Minimal modification of the network equations and connection matrix to

The work presented in this paper was sponsored by the Foundation for Research Science and Technology (FRST) Bright Future Scholarship. The authors would like to thank FRST for its financial support
The authors are with the Department of Electrical and Computer Engineering, University of Canterbury, Christchurch, New Zealand.

Presented at the International Conference on Power Systems Transients (IPST'05) in Montreal, Canada on June 19-23, 2005

Paper No. IPST05 - 063

cater for the new changes during fault is achieved by segregation of the sub-networks. To validate the proposed method, two different fault scenarios are simulated and the results are compared with those obtained using PSCAD/EMTDC.

II. TCS DIAKOPTICS FORMULATION

Consider a network with n nodes is systematically expanded into its elementary resistive r, inductive l, capacitive c and current source s branches. Diakoptical segregation of the network, subdivides the network nodes into three parts according to the type of branches connected to them:

α nodes: At least one capacitive branch.

β nodes: At least one resistive branch but no capacitive branches.

γ nodes: Only inductive branches.

The resulting branch-node incidence (connection) matrices for the r, l and c branches are K_{rn}^T, K_{ln}^T and K_{cn}^T respectively. The elements in the branch-node incidence matrices are determined as follows:

K_{bn}^T =1; If node n is the sending end of branch b

K_{bn}^T =-1; If node n is the receiving end of branch b

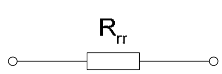
K_{bn}^T =0; If branch b is not connected to node n

By restricting the number of possible network configuration to those commonly encountered in practical systems, the efficiency of the solution can be improved significantly. The restrictions are [8]:

1. every capacitive branch sub-network has at least one connection to the system reference (ground node)
2. resistive branch sub-networks have at least one connection to either the system reference or an α node.
3. inductive branch sub-networks have at least one connection to the system reference or an α or β node.

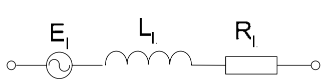
Neglecting current sources, the fundamental branches that result from the restrictions and their diakoptical network equations can be written as:

Resistive branches



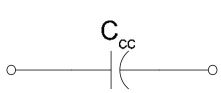
$$I_r = R_{rr}^{-1}\left(K_{r\alpha}^T V_\alpha + K_{r\beta}^T V_\beta\right) \quad (1)$$

Inductive branches



$$E_l - \frac{d}{dt}\left(L_{ll} I_l\right) - R_{ll} I_l + K_{l\alpha}^T V_\alpha + K_{l\beta}^T V_\beta + K_{l\gamma}^T V_\gamma = 0 \quad (2)$$

Capacitive branches



$$I_c = C_{cc} \frac{d}{dt}\left(K_{c\alpha}^T V_\alpha\right) \quad (3)$$

Applying the node type definitions, the nodal equations for each node type become:

$$\begin{aligned} &K_{\alpha c} + K_{\alpha r} + K_{\alpha l} = 0 \\ &K_{\beta r} + K_{\beta l} = 0 \\ &K_{\gamma l} = 0 \end{aligned} \quad (4)$$

Combining eqns. 1-4, a set of diakoptical network equations in the form of eqn. 5 and 6 can be formulated by taking capacitive node voltage V_α and inductive branch current I_l as the state variables. The state equations and dependent variable equations are expressed as in eqns. 7-11.

$$\dot{x} = [A]x + [B]u \quad (5)$$

$$y = [C]x + [D]u \quad (6)$$

State equation

$$\begin{bmatrix} \frac{dI_l}{dt} \\ \frac{dV_\alpha}{dt} \end{bmatrix} = \begin{bmatrix} -L_{ll}^{-1} M_{ll} R_{ll}^{'} & L_{ll}^{-1} M_{ll} A_{l\alpha}^T \\ -C_{\alpha\alpha}^{-1} A_{\alpha l} & -C_{\alpha\alpha}^{-1} G_{\alpha r} K_{r\alpha}^T \end{bmatrix} \begin{bmatrix} I_l \\ V_\alpha \end{bmatrix} + \begin{bmatrix} L_{ll}^{-1} M_{ll} \\ 0 \end{bmatrix} [E_l] \quad (7)$$

Dependent variables

$$V_\beta = -R_{\beta\beta}\left(K_{\beta l} I_l + K_{\beta r} R_{rr}^{-1} K_{r\alpha}^T V_\alpha\right) \quad (8)$$

$$\begin{aligned} V_\gamma = &-L_{\gamma\gamma} K_{\gamma l} L_{ll}^{-1}\left(-R_{ll} K_{l\beta}^T - R_{\beta\beta} K_{\beta l}\right) I_l + \ldots \\ &-L_{\gamma\gamma} K_{\gamma l} L_{ll}^{-1}\left(K_{\alpha l} - K_{l\beta}^T R_{\beta\beta} K_{\beta r} R_{rr}^{-1} K_{\alpha r}\right) V_\alpha + E_l \end{aligned} \quad (9)$$

$$\begin{aligned} I_r = &-R_{rr} K_{r\beta}^T R_{\beta\beta} K_{\beta l} I_l + \ldots \\ &R_{rr}^{-1}\left(K_{r\alpha}^T - K_{r\beta}^T R_{\beta\beta} K_{\beta r} R_{rr}^{-1} K_{r\alpha}\right) V_\alpha \end{aligned} \quad (10)$$

$$I_c = -C_{cc} K_{c\alpha}^T C_{\alpha\alpha}^{-1}\left(A_{\alpha l} I_l + G_{\alpha r} K_{r\alpha}^T V_\alpha\right) \quad (11)$$

where

$$M_{ll} = U_{ll} - K_{l\gamma}^T L_{\gamma\gamma} K_{\gamma l} L_{ll}^{-1}$$

$$A_{\alpha l} = K_{\alpha l} - K_{\alpha r} R_{rr}^{-1} K_{r\beta}^T R_{\beta\beta} K_{\beta l}$$

$$G_{\alpha r} = K_{\alpha r} R_{rr}^{-1}\left(U_{rr} - K_{r\beta}^T R_{\beta\beta} K_{\beta r} R_{rr}^{-1}\right)$$

$$C_{\alpha\alpha}^{-1} = \left(K_{\alpha c} C_{cc} K_{c\alpha}^T\right)^{-1}$$

$$L_{\gamma\gamma} = \left(K_{\gamma l} L_{ll}^{-1} K_{l\gamma}^T\right)^{-1}$$

$$R_{\beta\beta} = \left(K_{\beta r} R_{rr}^{-1} K_{r\beta}^T\right)^{-1}$$

$$R_{ll}^{'} = R_{ll} + K_{l\beta}^T R_{\beta\beta} K_{\beta l}$$

R, L and C are the resistance, inductance and capacitance matrices respectively. U_{rr} and U_{ll} are unit matrices of order r and l respectively

To solve for the state variables at each time step, implicit trapezoidal approximation is used owing to its good stability, accuracy and simplicity. The state equations can be written as in (12) and the change in the state variable Δx is defined in (13).

$$\dot{x} = f\left(I_l, V_\alpha\right) \tag{12}$$

$$\Delta x = \frac{h}{2}\left(\dot{x}_t + \dot{x}_{t+h}\right) \tag{13}$$

An iterative procedure can be used to determine x_{t+h} as follows:

1) For an initial estimates, it is assumed $\dot{x}_{t+h} = \dot{x}_t$

2) An intermediate x_{t+h} based on the $\dot{x}_{t+h}$ estimate is then calculated.

$$x_{t+h} = x_t + \frac{h}{2}\left(\dot{x}_t + \dot{x}_{t+h}\right)$$

3) An update of the $\dot{x}_{t+h}$ can be calculated from the intermediate x_{t+h} value from the state equation.

$$\dot{x}_{t+h} = [A]x_{t+h} + [B]u_{t+h}$$

Steps 2) and 3) are performed iteratively until convergence is reached. In the case that convergence fails, the step length is halved and the iterative procedure is restarted. Dependent state variables can be calculated at the end of each time step h or at the end of the simulation.

III. SIMULATION MODEL AND RESULTS

Each component in an a.c. power system can be modelled by its equivalent circuit model defined in terms of passive elements. These elementary branches form the basis of the state equations and dependent variable equations (3)-(7).

1. Generators: these are modelled by an e.m.f. source voltage with their equivalent R-R//L impedance.
2. Transformers: they are represented using two windings model depending on the type of magnetic circuit and on the connections of the terminals and the neutrals. i.e. delta or wye. Core losses are represented internally with an equivalent shunt resistance across each winding in the transformer.
3. Transmission lines: these are modelled rigorously by the three phase PI models and hence capable of incorporating non symmetric condition.
4. Loads: the real and reactive power components are represented by its equivalent resistance and inductance respectively.

The test system (Fig. 1) is taken from the Lower South Island of the New Zealand system. It consists of three generators, five delta/star-g transformers, three double circuit transmission lines, two single circuit transmission lines and three passive loads. The system contains 84 state variables correspond to the system's inductive branch currents and capacitive node voltages. For a credible validation of the proposed technique, the TCS based transient simulation is compared to those simulated using PSCAD/EMTDC. Two types of fault (three phase line-to-ground and single phase line-to-ground) are simulated at the fault location indicated by the black crosses in Fig. 1. The two crosses indicate the fault is simulated on a double cct. transmission line.

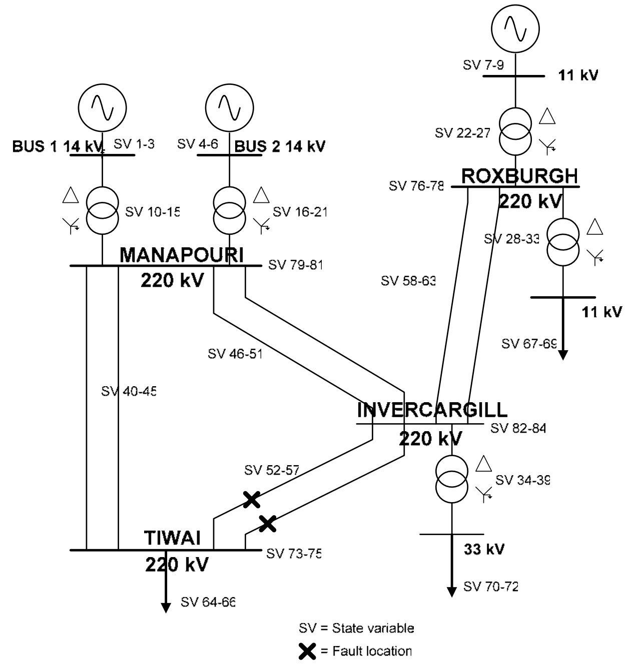


Fig. 1. Lower South Island of the New Zealand system

A. *Symmetrical Fault*

A duration of 2.5 cycles three-phase line-to-ground fault on the Invercargill-Tiwai transmission line is simulated. Complete collapse of all phase voltages at the receiving end and over-current at the corresponding sending and receiving end branches are expected.

Figs. 2-10 shows the Tiwai 220kV busbar voltages, Invercargill-Tiwai sending end branch currents and the fault currents. The TCS and PSCAD/EMTDC simulations are plotted in solid and dotted line respectively and their differences are shown at the bottom half of each figure. As indicated, the difference is relatively small during steady-state, but became larger when there is a step change. By comparing the simulation characteristics such as the dynamic response, indicates a close agreement between the TCS results and PSCAD/EMTDC simulations.

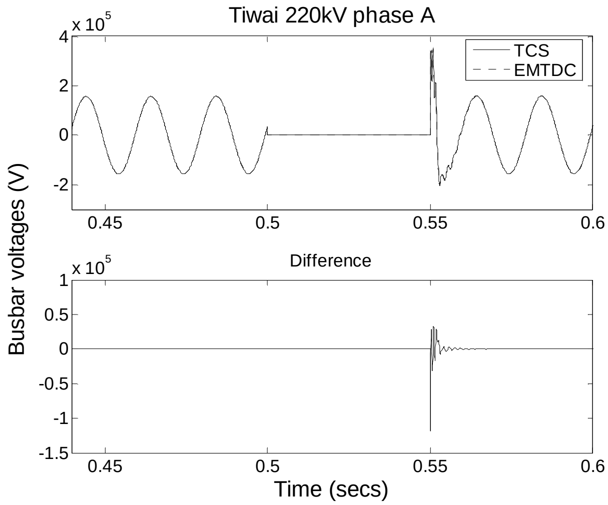


Fig. 2. Busbar voltage at Tiwai 220kV phase A

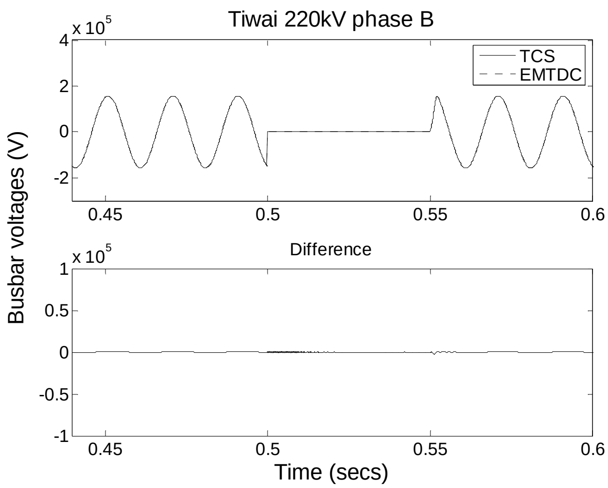


Fig. 3. Busbar voltage at Tiwai 220kV phase B

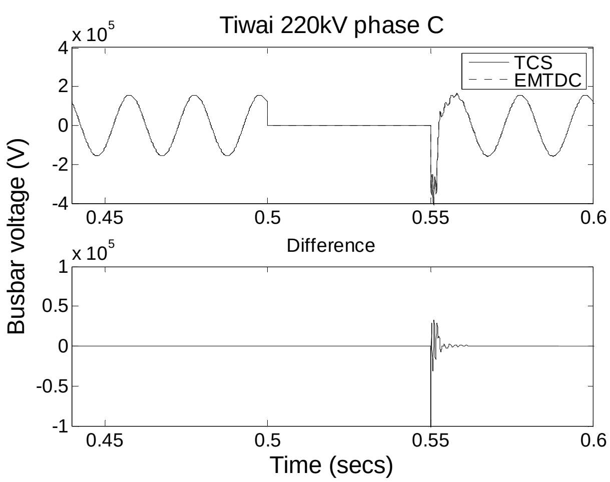


Fig. 4. Busbar voltage at Tiwai 220kV phase C

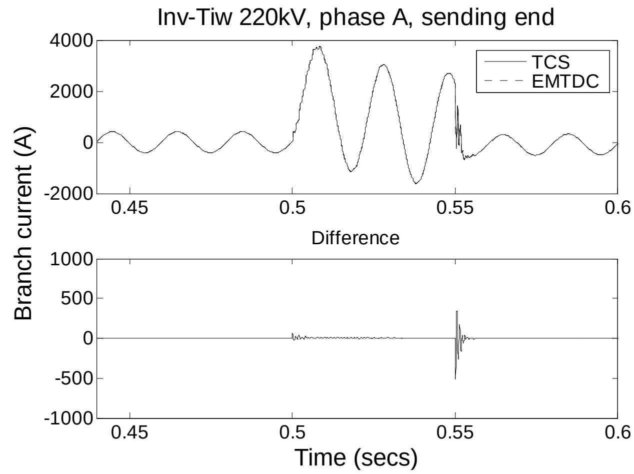


Fig. 5. Sending end Invercargill-Tiwai branch current, phase A

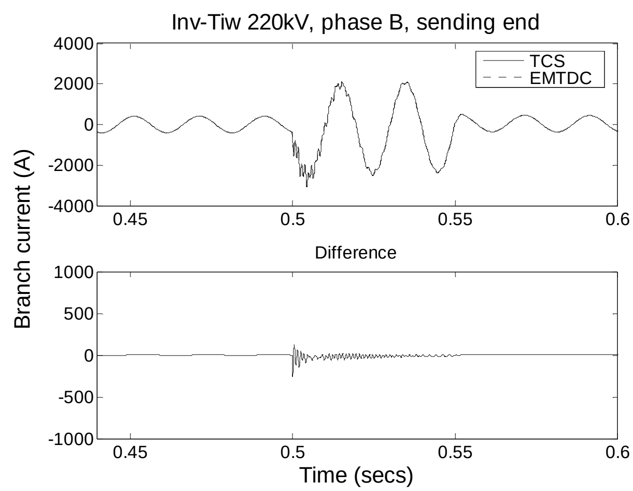


Fig. 6. Sending end Invercargill-Tiwai branch current, phase B

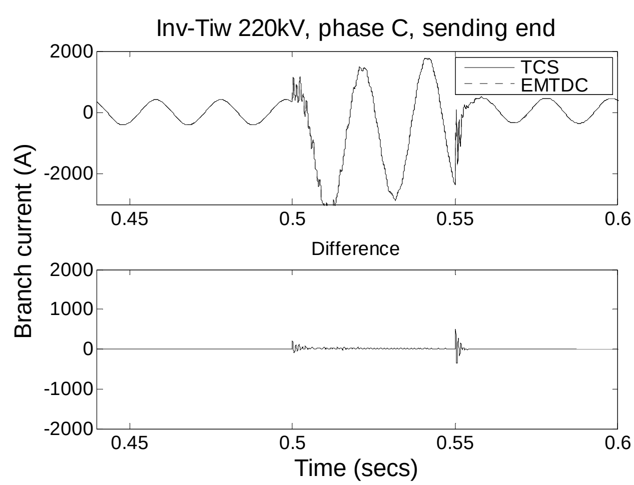


Fig. 7. Sending end Invercargill-Tiwai branch current, phase C

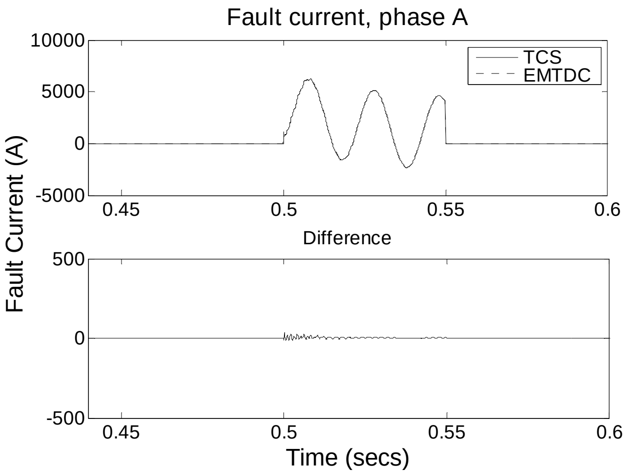


Fig. 8. Fault current, phase A

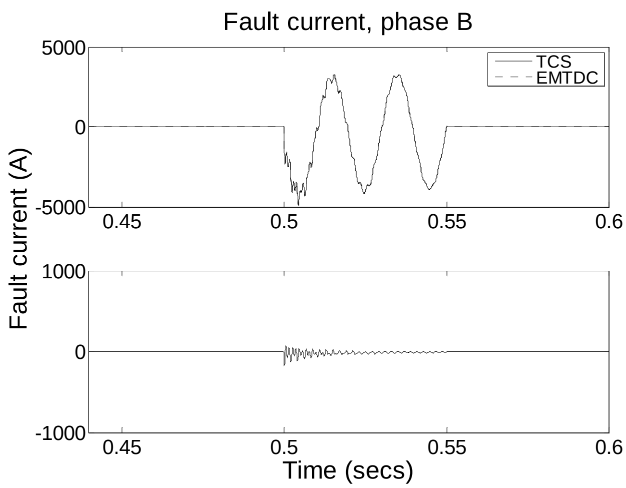


Fig. 9. Fault current, phase A

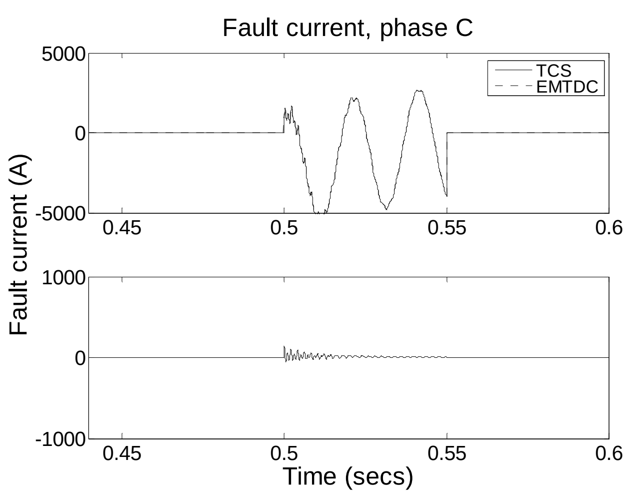


Fig. 10. Fault current, phase C

B. *Asymmetrical Fault*

While the results obtained under symmetric fault condition revealed good qualitative match, it is often necessary to look at the performance under unbalance condition as well before any conclusion can be made. A single phase line-to-ground fault of two and a half cycles duration at Invercargill-Tiwai transmission line is simulated in this case. Figs. 11-14 show the Tiwai 220kV busbar voltages and the Invercargill-Tiwai sending end currents. The three phase fault currents are depicted in Figs. 15-17. The results as shown are again in agreement with the PSCAD/EMTDC simulation despite small differences in the magnitude during step changes in the system topology. The ability in predicting asymmetrical transient response verifies the reliability of the proposed TCS algorithm in transient simulation.

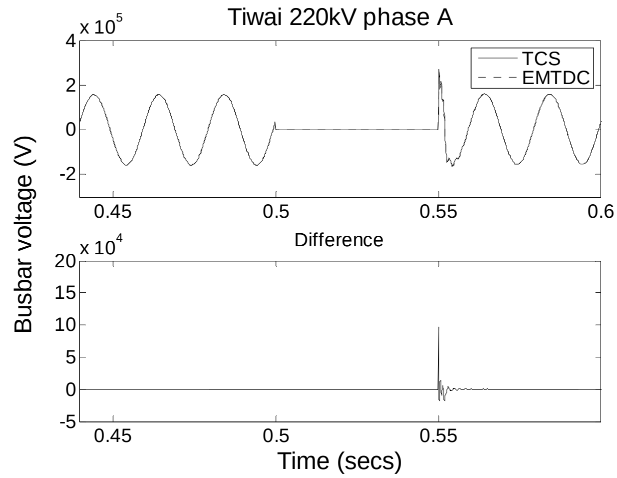


Fig. 11. Busbar voltage at Tiwai 220kV phase A

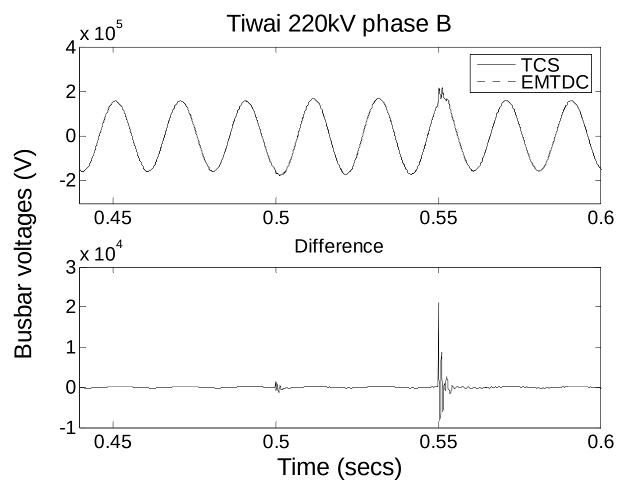


Fig. 12. Busbar voltage at Tiwai 220kV phase B

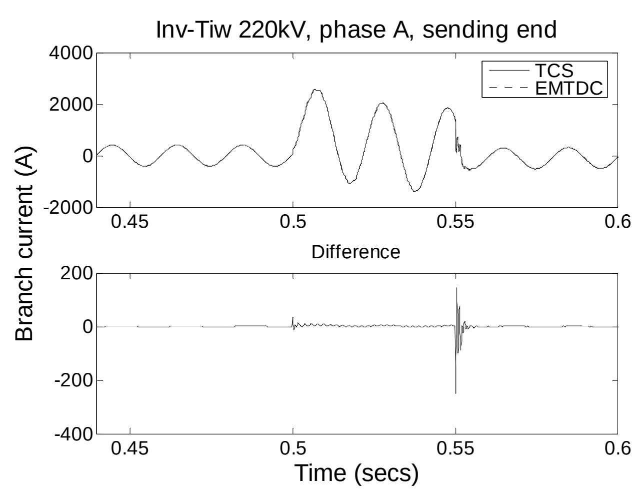


Fig. 13. Sending end Invercargill-Tiwai branch current, phase A

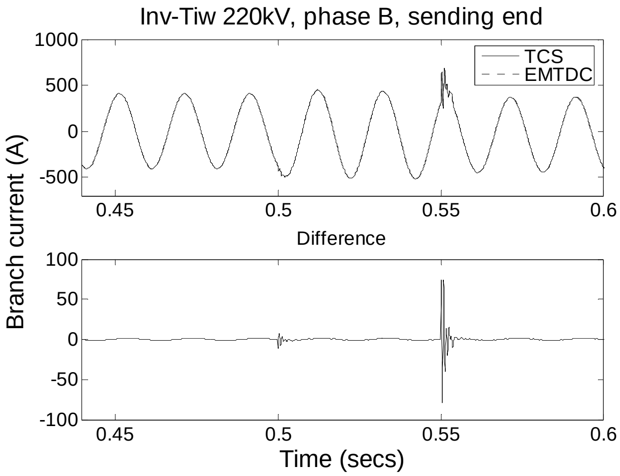


Fig. 14. Sending end Invercargill-Tiwai branch current, phase B

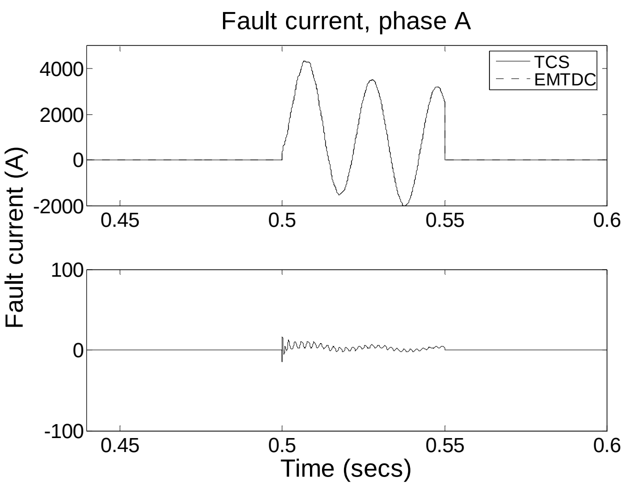


Fig. 15. Fault current, phase A

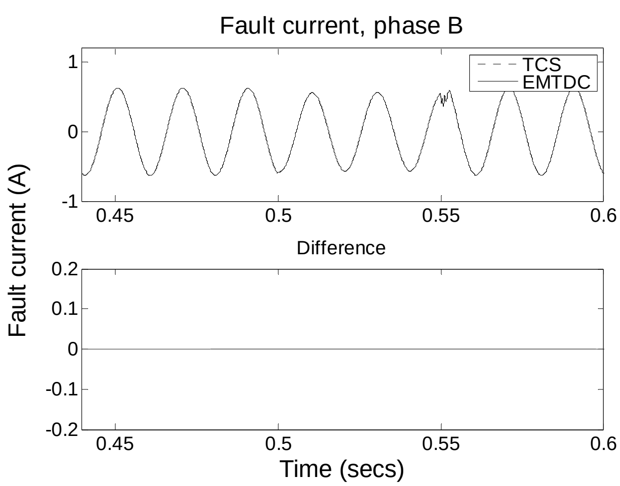


Fig. 16. Fault current, phase B

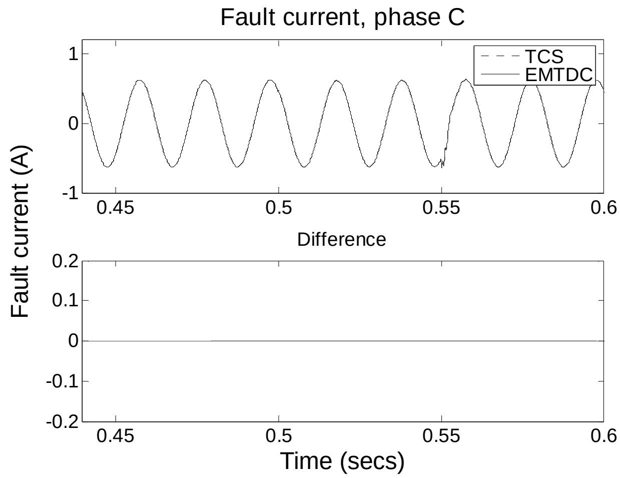


Fig. 17. Fault current, phase C

IV. Conclusion

In this paper, a diakoptical state variable approach is used to provide an efficient method of forming the system state equations. It overcomes the need to identify the set of independent state variables by diakoptical segregation of the sub-netowks and therefore is not affected by the presence of inductor cut-sets and loops of capacitors. Furthermore, for systems that undergo topological changes due to frequent switching, the diakoptical technique avoids changes of the whole network equations and only the relevant parts of the network equations are considered.

The performance of the proposed technique is compared with the PSCAD/EMTDC simulation based on the two test scenarios. The agreement between the two fundamentally different algorithms verifies that the new implementation of the TCS algorithm can be used with confidence in a.c. systems.

V. References

[1] J. Milias-Argitis, T. H. Zacharias, C. Hatziadoniu and G. Galanos, “An algorithm for transient simulation of power electronics systems,” *IEEE trans. circuits and systems*, vol. 34, no. 8, pp. 969-973, 1987

[2] G. V. Giannakopoulos, N. A. Vovos, T. I. Maris and A. D. Lygdis, “A fast and flexible method for transient simulation of integrated AC/DC systems,” *IEEE trans Power systems*, vol. 3, no. 4, pp. 1784-1792, 1988

[3] G. Galanos, “A mathematical model for dynamic simulation of HVDC systems,” *IEEE trans. Power apparatus and systems*, vol. pas-102, no. 8, pp. 1755-1763, 1983

[4] G. V. Giannakopoulos, N. A. Vovos, T. I. Maris and A. D. Lygdis, “A fast and flexible method for transient simulation of integrated AC/DC systems.” *IEEE Trans. Power System*, vol. 3, no. 4, pp. 1784-1792, 1988

[5] J. M. Zavahir, J. Arrillaga and N. R. Watson, “Hybrid electromagnetic transient simulation with the state variable representation of HVDC converter plant,” *IEEE Trans. Power Delivery*, vol. 8, no.3, pp. 1591-1598, 1993

[6] J. Arrillaga, C. P. Arnold and B. J. Harker, “Computer modeling of electrical power system,” J. Wiley & Sons, London, 1983

[7] R. A. Rohrer, "Circuit Theory: An Introduction to the State Variable Approach", McGraw-Hill, 1971.

[8] N.R. Watson and J. Arrillaga, “Power Systems ElectroMagnetic Transients Simulation”, IEE Books, 2003