Permanent Magnet Synchronous Machine Model for Real- Time Simulation

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Abstract--This paper presents the implementation of a model for a permanent magnet motor on a real time digital simulator. The conventional interfaced model approach popular in some emt-type programs is shown to be inappropriate for real-time simulators, because it is more susceptible to numerical instability. The paper introduces an 'embedded' phase domain model which reduces numerical instabilities.

Both the conventional as well as the embedded phase domain model models are shown to provide identical results in the steady state and transient situations; however the conventional model becomes unstable quickly if the time-step is increased.

This paper also presents the application of this new model to simulate the machine using the evaluated inductances that have been evaluated by (FEM) method based modeling.

Keywords: Permanent magnet machines, real time digital simulator, Electromagnetic transient analysis, embedded model of the machine, numerical instability.

I. Introduction

IN recent years, Permanent Magnet Synchronous Motors (PMSMs) are increasing applied in several areas such as traction, automobiles, robotics and aerospace technology [1]. Accurate digital simulation tools are necessary to evaluate their field performance particularly when they are driven with solid-state drives connected to larger electrical networks. One of the areas of interest is the design of controllers for these motor drives. In many applications the physical controls have to be designed and tuned for best performance. If the simulation of the motor and drive can be implemented in ***real-time,*** it becomes possible to interface the *physical* manufacturer-built controller (*not its model*) and protection equipment to the simulation using appropriate digital-analog and analog-digital converters. The real time digital simulator is a combination of specialized computer hardware and software designed specifically for the solution of power system electromagnetically transients in real-time. One such real-time digital simulator is the RTDS®. Since the RTDS combines the real time operation properties of analogue simulators with the flexibility and accuracy of digital simulation programs, there are many areas where this technology has been successfully applied [2].

Traditionally transient simulation programs use the dq0 based modeling method to model different types of machines and the machines are interfaced to the network. These interfaced models have interfacing delays which can cause numerical stability problems in the presence of other interfaced components, particularly when the case is running in real time. Modeling the permanent magnet machines in RTDS has important industrial applications such as maglev trains and new generation of automobiles.

This paper presents two different approaches to model the permanent magnet synchronous machine in RTDS, the traditional dq0 model of the machine and the embedded phase domain model. According to second method the machine is modeled as a set of time-variant mutual inductances. The machines are compared in the transient and steady state situations, and in the final step their performance is observed with an inverter, supplying the AC voltage for the terminals of the permanent magnet synchronous machines.

II. Equivalent Circuit Model of the PMSM

A. Structure of the Permanent Magnet Machine

The cross-sectional layout of a surface mounted permanent magnet motor is shown in Fig.1.

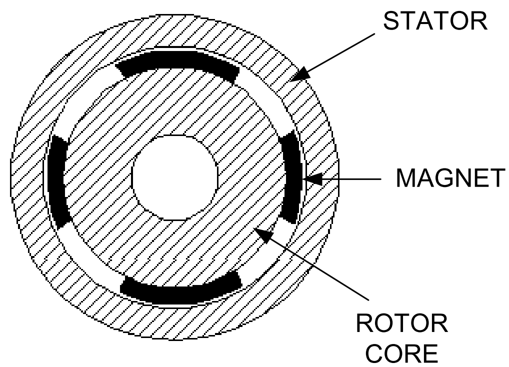

Fig. 1. Structure of the permanent magnet synchronous machine [3]

The stator carries a three-phase winding, which produces a near sinusoidal distribution of magneto motive force based on the value of the stator current. The magnets are mounted on the surface of the motor core. They have the same role as the

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field winding in a synchronous machine except their magnetic field is constant and there is no control on it [3].

B. Equivalent dq0 model of the PM machine

The dq0 equivalent circuit of the PM machine shown in Fig.2 is similar to the one for the synchronous machine; it has the armature resistance R_s, d and q axis leakage and mutual inductances l_s, L_{md} and L_{mq}.

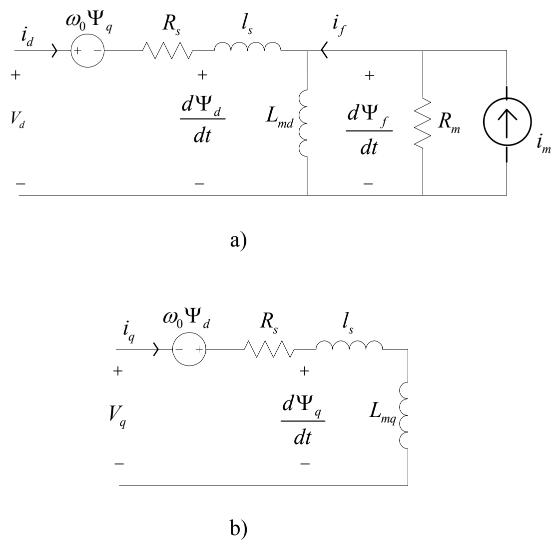

Fig. 2. Dq0 Equivalent circuit model of the PMSM a) D axis b) Q axis

The rotor magnet can be considered as a loop of constant current source, i_m located at the stator direct axis. Any change in the magnetic flux of the rotor magnet will cause an induced electromagnetic force, resulting in a circulating current in the magnet [3].Essentially resistance R_m connected across the direct-axis magnetization inductance L_{md} shows this effect [3]. There is no leakage inductance in the field. The permeability of the magnet material is almost unity so the air gap inductance seen by the stator is the same in direct and quadrature axes and also no saturation will happen inside the machine.

Fig. 3 shows the demagnetization curve of the magnet that can be divided into three regions by three lines, called no-load, rated-load and excessive-load lines [4].

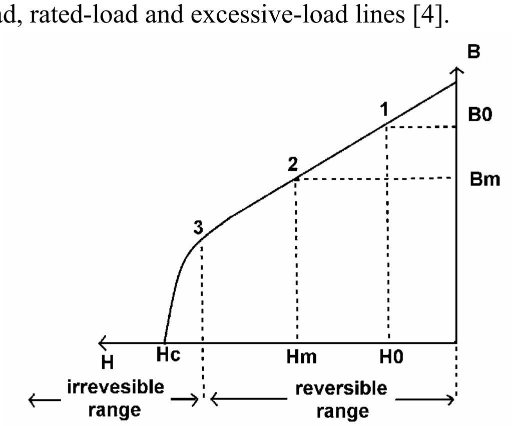

Fig. 3. Demagnetization curve of the permanent magnet [4]

We always try to not enter the excessive load region; otherwise the magnet is in danger of being damaged.

The equations for the dq0 model of the Permanent Magnet Synchronous Machine are:

$$\begin{aligned} \frac{d\Psi_d}{dt} &= V_d - R_s i_d - \omega\Psi_q \\ \frac{d\Psi_f}{dt} &= R_m i_m - R_m i_f \\ \frac{d\Psi_q}{dt} &= V_q - R_s i_q + \omega\Psi_d \end{aligned} \tag{1}$$

$$\begin{bmatrix} \Psi_d \\ \Psi_f \\ \Psi_q \end{bmatrix} = \begin{bmatrix} L_s + L_m & L_m & 0 \\ L_m & L_m & 0 \\ 0 & 0 & L_s + L_m \end{bmatrix} \begin{bmatrix} i_d \\ i_f \\ i_q \end{bmatrix} \tag{2}$$

The magnet is modeled by a current source i_m parallel to the resistance R_m. This part of the circuit can be modeled as a thevenin equivalent circuit, so that the direct axis equivalent circuit of the machine will be the circuit shown in Fig. 4.

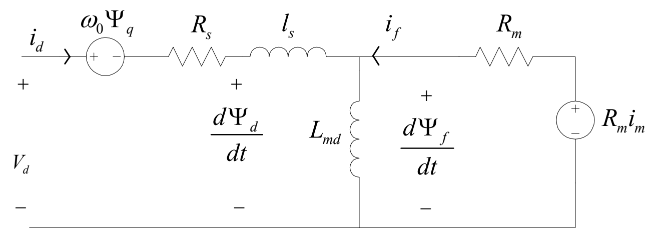

Fig. 4. D axis thevenin equivalent circuit of the PMSM

This model is similar to the conventional equivalent circuit of the synchronous machine, except there is no leakage inductance on the field.

C. Typical data for the PM motor

The data used for the simulations in this paper is from a 3-phase 208-volt, 6 kW test motor used by the Sebastian and Slemon [3]. Table 1 shows the value of the machine parameters. The base values of the voltage and current are RMS values of the stator phase voltage and current.

III. Interface Model of the PMSM in RTDS

Equations (1) and (2) can be used in the diagram of Fig.5 to interface the machine to the network. In every time step the program reads the voltage from the previous time step and applies the Park transform to get the dq0 components of the voltage, then using (1) it calculates the derivatives of the fluxes. Predictor corrector integration calculates the new values of the fluxes and by multiplying the inverse of inductance matrix to the fluxes, dq0 component of the currents can be obtained. Using the inverse Park transform we have the phase currents i_a, i_b and i_c that can be injected to the network.

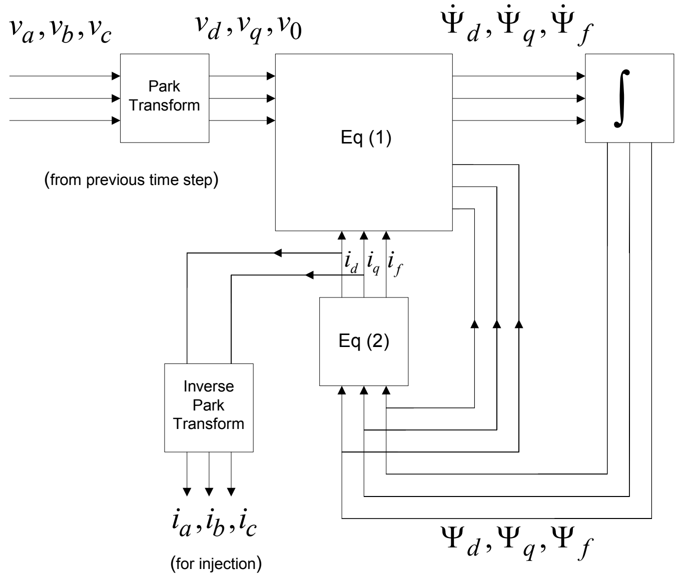

Fig. 5. Interfacing the machine to the network.

IV. Embedded Model of the PMSM in RTDS

The machine can be modeled as set of mutual inductances that change in value with time. In this case the model doesn't have the problem of interface that may cause numerical instabilities. This model doesn't use the Park transform and directly solves the machine equations in phase domain. For a machine, or in general, a set of time-varying mutual inductances can be written:

$$\underline{v}(t) = \frac{d}{dt}\left(\left[L(t)\right]\cdot \underline{i}(t)\right) \tag{4}$$

Where $\underline{v}$ and $\underline{i}$ are vectors of node voltages and branch currents and $[L]$ is the inductance matrix of the set. Using the trapezoidal integration we have:

$$\underline{i}(t) = \frac{\Delta t}{2}\left[L(t)\right]^{-1}\underline{v}(t) + \underbrace{\frac{\Delta t}{2}\left[L(t)\right]^{-1}\underline{v}(t-\Delta t) + \left[L(t)\right]^{-1}\left[L(t-\Delta t)\right]\underline{i}(t-\Delta t)}_{\underline{Ih}} \tag{5}$$

The machine can be modeled as set of current sources $\underline{Ih}$ parallel to a network of g values that can be obtained from the matrix $GL = \frac{\Delta t}{2}\left[L\right]^{-1}$.

A. Calculating the Phase Domain Inductances of the Permanent Magnet Machine

Equation (5) can be directly used to model the machine; however we need to have the value of the inductances as a function of time. Using an orthogonal transformation [5]; $T^{-1}(\theta) = T^{t}(\theta)$ the data in table I can be used to write:

$$\left[L_{abc}(\theta)\right] = T^{-1}(\theta)\begin{bmatrix} L_d & 0 & 0 \\ 0 & L_q & 0 \\ 0 & 0 & L_0 \end{bmatrix} T(\theta) \tag{6}$$

$$\begin{bmatrix} L_{af}(\theta) \\ L_{bf}(\theta) \\ L_{cf}(\theta) \end{bmatrix} = T^{-1}(\theta)\begin{bmatrix} L_{md} \\ 0 \\ 0 \end{bmatrix} \tag{7}$$

From (6) and (7) as an example we have:

$$L_{aa}(\theta) = L_s + L_m \cos 2\theta \ (\mathrm{H}) \tag{8}$$

$$L_{ab}(\theta) = L_{ba}(\theta) = -M_s - L_m \cos 2\left(\theta + \frac{\pi}{6}\right) \ (\mathrm{H}) \tag{9}$$

$$L_{af}(\theta) = L_{fa}(\theta) = M_f \cos\theta \ (\mathrm{H}) \tag{10}$$

The rest of inductances can be calculated in a similar way. The self inductance of the filed is the same in phase domain and dq0 domain:

$$L_f(\theta) = L_f \tag{11}$$

Where:

$$L_s = \frac{1}{3}\left(L_0 + L_d + L_q\right) \qquad L_m = \frac{1}{3}\left(L_d - L_q\right)$$
$$M_s = -\frac{1}{3}\left(-\frac{L_d + L_q}{2} + L_0\right) \qquad M_f = \sqrt{\frac{2}{3}}L_{md} \tag{12}$$

Because the matrix has to be inverted in every time step, it's important to have a non-singular inductance matrix, so the above parameters must satisfy this condition.

B. Inverting the Inductance Matrix of the Machine

The inductance matrix $[L]$ of the machine has to be inverted in every time step to calculate the G matrix of the machine. In RTDS this task has to be done in a limited amount of time. As a result having a fast routine for inverting the inductance matrix is important. Cholesky decomposition can be used for inductance matrices generated by FEM programs.

An analytical inverse of the inductance matrix can be prepared when this matrix is as defined as the equations in the preceding section.

V. Comparing Different Models of the PMSM in RTDS

The dq0 and embedded model of the PM machine have been implemented in RTDS in addition to the normal RTDS synchronous machine model. Both of the machine models are running at rated speed. It is possible to add a mechanical load to the machine and project ω from the previous time step [6].

A. Performance of the Machines in Steady State

The circuit of the machine is shown in Fig. 6. A 60 Hz three phase voltage source with 208 V line-line voltage supplies the machines.

The embedded model of the machine contains a component with 4 coupled windings. The model allows entering the parameters of the PM machine as input. The windings A, B and C are Y connected armature windings with the neutral isolated from the ground. A very large resistance provides the isolation. The equivalent current source and resistance of the magnet is modeled by a Thevenin equivalent circuit with the 1.272 kV voltage source in series with the magnet resistance. The armature resistances are in series with the inductances.

Dq0 PM machine is connected to a similar AC source and is a component with three nodes. Parameters of the machine are identical, and the initial rotor angle (θ_0) for both of them is set to 0°.

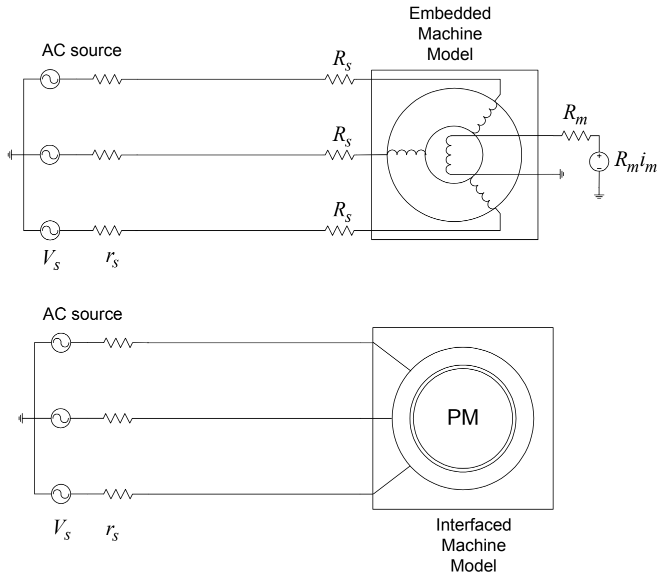

Fig. 6. Test circuit for comparing the new machine models

There is an interfaced model of the synchronous machine in RTDS. By setting the parameters, this machine is used as a permanent magnet machine and the simulation results were compared with the embedded and interfaced models of the machine in Fig. 7.

Fig. 7 shows the steady state waveform of the phase “A” voltage and current. The currents of the three machines are exactly the same; although there is a 5μS delay in the current of dq0 machine model, which is the error of projection.

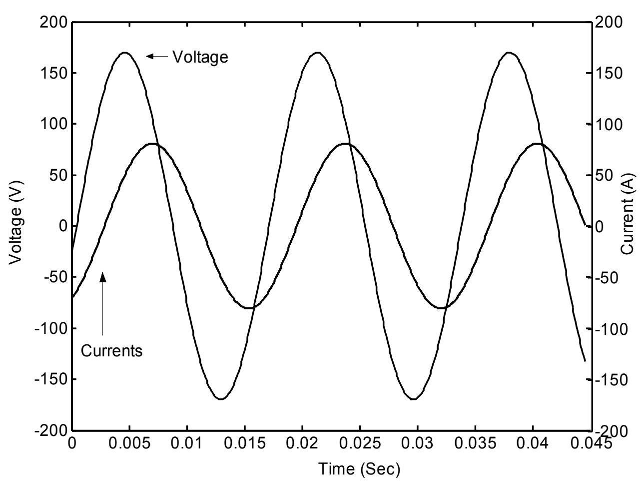

Fig. 7. Performance of the Machines in Steady State

The equivalent circuit of the machine in steady state is shown in Fig. 8. The term $\sqrt{2/3}$ at the voltage source is because of the Park transform that we used.

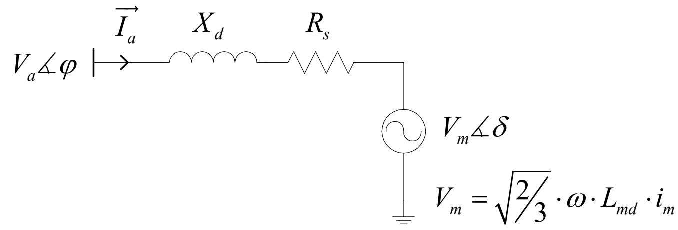

Fig. 8. Equivalent circuit of the machine in steady state

Solving the circuit gives us the values of the armature current at the steady state which agrees with the simulation results.

$$I_{a,peak} = \frac{V_{a,peak}\measuredangle\varphi - \sqrt{2/3}\cdot\omega\cdot L_{md}\cdot i_m\measuredangle\delta}{jXd + R_s} = 79.953\ \text{(A)} \tag{13}$$

B. Three Phase Short Circuit on the terminals of the Machine

Fig. 9. shows the current waveforms of the machines after the short circuit. The currents of the machines are identical however the delay of the current in dq0 model is more after the short circuit.

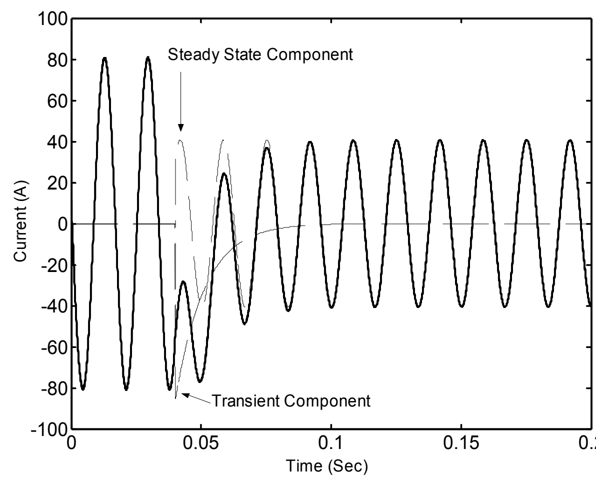

Fig. 9. Three phase short circuit current of the machines

Equation (14) gives the peak value of the short circuit current at the steady state which agrees with the simulation result.

$$I_{sc,peak} = \sqrt{\frac{2}{3}}i_m \frac{\omega L_{md}}{\omega^2 L_d L_q + R_s^2}\sqrt{\left(\omega L_q\right)^2 + R_s^2} = 40.721\ \text{(A)} \tag{14}$$

The transient part of the short circuit current contains three components: steady state AC current, transient exponentially damping AC current with the time constant T_d', and the transient exponentially damping DC component with the time constant T_a, where:

$$T_d' = \frac{l_s \parallel L_{md}}{R_m} = 0.084\ \text{(ms)} \tag{15}$$

$$T_a = \frac{1}{\frac{R_s}{2}\left(\frac{1}{L_d'} + \frac{1}{L_q'}\right)}\ \text{[7]} \tag{16}$$

We ignored the dampers in the machine so:

$$L'_d = L'_q = L_d \tag{17}$$

And

$$T_a = \frac{L_d}{R_s} = 11.253 \text{ (ms)} \tag{18}$$

Equation (18) shows that T'_d the transient time constant of the machine is very small, and it's because R_m the magnet resistance is very large. This interesting fact removes the transient AC component of the short circuit current, and we just deal with the damping DC current and steady state current. Fig. 9 verifies the time constants of the waveforms obtained from the short circuit.

C. *Numerical Stability of the Machines*

Numerical stability of the different types of the permanent magnet synchronous machine was studied by running the case with different time steps. The embedded model of the machine has the best performance among these models and is stable even for very large time steps. This model is accurate and the results are the same in different time steps. The RTDS synchronous machine is unstable with the time-steps larger than $169\mu S$, the same thing happens with the interface model with the time-steps larger than $167\mu S$. Typical time-steps are $50\mu S$. Similar results obtained when an inductive source supplied the machines. The conventional model of the machine becomes numerically unstable at large time-steps with the inductive source. Poor performance of interface machine in larger time steps is shown in Fig. 10.

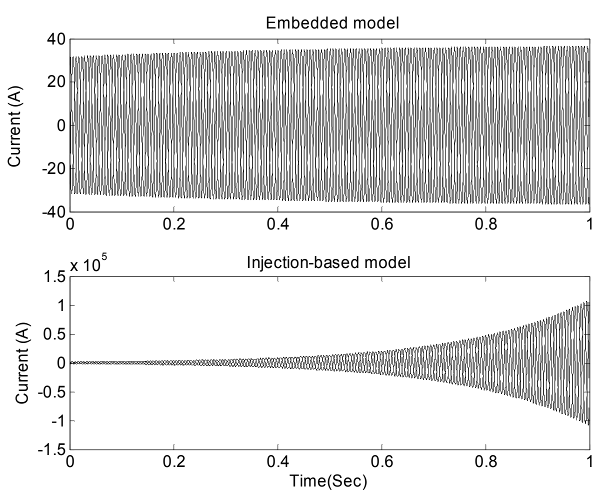

Fig. 10. Armature currents of the machines in time step 167 µS

VI. Finite Element Approach for Modeling the Electric Machines in RTDS

Equations (9)-(12) are approximate formula for calculating the inductances of the synchronous machine however, in a real machine inductances change as a more sophisticated function of rotor position. Numerical methods like FEM base methods can be used to model the electromagnetic phenomena in the electric machines and also calculate the inductances, however these programs are very slow and are not practical for network solution.

In this paper the calculated inductances from [8] are tabulated and used to run the embedded phase domain case of the synchronous machine with more accurate inductances. The machine is a 90kVA, 200V (L-L), 400 Hz, 4 pole, 3-phase salient pole synchronous generator (aircraft generator). In another case the traditional dq0 model of the synchronous machine uses the simplified dq0 equivalent circuit of this machine:

Fig. 12 shows the phase "A" open circuit voltage of both machines which has a good agreement with the results in [9].

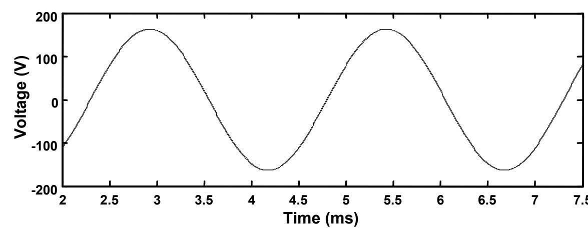

Fig. 12. Phase A open circuit voltages of the machines

Phase A armature current and the field current of the machine after a 3 phase symmetric short circuit is shown in Fig. 13. Fig. 13. (a) is the stator current of the dq0 machine and Fig. 13. (b) is for the embedded phase domain machine with real inductances.

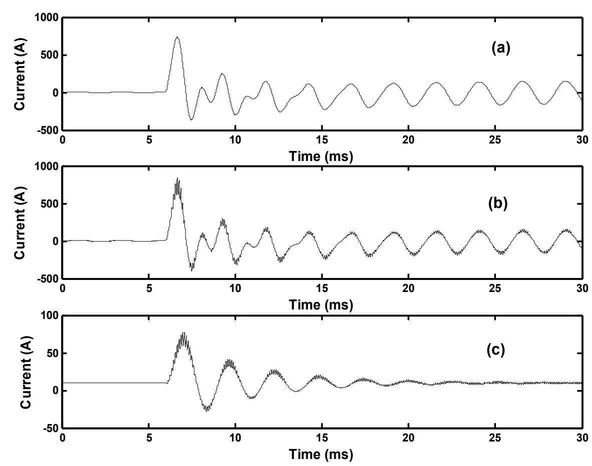

Fig. 13. The currents of the machine in short circuit a) phase current of dq0 model of the machine b) phase current of embedded model of the machine c) Field current of embedded model of the machine

VII. Conclusion

The dq0 and embedded model of the permanent magnet machine has been implemented and verified in RTDS. The models have identical results. The embedded model of the machine is more stable with large time-steps than the conventional model of the machine. The problem of the embedded model of the machine is the calculation load of the model which is higher than the conventional model, because the inductance matrix of the machine has to be inverted in every time step. With the new RPC cards in RTDS, this task is not a significant problem anymore. Coupling the FEM based programs and transient simulation programs can be a very successful step in modeling the power system network with more accuracy.

VIII. Appendix

TABLE I
TYPICAL DATA FOR THE PMSM [SLEMON]

Parameter	Symbol	Typical value	
Line-line rated voltage	V_{ll}	208 V	
Rated stator current	I_l	16.7 A	
Rated power	P_{rated}	6016 W	
D-axis inductance	L_d	.249 pu	4.76 mH
Q-axis inductance	L_q	.249 pu	4.76 mH
Stator leakage inductance	l_s	.109 pu	2.09 mH
Stator resistance	R_s	.059 pu	0.423 Ω
Magnet resistance	R_m	1.94 pu	13.92 Ω
Equivalent magnet current	i_m	5.47 pu	91.35 A

IX. Acknowledgement

The authors gratefully acknowledge the support of RTDS Technologies Inc.

X. References

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XI. Biographies

A. B. Dehkordi (M'04) was born in Shahrekord, Iran in 1977. He received his B.Sc. and M.Sc. degrees in Electrical Engineering from Sharif University of Technology, Tehran, Iran in 1999 and 2002 respectively. He worked as a research engineer for Sharif University of Technology in power quality projects in 2002. He is currently a Ph.D. student at the Department of Electrical and Computer Engineering at the University of Manitoba. His research is on the modeling of electric machines for the Real Time Digital Simulation.

A. M. Gole (M'82, SM'04**)** obtained the B.Tech. (EE) degree from IIT Bombay, India in 1978 and the Ph.D. degree from the University of Manitoba, Canada in 1982. He is currently a Professor of Electrical and Computer Engineering at the University of Manitoba. Dr. Gole's research interests include the utility applications of power electronics and power systems transient simulation. As an original member of the design team, he has made important contributions to the PSCAD/EMTDC simulation program. Dr. Gole is active on several working groups of CIGRE and IEEE and is a Registered Professional Engineer in the Province of Manitoba.

T. L. Maguire (M'84, SM'04) was born in Canada in 1951. He received a Ph.D. degree in 1992 at the University of Manitoba. Dr Maguire is with the RTDS technologies Inc. He is experienced with FACTS and HVDC as well as with power system control application and simulations.