Numerical Modeling of Vacuum Arc Dynamics at Current Zero Using ATP

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Abstract: Simulation of the so called Continuous Transition Model of low pressure vacuum arcs, during current interruption, was developed using the Alternative Transients Program (ATP). A special routine was implemented in the Transient Analysis of Control Systems (TACS) within ATP to simulate the performance of a vacuum interrupter and its interaction with the power system at current zero (CZ). Comparison with test results indicates the validity of this numerical model. This numerical model is of great importance for the prediction of the vacuum circuit breaker performance under different electrical circuit conditions. Also, it has helped to analyze breakdown phenomena in vacuum switches. In that way, the model presented is a useful tool for analysis of specific switching operations and for the understanding of the interruption process in vacuum devices.

Keywords: ATP, TACS, vacuum arcs, low pressure plasmas, current interruption, current zero.

I. INTRODUCTION

Vacuum is known as a very good insulating environment for medium voltage circuit breakers due to its excellent arc quenching capabilities. The physics of the arc in vacuum and its behavior under certain circuit conditions are reasonably known at the present time. A few mathematical models have been developed to describe that behavior [1-5]. In this paper the development of a numerical model of the vacuum arc based on such mathematical description is presented. The model has been implemented in the Alternative Transients Program (ATP) using Transient Analysis of Control Systems (TACS) to model the arc plasma dynamic equations. In contrast to other work reported [5, 8, 11], the model presented has been made using a practical approach in which simulations are helpful for analysis of switching operations with vacuum devises or for the analysis of the plasma development in such vacuum devises. Comparison of results from simulations of the numerical model with actual interrupter's test is also presented.

II. THE VACUUM ARC MODEL

A. System of equations describing the arc plasma dynamics.

The mathematical model describing the interruption process of the arc in vacuum is based on the continuous transition model by Andrews and Varey [1]. The model is represented by the following equations:

$$l^2 = \frac{4\,\varepsilon_0\,U_0}{9eZN_i}\left[\left(1+\frac{u(t)}{U_0}\right)^{\frac{3}{2}}+\frac{3u(t)}{U_0}-1\right] \qquad (1)$$

$$U_0 = \frac{M_i}{2e}\left(v_i - \frac{dl}{dt}\right)^2 \qquad (2)$$

$$i(t) = \frac{\pi D^2 ZN_i e}{4}\left(v_i - \frac{dl}{dt}\right) \qquad (3)$$

Where,

- l – length of the positive ion sheath
- U_0 – electric potential at the sheath edge
- Z - average charge multiplicity constant, it varies from 1.3 to 1.5 for Copper based materials in diffuse arcs.
- N_i - ion density at the sheath edge, values in the range from 10^{18} to 10^{23} are known for diffuse vacuum arcs
- $u(t)$ – voltage across the vacuum contacts
- ε_0 – permittivity of free space ($8.8419\text{x}10^{-12}$ Farads/m)
- e – electron charge ($1.6\text{x}10^{-19}$ Coulombs)
- M_i - ion mass, $M_i \cong$ atomic weight times the proton mass, 1.062×10^{-25} for Cu
- v_i - velocity of the ions at the sheath edge and in the quasi-neutral plasma, it varies in order of magnitude from 10^3 to 10^4 m/s for Copper based materials in diffuse arc mode
- $\frac{dl}{dt}$ – velocity of sheath development
- $i(t)$ – post arc current through the vacuum contacts
- D – Arc column diameter, in diffuse low pressure arcs this value is approximately equal to the contact diameter

Equation (1) describes the growth of the positive ion sheath (l), moving from the anode to the cathode a few nanoseconds after current zero (CZ), when the transient recovery voltage ($u(t)$) starts developing across the contacts. A description of this interaction will be presented in section III. When a successful interruption occurs, this sheath "sweeps" the quasi-

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neutral plasma from the inter-electrode space, taking away the free electrons and leaving the positive heavy metal ions behind. The thickness of the sheath (l) is a function of the recovery voltage applied to the electrodes as well as the electric potential at the edge of the sheath.

Equation (2) is a measure of the electric potential of the ions at the sheath edge (U_0). The parameter U_0 depends on the rate at which the sheath is developing (dl/dt) and the drift velocity of the metal ions (v_i).

Finally, equation (3) describes the post-arc current ($i(t)$) that flows through the interrupter after the sheath started to grow but before the interruption process is completed. The $i(t)$ has two components, a displacement current that is proportional to the sheath velocity (dl/dt), which is similar to the displacement current caused by the electric field in the Maxwell's equation, and a conduction component which depends on the velocity of the ions (v_i).

At this point, the system of nonlinear equations (1-3) consist of five unknowns l, Uo, $u(t)$, N_i and $i(t)$. Since the system has five variables and only three equations it would have an infinite number of solutions. Consequently, the system needs two equations more in order to find a unique solution.

In reality, v_i is also a variable but in this simplified approach it is taken as a constant. Reference [1] reported a velocity with an approximately constant value of 930 m/s for mercury arcs. For copper based arcs the velocities range from 10^3 to $2x10^4$ m/s [6, 7].

An expression for N_i will be discussed in section II.B, below. The voltage across the contacts $u(t)$ can be found with the addition of equations for the external electrical circuit connected to the vacuum interrupter. In the present model the variable $u(t)$ is computed using the electrical system modeled in ATP, as will be clear in due course.

The model described in this paper is considered a black box model, since the only signals that the model receives externally, from ATP, are the voltage and current. In the same manner, the only characteristic that the external circuit receives from the vacuum interrupter, in TACS, is an equivalent time dependent resistance $R(t)$ computed from the solution of the continuous transition model.

B. Ion density distribution applied to the continuous transition model.

The author has named the model having a description of the ion density distribution (N_i) the Enhanced Continuous Transition Model (ECTM). From reference [8] an equation describing the ion density as a function of time and space was adopted as follows,

$$N_i = N_{io} \exp\left(-\frac{t - t_o}{\tau}\right)\left(AMP\,\frac{l^2}{gap^2} + 1\right) \qquad (4)$$

The exponential function with the time constant τ reflects the plasma diffusion process decaying as a function of time. This time constant τ is adjusted to calibrate the results of the simulation with the results from tested devices. The constant *AMP* controls the ion space charge distribution from one electrode to the other. After CZ, at the new cathode, where the sheath l is equal to zero, the factor

$$\left(AMP\,\frac{l^2}{gap^2} + 1\right)$$

is equal to 1, whereas at the new anode, when l is equal to the gap length, the factor is equal to *AMP*+1.

The constant N_{io}, the initial ion density when $t=t_0$ and $l=0$, depends on the rate of decay of the arcing current prior to current zero ($t<t_0$). Figure 1 helps visualize the current and voltage curves around CZ in a vacuum interrupter. Notice the pause between CZ and the beginning of the transient recovery voltage (TRV). This pause may have values between 40ns and 150ns.

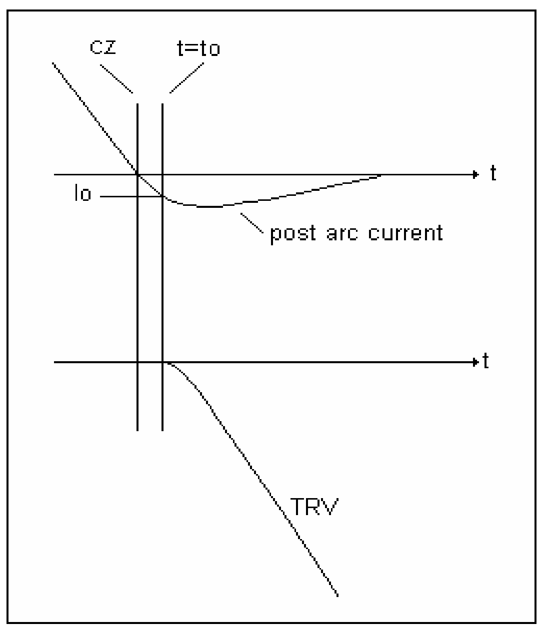

Figure 1. Beginning of the TRV after CZ.

In figure 2 the development of the sheath can be visualize. At CZ the drift velocity of the electrons is equal to the drift velocity ions. However, the electrons' velocity is declining very rapidly due to the voltage imposed by the system at that instant. Until the velocity of the electrons is instantaneously zero, no TRV appears across the contacts and the inter-electrode space is full with quasi-neutral plasma. That is the reason for the pause between the CZ and the beginning of the TRV as seen in figure 1, while current is still flowing, no appreciable voltage can developed. Suddenly the electrons reverse their direction towards the new anode, leaving a cloud of heavy metal ions called the ion sheath. After that moment the TRV starts developing between the contacts of the interrupter. This TRV only appears across the sheath which is free of electrons as can be realize in figure 2.

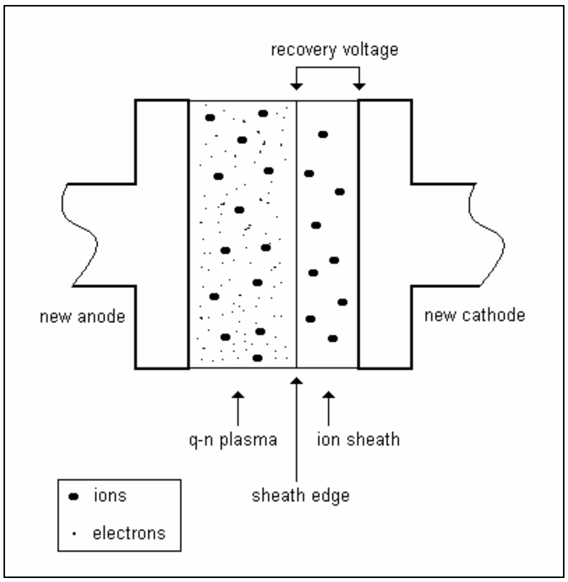

Figure 2. Visualization of sheath development during post-arc period.

The initial density of ions can be evaluated from equation (3) knowing the current I_0, the post-arc current just after the pause as:

$$N_{io} = \frac{4I_o}{v_i \pi \ D^2 \ Ze}$$

Until the time t_0, beginning of the TRV after CZ in figures 1 and 2, the current is of a conduction type only. Hence, during the pause from CZ to time t_0 the displacement component (dI/dt) of equation 3 is zero [9]. At the instant t=t_0, $i(t)$ is equal to I_0 and the value of N_{io} can be found. The initial ion density N_{io} therefore reflects the past-history of the arc, i.e. the plasma that was generated at the inter-electrode space (gap) some flight time earlier. This N_{io} depends on the rate at which the current is approaching CZ and the dimensions of the arc discharge about t_0. Reference [9] shows that t_0 varies from 100 to 150ns for gaps of 10mm and different dV/dt. A later work [10] shows values for t_0 between 40 and 50ns after CZ for shorter gaps (1mm and less).

III. MODEL IMPLEMENTATION IN ATP-TACS

Equations 1 to 4 were implemented in a specially designed TACS routine. Figure 3 shows a diagram of the model for the simulation of a single-phase breaker terminal fault. At every time step, ATP calculates the voltage $u(t)$ and the current $i(t)$ from the Thevenin equivalent of the power system. These values are imported by TACS for the calculation of the ECTM.

The initial ion density (N_{io}) becomes a constant after the first time step computation. The resistance $R(t)$, the interface between ATP and TACS, is initially set to a very small value, and will start changing when the simulation time is equal to t_0. After that point in the simulation the TACS routine computes the post-arc current from equation 3. An equivalent resistance $R(t)$ is computed as the ratio of the voltage across the switch $u(t)$ and the post-arc current $i(t)$. This $R(t)$ is transferred to ATP for the next time step calculation [12]. It is important to mention that the equivalent resistance $R(t)$ of the vacuum arc is somewhat artificial and, in the case of vacuum circuit breakers, does not correspond to any physical resistance of the arc plasma. Therefore, $R(t)$ is only a technique to implement the model in TACS.

The ion density distribution (N_i, *equation 4*) is also computed at every time step after the simulation time reach t_0. This makes this equation part of the system of four equations (1 to 4). The process is repeated until the end of the simulation.

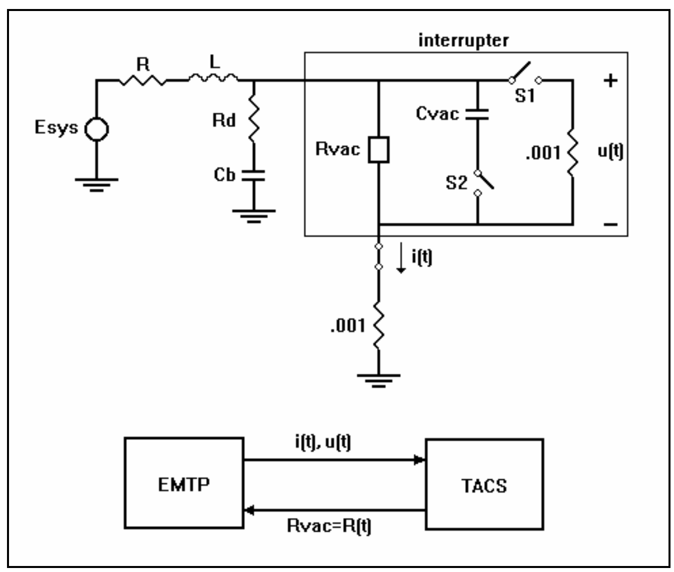

Figure 3. Circuit diagram for the simulation of a breaker terminal fault and block diagram showing the flow of variables between ATP and TACS.

IV. VERIFICATION OF THE MODEL

The validation of the model was performed using experimental results provided by references [13, 14]. Figures 4 and 5 show traces of sample simulations made with the numerical model. The traces labeled “simulation” are the output from the ATP/TACS model after the parameters of such a model were extracted with system identification methods.

From the oscillograms, the values of *dI/dt* before CZ, the peak transient recovery voltage, the rate of rise of recovery voltage and the damping ratio were estimated. The author received the value of power frequency for each test. With these values the author calculated the Thevenin equivalent circuit for each individual test to implement the ATP simulation.

In the case of figure 4, values of TRV=66kV peak, *dV/dt*=2kV/µs, *dI/dt*=18.98A/µs at 50Hz and a damping ratio of 10 gave the following lumped circuit parameters:

- E_{sys}=35.68 kV@50Hz
- L=1.88 mH
- Cb=110 nF
- Rd=13.07 Omhs

In figure 5, the TRV=55kV peak, *dV/dt*=1.25kV/µs, *dI/dt*=5A/µs at 60Hz and the same damping ratio gave the following lumped circuit parameters:

- E_{sys}=33 kV@60Hz
- L=1.87 mH
- Cb=8.5 nF
- Rd=450 Omhs

Equations 1 to 4 have four constants (D, v_i, τ, AMP) that change depending on the vacuum devise and the switching conditions. For each individual oscillogram such constant values were found with the aid of parameter estimation routines developed by the author using MATLAB [15].

Using the digital oscillograms of the tests and with the Thevenin equivalent parameters reported above in the current section, the optimization routine gave the following values for the case presented in figure 4:

- D=3 mm
- v_i=2250 m/s
- τ=10 µs
- AMP=5

For the case presented in figure 5, and reported previously in reference [16], the values of the constant parameters are:

- D=5.3 mm
- v_i=1000 m/s
- τ=4.09 µs
- AMP=1.447

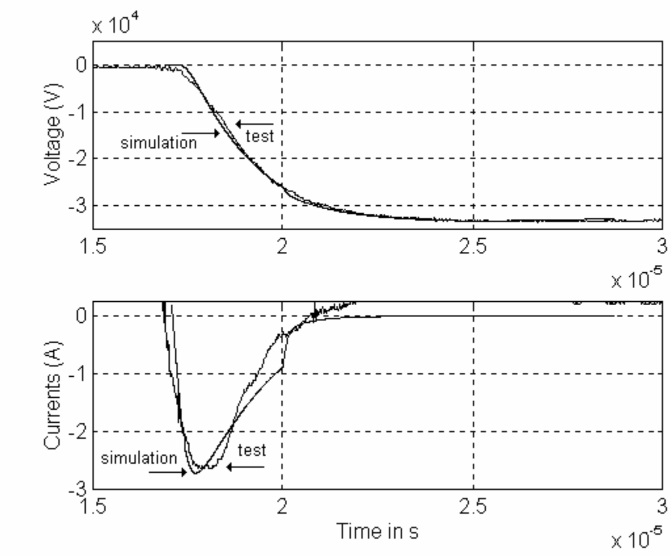

Figure 4 Traces for comparison of test with simulation, after use of the extracted constants from the parameter extraction and optimization program.

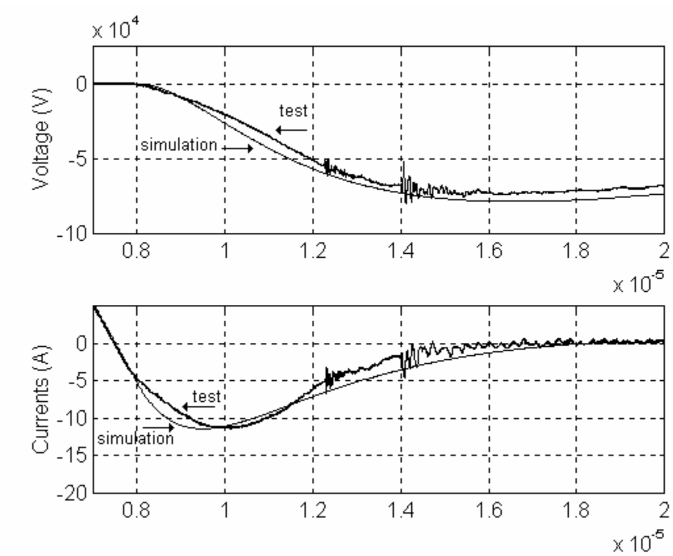

Figure 5 Other traces for comparison of test with simulation, after use of the extracted constants from the parameter estimation program.

In the example of figure 4 the post arc current peak is 2.7A for the simulation and 2.6A for the test. In the figure, the simulation current drops from 0.7A to zero within a time of approximately 20µs. This is due to the clearance of the residual plasma by the ion sheath. In other words, the sheath has grown to the full length of the gap. The test current contains a capacitive component that causes the zero crossing at 21µs.

In figure 5 the post arc current peak for the simulation is 12A and 11.8 for the test. Both simulation and test currents return to zero at the same time. There is a time delay between the peaks of the currents for the simulations and tests of approximately 1μs and 0.5μs for both cases, respectively. However, both cases show good agreement between the simulation results and test for the post arc current as well as for the TRV.

It is worth to note that specific information of the tested interrupter is not needed for the estimation of the constant parameters using the optimization routine. It has to be clarified that the solution provided by the parameter estimation routine is not unique. Different sets of parameters can be found from this routine that give close approximations. Current research aims to develop better parameter estimation routines to identify such constant parameters.

The reader must understand that the constant parameters just reported are unique for a specific devise under certain switching conditions, and should not be use as universal constants for the simulation of the ECTM. Different devices will have different parameters.

V. CONCLUSIONS

A simplified numerical model of the post arc current in vacuum based on the continuous transition model has been successfully implemented in the ATP using TACS routines. The model in TACS is independent of the electrical circuit (black box model). Different electrical systems conditions can easily be tested with the same interrupter by changing the parameter of such systems in ATP. This black box model can be used for analysis of the interaction between the circuit breaker arc and the electric power network in which it resides.

This model can also be used to study the behavior of the post arc plasma. It has been shown [16] that this model helps to better understand the mechanisms that could affect the interruption and breakdown processes in vacuum interrupters.

The figures of two different tests show a very good agreement between the simulations using the model and the tests.

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VII. BIOGRAPHY

Lionel R. Orama (S'91-M98) was born in Rio Piedras, Puerto Rico, on 1967. He received the B.S. in electrical engineering from the Polytechnic University of Puerto Rico, San Juan, P.R, in 1992, and the M.E. degree in electric power engineering from Rensselaer Polytechnic Institute, Troy, NY, in 1994. He received his D.Eng. degree also from Rensselaer in October of 1997, and became an Assistant Professor at the University of Puerto Rico, Mayagüez, Puerto Rico. Nowadays he is an Associate Professor of Electrical Engineering doing research in electrical discharges and transients.