Improvements of an FDTD-Based Surge Simulation Code and Its Application to the Lightning Overvoltage Calculation of a Transmission Tower

Taku Noda, *Member, IEEE*, Akiyoshi Tatematsu, *Member, IEEE*, and Shigeru Yokoyama, *Fellow, IEEE*

Abstract— **This paper presents new features recently added to a general-purpose surge simulation code based on the FDTD (Finite Difference Time Domain) method. The added features include various-shape conductor models, lumped-parameter circuit-element models, a lightning-channel model, and an integrated analysis environment (IAE). For precisely modeling the shapes of various conductors, the following conductor models have been added: inclined thin wire, disc, square plate, cylinder, cone, and quadrangular pyramid. The lumped-parameter circuit-element models allow the user to represent the lumped impedance of an apparatus placed inside the analysis space. The lightning-channel model realizes a return-stroke development at a speed slower than the light speed. The IAE includes a GUI (Graphical User Interface) which allows the user to enter geometrical data in a visual way. It also provides a waveform plotting program for viewing voltage, current, electric-field, and magnetic-field waveforms and a movie program for displaying the animation of a transient electric/magnetic-field intensity distribution. For an illustrative example, the lightning overvoltage calculation of a transmission tower is presented.**

Index Terms— **Electromagnetic transient analysis, FDTD methods, Graphical user interfaces, Lightning, Poles and towers, Simulation software, and Surges.**

I. INTRODUCTION

VERY-FAST surge phenomena on three-dimensional (3-D) structures cannot be simulated accurately by conventional circuit-theory-based techniques. For instance, a traveling wave on a tall vertical structure propagating downward cannot be analyzed by the distributed-parameter-line theory due to the following reasons: (i) The lines of electric force from the traveling wave have not yet reached the ground surface, and thus, defining the capacitance per unit length to the ground is meaningless. (ii) The magnetic fluxes generated by the traveling wave stay in the vicinity, and the resultant effective inductance p.u.l. is different from that obtained assuming a quasi-static magnetic field. This suggests that these kinds of very-fast surge phenomena should be analyzed by numerical electromagnetic (EM) field simulations such as the Finite Difference Time Domain (FDTD) method and the Method of Moments (MoM). Ishii and Baba used MoM to analyze the lightning response of a transmission tower [1], and Tanabe used FDTD for the simulation of transient grounding impedance [2]. The authors developed a general-purpose surge simulation code based on FDTD [3] and proposed a method to represent the radius of a thin wire in the FDTD surge simulation [4]. The developed simulation code, called VSTL (Virtual Surge Test Lab.), is now used at electric power companies and universities for practical and research-purpose studies.

This paper presents new features recently added to VSTL. The added features include various shape conductor models, lumped-parameter circuit-element models, a lightning-channel model, and an integrated analysis environment (IAE). For precisely modeling the shapes of various conductors, the following conductor models have been added: inclined thin wire, disc, square plate, cylinder, cone, and quadrangular pyramid. The lumped-parameter circuit-element models allow the user to represent the lumped impedance of an electric/electronic apparatus placed inside the analysis space. The lightning-channel model realizes a return-stroke development at a speed slower than the light speed. The IAE includes a GUI (Graphical User Interface) which allows the user to enter geometrical data in a visual way. It also provides a waveform plotting program for viewing voltage, current, electric-field, and magnetic-field waveforms and a movie program for displaying the animation of a transient electric/magnetic-field intensity distribution. For an illustrative example, the lightning overvoltage calculation of a 500-kV transmission tower is presented. The overvoltages at the insulator strings of the tower are calculated using VSTL with the new features.

II. VARIOUS-SHAPE CONDUCTOR MODELS

Since a conductor system of interest often has a complicated structure, conductor models with various shapes have been added to the simulation code for precisely modeling the complicated shapes. The added conductor models are (a) thin wire which is not parallel to any of the coordinate axes (inclined thin wire), (b) disc, (c) square plate, (d) cylinder, (e) cone, and (f) quadrangular pyramid, while a rectangular-parallelepiped conductor model had already existed in the first version of VSTL.

The inclined thin wire model was implemented with a staircase approximation. In [5] the error in propagation velocity due to the staircase approximation is investigated, and it is shown that the error, a slower propagation speed due to

---

All authors are with the Electric Power Engineering Research Lab., CRIEPI (Central Research Institute of Electric Power Industry), 2-6-1 Nagasaka, Yokosuka-shi, Kanagawa-pref. 240-0196, Japan (e-mails: takunoda@ieee.org; akiyoshi@criepi.denken.or.jp; yokoyama@criepi.denken.or.jp).

Presented at the International Conference on Power Systems
Transients (IPST'05) in Montreal, Canada on June 19-23, 2005
Paper No. IPST05 – 138

the zigzag path as a result of the staircase approximation, is generally small. The details are not repeated here.

The conductor models (b)–(f) were implemented by the algorithm below. Let us assume that the conductor shape (and its inner part) that we like to represent in the analysis space is described by the set of inequalities

$$\begin{aligned} f_1(x,y,z) &\le c_1, \\ f_2(x,y,z) &\le c_2, \\ &\vdots \end{aligned} \tag{1}$$

where $f_1(x,y,z)$, $f_2(x,y,z)$, $\cdots$ are functions of coordinates and c_1, c_2, $\cdots$ are constants. When the midpoint P of a side of a cell[1] in the analysis space satisfies (1), the electric field of the side is forced to be zero. The midpoint P is illustrated in Fig. 1. If this procedure is repeated so that the electric fields of all sides that satisfy (1) are forced to be zero, the conductor shape is now represented in the analysis space. For example, the cylinder conductor specified by

- (x_0, y_0, z_0) : the coordinates of the center of the base
- r : the radius of the base
- h : the height

can be represented by forcing all electric fields whose midpoints satisfy the following inequalities to be zero:

$$\begin{aligned} (x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 &\le r^2, \\ -z &\le -z_0, \\ z &\le z_0 + h, \end{aligned} \tag{2}$$

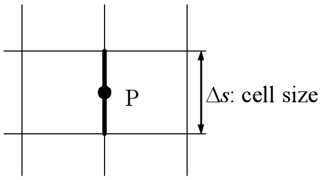
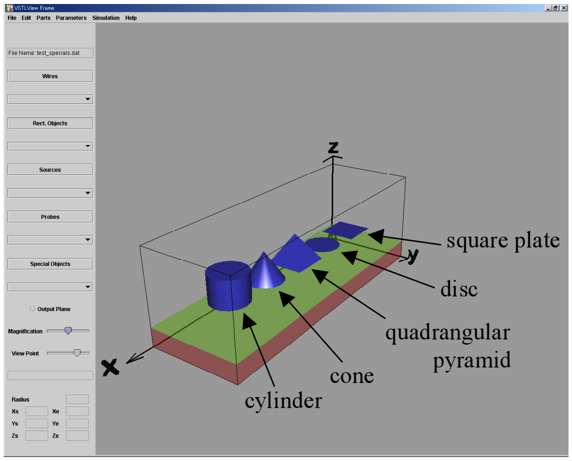


Fig. 1. Midpoint P of a side of a cell.



File Name: test_specials.dat

Fig. 2. Visualization of various-shape conductor models using the GUI.

where (x, y, z) is the coordinates of the midpoints. Other conductor models are represented in the same way. The added conductor models are shown in Fig. 2 using the GUI which will be described in Section IV-A.

III. LUMPED-PARAMETER CIRCUIT-ELEMENT MODELS

A. RLC Elements

In order to represent the lumped impedance of an electric/electronic apparatus placed in the analysis space, the lumped-parameter R, L, and C models shown in Fig. 3 are added to the simulation code using the method described in [6]. Any of the R, L, and C models occupies a side of a cell in the analysis space. Here, formulas for a circuit-element model which occupies a side in the z direction are shown, but those for models in other directions can also be implemented using similar formulas.

For the lumped-parameter R model (the resistance model), the following formula is obtained.

$$E_z^n = \frac{1 - \frac{\Delta t}{2R\varepsilon\Delta s}}{1 + \frac{\Delta t}{2R\varepsilon\Delta s}} E_z^{n-1} + \frac{\Delta t/\varepsilon}{1 + \frac{\Delta t}{2R\varepsilon\Delta s}} \left(\nabla \times \boldsymbol{H}^{n-\frac{1}{2}} \right)_z, \tag{3}$$

where

E_z^n : z-direction electric field at the n-th time instance at the side where the R model is placed,
R : resistance value,
ε : permittivity of the space,
Δt : time step,
Δs : space step (cell size),
$\boldsymbol{H}^{n-\frac{1}{2}}$: magnetic field (vector) around the R model at the $(n-\frac{1}{2})$-th time instance,

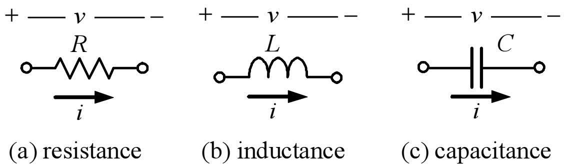
and $()_z$ denotes the z-component of the argument. The equation above can be derived by Ohm's law $V = RI$ with substituting the voltage of the side $V = -E_z \Delta s$ and the current I given by Ampare's law. Since (3) is in the same form as the fundamental FDTD formula for the electric-field calculation, (3) can readily be implemented by modifying the coefficients when calculating the electric field at the side of the R model in the FDTD calculation.

The formula for the L model (the inductance model) is derived from $V = Ldi/dt$:

$$E_z^n = E_z^{n-1} + \frac{\Delta t}{\varepsilon} \left(\nabla \times \boldsymbol{H}^{n-\frac{1}{2}} \right)_z - E_{hist}^{n-1}, \tag{4}$$

where the history term is given by

$$E_{hist}^{n-1} = E_{hist}^{n-2} + \frac{\Delta t^2}{L\varepsilon\Delta s} E_z^{n-1}. \tag{5}$$



(a) resistance
(b) inductance
(c) capacitance

Fig. 3. Lumped-parameter circuit-element models.

---

[1] a "cell" is a unit cube obtained as a result of space discretization of the FDTD method.

For the C model (the capacitance model), $I = C dv/dt$ gives

$$E_z^n = E_z^{n-1} + \frac{\Delta t/\varepsilon}{1+\frac{C}{\varepsilon \Delta s}} \left(\nabla \times \boldsymbol{H}^{n-\frac{1}{2}} \right)_z . \qquad (6)$$

B. Lightning-Channel Model

In lightning-overvoltage studies, the return-stroke current of a lightning is considered. A return stroke develops, or propagates, along a lightning channel upward. Observation results [7], [8] show that the development speed of a return stroke is generally slower than the light speed, and it is known that the development speed has an effect on calculated overvoltages. Therefore, a lightning-channel model which realizes a slower development speed compared with the light speed is desired.

Because a lightning channel is a vertical current path, the wave propagation does not follow the TEM (Transverse Electro-Magnetic) mode. To obtain qualitative insight, however, we use the following equation that gives the propagation speed v of the TEM mode:

$$v = \frac{1}{\sqrt{LC}}, \qquad (7)$$

where L and C are the inductance and the capacitance per unit length. As mentioned in the introduction section of this paper, it is impossible to identify L and C for such a vertical path. Nevertheless, (7) qualitatively indicates that if the inductance L is increased we may obtain a slower propagation speed.

Based on the above insight, we have proposed a lightning-channel model utilizing the lumped-parameter inductance model described in Section III-A [9]. The proposed lightning-channel model consists of a series connection of lumped-parameter inductance models. The development speed is controlled by the value of the inductance per unit length. Fig. 4 shows the arrangement of a simulation case for testing the proposed lightning-channel model. On the ground soil with a resistivity of 100 Ωm a current source is vertically placed, and the proposed lightning-channel model is also vertically placed on top of the current source. The current source injects the current waveform shown in Fig. 5 (a) from the ground to the bottom of the lightning-channel model. The analysis space is discretized with a space step of 2 m, and all boundaries are treated as absorbing boundaries using Liao's second-order formulation. The current which develops along the lightning-channel model was calculated by the FDTD method. The result obtained when each lumped-inductance element was set to 20 μH, that is, 10 μH/m is shown in Fig. 5 (b). This inductance value, which has been found through a trial-and-error process, realizes a development speed close to about 1/3 of the light speed. The observation results in [7], [8] suggests 1/3 of the light speed as an average value.

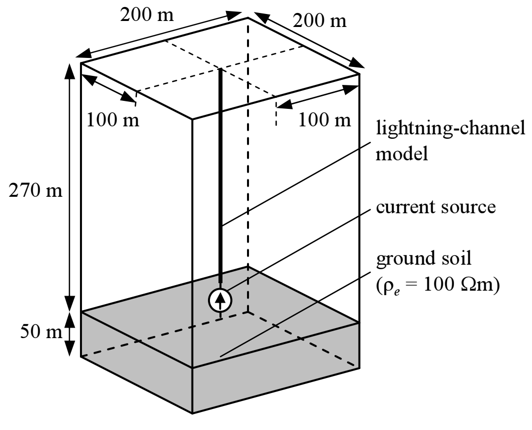


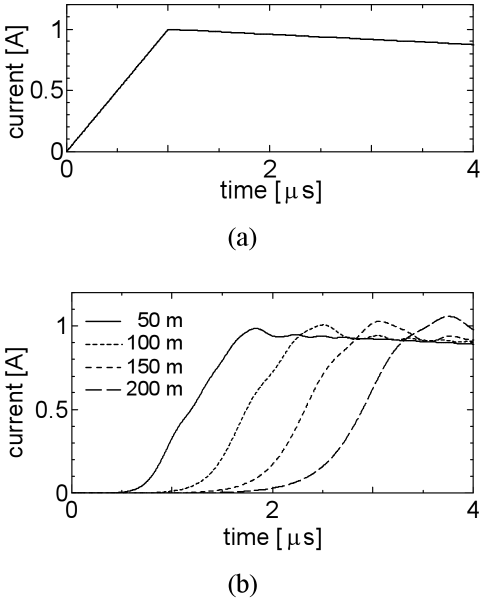
Fig. 4. Simulation case for testing the proposed lightning-channel model.

IV. IAE: INTEGRATED ANALYSIS ENVIRONMENT

To increase productivity of analysis using the developed simulation code, an integrated analysis environment (IAE), which includes a graphical user interface (GUI) for efficient data entry, a waveform plotting program, and a movie program for viewing the animation of a transient electric/magnetic-field intensity distribution, has been developed.

A. GUI

Prior to carrying out an FDTD simulation, the 3-D arrangement of a target conductor system has to be described by some means to pass the data to the simulation code. To this end, it is generally used to write the conductor arrangement data in a text file following a specific format. However, when the conductor arrangement is complicated, the process of preparing such data can be intricate and may lead to mistakes. To solve this problem, the GUI shown in Fig. 6 has been developed. Since the GUI displays a conductor arrangement as a 3-D picture and is able to magnify, reduce, and rotate the picture, it allows the user to enter data in a visual way. Using the GUI, a complicated conductor structure such as the one shown in Fig. 6 — the detailed structure of a 500-kV



(a)

(b)

Fig. 5. Current propagating along the lightning-channel model. (a) injected current waveform. (b) current waveforms along the lightning-channel model at different heights from the ground surface.

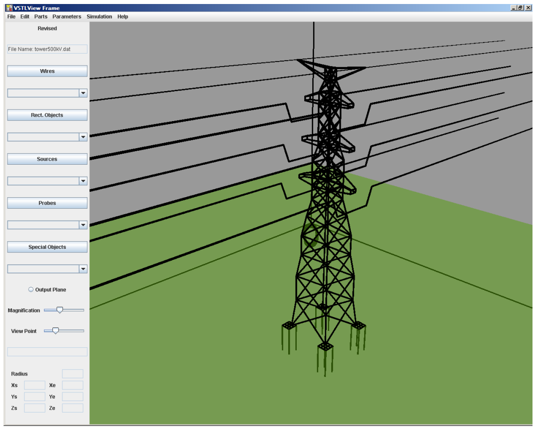
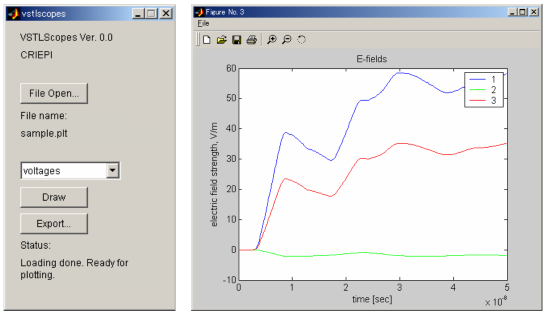
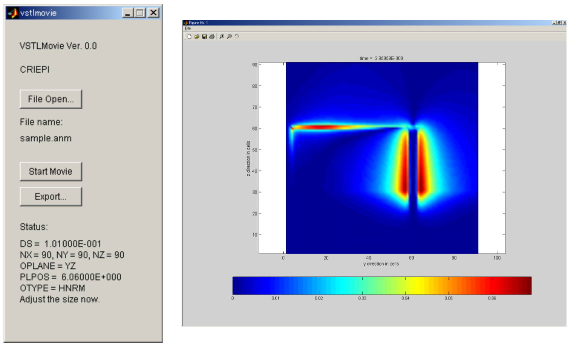


Fig. 6. GUI (Graphical User Interface).



VSTLScopes Ver. 0.0
CRIEPI
File name:
sample.plt
Status:
Loading done. Ready for plotting.

Fig. 7. Waveform plotting program.



VSTLMovie Ver. 0.0
CRIEPI
File name:
sample.anm
Status:
DS = 1.01000E-001
NX = 90, NY = 90, NZ = 90
OPLANE = YZ
PLPOS = 6.06000E+000
OTYPE = HNRM
Adjust the size now.

Fig. 8. Movie program.

transmission tower — can be entered efficiently. The data is for calculating a lightning-surge response of the tower.

B. Waveform Plotting Program

Fig. 7 is the waveform plotting program developed for showing waveforms of voltages, currents, electric-fields, and magnetic-fields. When preparing data using the GUI, positions and directions where waveforms are required to be recorded can be specified using "probe" parts. After the simulation, the recorded waveforms can be viewed using this program.

C. Movie Program

For visually understanding surge propagation phenomena, a movie program, which shows the transient variation of electric- or magnetic-field intensity distribution in a cross section of an analysis space in the form of an animation, has been developed. Fig. 8 shows the program. A cross section can be specified in the GUI when preparing data. In the case of an electric-field intensity distribution the tangential intensity is shown, and in the case of a magnetic-field distribution the normal intensity. The distribution is two-dimensionally interpolated and shown as a color gradation which uses warmer colors at intenser portions and colder colors at weaker portions.

V. LIGHTNING OVERVOLTAGE CALCULATION OF A TRANSMISSION TOWER

For an illustrative example, the lightning overvoltage calculation of a 500-kV transmission tower is presented. Fig. 9 illustrates the arrangement of the simulation. The dimensions of the analysis space are 250 m in the line direction, 150 m in the other horizontal direction (perpendicular to the line), and 250 m in the vertical direction. The space step is set to 1 m. For assuming an open space, all boundaries are treated as absorbing ones using Liao's second-order formulation. The representation of ground soil in the FDTD method is straightforward. The resistivity of the space assigned to be ground soil is simply set to a desired value. In this simulation case, it is set to 250 Ωm.

An actual return stroke generates opposite-polarity current waves at a tower top. One of the current waves propagates upward along the lightning channel and the other downward along the tower. To simulate this, a current source is placed on top of the tower. It injects, from the bottom of the lightning-channel model to the tower top, the ramp-wave current shown in Fig. 10 with an amplitude of 1 A and a wavefront time of 1 μs. The lightning-channel model realizes a development speed

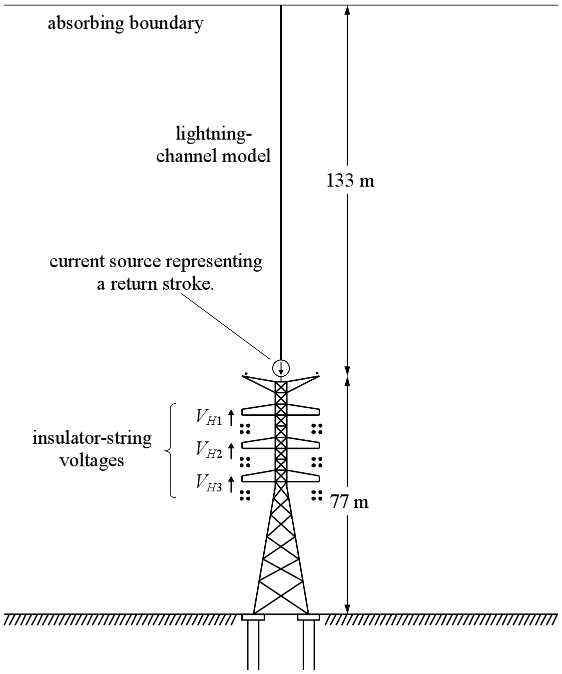


Fig. 9. Arrangement for the lightning overvoltage calculation of a 500-kV transmission tower.

of about 1/3 of the light speed as described in Section III-B. The tower body, of which the height is 77 m, is represented in detail as shown in Fig. 6. The arms, the slant elements, and the foundation structures are represented as in detail as possible under limitations due to the space step used. The tower supports two ground wires of OPGW 290 and six bundled phase wires of four TACSR 810 wires with a 50 cm spacing. The radius of the ground wires and the equivalent radius of the bundled phase wires are taken into account using the method proposed in [4]. The equivalent radius was calculated by the GMD (Geometrical Mean Distance) technique.

With the conditions mentioned above, the simulation was carried out using VSTL, and the result shown in Fig. 11 was obtained. A similar simulation was carried out using the method of moments (MoM) in [10]. In the MoM simulation, a tower is represented by a simple structure consisting of only four main poles, where arms, slant elements, and foundation structures are neglected. Also, lightning channel was represented by a simple straight wire and a perfectly-conducting ground was assumed. Therefore, the result presented here is obtained with more practical conditions (for more discussion see Appendixes A and B).

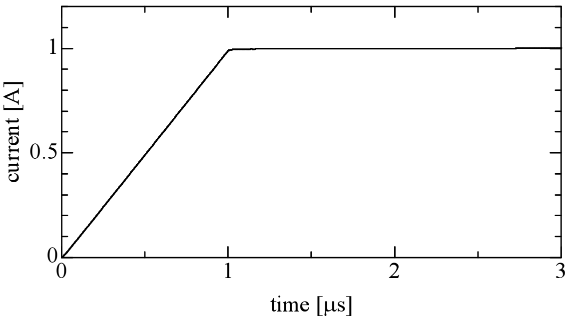
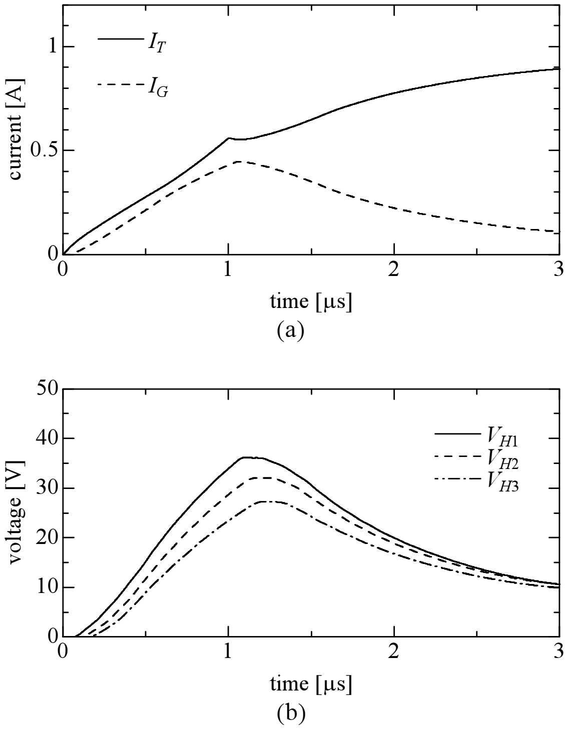


Fig. 10. Waveform of the current source representing a return stroke.



(a)

(b)

Fig. 11. Result of the lightning overvoltage calculation of the 500-kV transmission tower. (a) Waveforms of the currents flowing into the tower top I_T and flowing into the ground wires I_G. (b) Waveforms of the upper-phase, middle-phase, and lower-phase insulator-string voltages V_{H1}, V_{H2}, V_{H3}.

VI. CONCUSIONS

This paper has presented important features newly added to VSTL, a general-purpose surge simulation code based on the FDTD method. The added features include various-shape conductor models, lumped-parameter circuit element models, a lightning-channel model, and an integrated analysis environment (IAE). With the new models, a target conductor system can be represented more in detail, in other words, more practical. The simulation cycle of data entry, simulation, and viewing results has been made simplified and efficient by the IAE. The lightning overvoltage calculation of a 500-kV transmission tower was illustrated to show a complicated simulation can be performed using VSTL with the new features.

REFERENCES

[1] M. Ishii and Y. Baba, “Numerical Electromagnetic Field Analysis of Tower Surge Response,” *IEEE Trans. on Power Delivery*, vol. 12, no. 1, pp. 483–488, Jan. 1997.

[2] K. Tanabe, “Novel Method for Analyzing the Transient Behavior of Grounding Systems Based on the Finite-Difference Time-Domain Method,” *Proc. 2001 IEEE Power Engineering Society Winter Meeting*, vol. 3, pp. 1128–1132.

[3] T. Noda and S. Yokoyama, “Development of Surge Simulation Code Based on Finite-Difference Time-Domain (FDTD) Approximation of Maxwell’s Equations,” *Proc. Int. Conf. on Power Systems Transients (IPST) 2001*, Rio de Janeiro, Brazil, 2001.

[4] T. Noda and S. Yokoyama, “Thin Wire Representation in Finite Difference Time Domain Surge Simulation,” *IEEE Trans., Power Delivery*, vol. 17, no. 3, pp. 840–847, July 2002.

[5] T. Noda, R. Yonezawa, S. Yokoyama, and Y. Takahashi, “Error in Propagation Velocity Due to Staircase Approximation of an Inclined Thin Wire in FDTD Surge Simulation,” *IEEE Trans., Power Delivery*, vol. 19, no. 4, pp. 1913–1918, Oct. 2004.

[6] K. S. Kunz and R. J. Luebbers: *The Finite Difference Time Domain Method for Electromagnetics*, CRC Press, 1993.

[7] B. F. J. Schonland et al.: *The Lightning Discharge*, Handbuch der Physic, vol. 22, Springer-Verlag, OHG, Berlin, 1956 (p. 576).

[8] V. P. Idone et al., “Lightning Return Stroke Velocities in the Thunderstorm Research Internatinal Program (TRIP)”, *J. Geophys. Res.*, vol. 87, No. C7, pp. 4903-4915, 1982.

[9] A. Tatematsu, T. Noda, and S. Yokoyama, “Development of Simulation Technique of Lightning-Induced Voltages on Distribution Lines Using the FDTD Method,” *CRIEPI Reports*, No. H04009, 2005.

[10] Y. Baba and M. Ishii, “Numerical Electromagnetic Field Analysis on Lightning Surge Response of Tower with Shield Wire, *IEEE Trans., Power Delivery*, vol. 15, no. 3, pp. 1010–1015, July 2000.

[11] M. Ishii, et al., “Multistory Transmission Tower Model for Lightning Surge Analysis,” *IEEE Trans., Power Delivery*, vol. 6, no. 3, pp. 1327–1335, July 1991.

APPENDIX

A. Comparison between FDTD and MoM

The FDTD method is based on direct time and space discretization of Maxwell’s equations, while MoM uses a frequency-domain boundary-element formulation derived from scalar- and vector-potential expressions of Maxwell’s equations. FDTD is generally more computationally intensive than

MoM, but due to recent progress of computers in terms of execution speed and memory capacity this is not an important issue. More importantly, if Courant's condition is satisfied FDTD is always numerically stable, while MoM is often unstable at low frequencies.

Due to the direct time and space discretization, FDTD is more advantageous to handle three-dimensionally distributed currents in an imperfectly conductive medium like ground soil. MoM has severe limitations to handle such a medium.

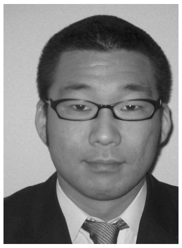
MoM can rigorously represent the radius of a thin wire and a thin wire with any direction can be handled, while FDTD had limitations for such thin wire modeling issues. However, these issues have been solved by the authors as reported in [4] and [5], and thus, we have concluded that FDTD is more suitable than MoM for the simulation of a very-fast surge phenomenon.

B. Numerical Electromagnetic Simulation versus Distributed-Parameter Line Modeling

As mentioned at the beginning of the introduction section, it is theoretically impossible to represent a vertical conductor by a distributed-parameter line model which assumes a TEM mode of propagation, since we cannot define the inductance and the capacitance (to ground) per unit length. Therefore, it should be noted that a distributed-parameter line modeling inherently involves an audacious approximation. In observation results, we obtain larger voltages at upper insulator strings (for instance see [11]). Such a result cannot be reproduced using a distributed-parameter line modeling with a constant surge impedance. If a non-uniform modeling is applied by adjusting surge impedance values with respect to height, we may obtain a reasonable result but the adjustment of surge impedance values cannot be theoretically justified. Also, the effects of electric and magnetic fields generated by the tower currents to the insulator strings cannot be taken into account. On the other hand, since the FDTD method (or MoM) rigorously solves Maxwell's equations, all electromagnetic phenomena are taken into account and can reproduce observation results without any artificial adjustment.

Taku Noda (M'97) was born in Osaka, Japan in 1969. He received his Bachelor's, Master's, and PhD degrees all in engineering from Doshisha University, Kyoto, Japan in 1992, 1994, and 1997 respectively. In 1997, he joined Central Research Institute of Electric Power Industry (CRIEPI), where he is currently a Research Scientist. From January 2001 to March 2002 he was a Visiting Scientist at the University of Toronto, Toronto, Ontario, Canada. From April 2005, he also has the position of Visiting Professor at Doshisha University.

His primary research interest is the transient analysis of power systems. Dr. Noda is a member of IEE (UK) and IEE of Japan.



Akiyoshi Tatematsu (M'04) was born in Aichi, Japan on 2 June, 1975. He received his Bachelor's, Master's, and PhD degrees all in engineering from Kyoto University, Kyoto, Japan in 1999, 2001, and 2004, respectively. In 2004, he joined Central Research Institute of Electric Power Industry (CRIEPI), where he is currently a Research Associate. He has been engaged in the study of electromagnetic field calculation. Dr. Tatematsu is a member of IEE of Japan.

Shigeru Yokoyama (M'83, S'91, F'96) was born in Miyagi, Japan on March 5, 1947. He received his Bachelor's and PhD degrees in engineering from the University of Tokyo, Tokyo, Japan in 1969 and 1986 respectively. In 1969, he joined CRIEPI, where he is currently an Executive Research Scientist. He is also a Visiting Professor at Kyushu University, Fukuoka, Japan.

His primary research interest is the lightning protection and the insulation co-ordination of transmission and distribution lines. Dr. Yokoyama is a member of IEE of Japan.