

# A method for coupling electromagnetic transients programs with finite element magnetic field solvers

E. Melgoza, *member, IEEE*, J. L. Guardado, *Senior member, IEEE*, V. Venegas

**Abstract**—A method for coupling electromagnetic transients programs and finite element field solvers is introduced. The field model takes into account the detailed structure of the magnetic device, including saturation effects, and provides a set of parameters in the form of an inductance matrix, which is then communicated to the transients program. The parameters are updated at each time step to reflect the true operating point of the device. The effect of the time delay between the computation of the parameters and their use in the transients program is investigated.

**Keywords:** Inductance matrix, magnetic field, finite elements, coupled problems

## I. INTRODUCTION

The detailed modeling of saturable components is often required in transient simulations to accurately reproduce phenomena such as inrush currents, ferro-resonance, harmonic generation and propagation and others. Similarly, it is necessary to predict the performance of a magnetic device at the design stage, taking into account system characteristics and the external network. These analysis tasks have been tackled by two kinds of numerical codes, namely circuit simulators and field solvers.

Circuit simulators perform an efficient solution provided a circuit model can be specified for the device. In the case of one-winding devices, it is not difficult to obtain a set of current-flux linkage pairs, thus characterizing the nonlinear behavior of the component. Transients programs provide support for these two-terminal saturable components. Things get complicated, though, for multi-winding devices: the off-line calculation of current-flux linkage tables is impractical when the number of windings exceeds two or three [1]; moreover, transients programs do not support multi-winding table models of saturable components and as a result, the nonlinear behavior is modeled by using a single two-terminal nonlinear inductance. Thus, although a fair set of tools for modeling magnetic components exist in transient programs, there is still the need to consider more detailed models, as in the case of zero-sequence flux in multiphase transformers.

Consideration of magnetic saturation is provided by magnetic circuit models [2]. In this approach, each section of the core and the air leakage paths are modeled as a network of magnetic reluctances; the magnetic circuit model is coupled to the electrical network using duality concepts. Some simplifications are necessary in order to construct magnetic circuit models: uniform flux distribution in core sections and assumed flux trajectories in the air.

Field solvers, for instance one based on the finite element method (FEM), can accurately take into account the material nonlinearity and winding connections to yield a solution for the magnetic field distribution. Field solvers have evolved to take into account the effect of eddy currents, material anisotropy and hysteresis. One can distinguish between static and transient field solvers, where in the latter case a time stepping process is performed. The natural source of excitation for magnetic field problems is a current density and therefore the incorporation of voltage sources into the model is only possible by constructing an augmented system of equations, where the field and circuit equations are solved simultaneously. However, field solvers do not provide the variety of power system components needed for a full-scale simulation, like multiphase transmission lines, circuit breakers, control systems, etc. Addition of the necessary components to the coupled field-circuit codes is a possibility, hampered by several factors such as unavailability of source code for the circuit solver and effort duplication. Another approach, explored in this contribution, is the coupling of separate circuit and field simulators, communicating through a set of variables at each time step; in this way, existing codes are used and the experience accumulated in their development is preserved.

In order to achieve the communication between field and circuit solvers, the huge amount of information provided by the field solution must somehow be compressed into a meaningful way before it is fed back to the circuit solver. The most natural way of doing this is through the well known concept of inductance, a quantity associated with each terminal pair by means of self and mutual coefficients. The rest of the paper is organized as follows: first, the concept of differential inductance and an efficient method for its computation are reviewed. The mechanism for the communication between the field solver and the transients program is then discussed, with special reference to ATP (Alternative Transients Program); finally, simulation results obtained with the proposed method are presented.

The authors are with Instituto Tecnológico de Morelia, Av. Tecnológico 1500, Morelia, Mich. México, C.P. 58120.  
E-mail: emelgoza@itmorelia.edu.mx

---

Presented at the International Conference on Power System Transients (IPST'05), Montreal, Canada on June 19–23, 2005.  
Paper No. IPST05-179.

## II. APPARENT AND DIFFERENTIAL INDUCTANCE

We must distinguish between two definitions of inductance [3]. The apparent inductance is the usual relation:

$$L^a = \frac{\lambda}{i}. \quad (1)$$

On the other hand, the incremental or differential inductance is defined as:

$$L^d = \frac{\partial \lambda}{\partial i}. \quad (2)$$

Each of these definitions leads to a different simulation scheme.

### A. Apparent inductance

If the inductance is a function of the current given by

$$L^a = L^a(i), \quad (3)$$

then the voltage on the device terminals is

$$v = \frac{d}{dt}(L^a i) = L^a \frac{di}{dt} + i \frac{dL^a}{dt}. \quad (4)$$

The last term can be written as

$$\frac{dL^a}{dt} = \frac{dL^a}{di} \frac{di}{dt}, \quad (5)$$

so that

$$v = \left( L^a + i \frac{dL^a}{di} \right) \frac{di}{dt}, \quad (6)$$

### B. Differential inductance

If the flux linkage is a function of current given by

$$\lambda = \lambda(i) \quad (7)$$

then

$$\Delta \lambda = \frac{\partial \lambda}{\partial i} \Delta i. \quad (8)$$

Dividing by a small  $\Delta t$  and taking the limit leads to

$$\frac{d\lambda}{dt} = \frac{\partial \lambda}{\partial i} \frac{di}{dt}, \quad (9)$$

from where

$$v = L^d \frac{di}{dt}. \quad (10)$$

A comparison of expressions (6) and (10) reveals that the latter is more easily incorporated into existing codes, since it takes the same form as that of a constant inductance. Therefore, the differential inductance will be used as the basis for the field-circuit coupling.

## III. INDUCTANCE MATRIX COMPUTATION

The key element in the coupling method is the determination of the (differential) inductance matrix. Several schemes have been proposed for its computation [3], [4]. In the following paragraphs, we review the method proposed in [4].

Assuming a plane Cartesian geometry, the resulting magnetic field equation is:

$$\frac{\partial}{\partial x} \left( \nu \frac{\partial A_z}{\partial x} \right) + \frac{\partial}{\partial y} \left( \nu \frac{\partial A_z}{\partial y} \right) + J_z = 0, \quad (11)$$

where  $\nu$  is the material reluctivity,  $A_z$  is the normal component of the magnetic vector potential and  $J_z$  is the normal (and only) component of current density. Equation (11) is discretized by the finite element method, resulting in the nonlinear algebraic equation

$$\mathbf{K}(\mathbf{a})\mathbf{a} + \mathbf{f}(\mathbf{i}) = \mathbf{0}, \quad (12)$$

where  $\mathbf{K}$  is the stiffness matrix,  $\mathbf{a}$  is the vector of unknown nodal magnetic vector potentials and  $\mathbf{f}$  is the force vector, which is a function of the current  $\mathbf{i}$  in the windings. In (12), the current  $\mathbf{i}$  is the independent variable, and the computation yields  $\mathbf{a}$  as a dependent variable. Equation (12) has the form

$$\mathbf{F}(\mathbf{a}, \mathbf{i}) = \mathbf{0}. \quad (13)$$

The inductance matrix computation proceeds by calculating the sensitivity of the magnetic vector potential  $[\mathbf{a}]$  to a change in the current  $i_w$  in the windings:

$$\frac{\partial \mathbf{F}}{\partial \mathbf{a}} \frac{\partial \mathbf{a}}{\partial i_w} = - \frac{\partial \mathbf{F}}{\partial i_w}, \quad (14)$$

where  $\frac{\partial \mathbf{F}}{\partial \mathbf{a}}$  is the Jacobian matrix of system (13),  $\frac{\partial \mathbf{F}}{\partial \mathbf{a}}$  can be calculated from the geometry of the model and  $\frac{\partial \mathbf{a}}{\partial i_w}$  is obtained by solving the indicated linear equation. On the other hand, the flux linkage is a function of the magnetic vector potential, from where  $\frac{\partial \lambda}{\partial \mathbf{a}}$  can be calculated. Combining the two vectors one has

$$\frac{\partial \lambda_v}{\partial i_w} = \frac{\partial \lambda_v}{\partial \mathbf{a}} \frac{\partial \mathbf{a}}{\partial i_w}, \quad (15)$$

which is an entry of the differential inductance matrix. For a more detailed treatment of the inductance computation process for the Cartesian case, as well as for the axisymmetric case, see [4].

It is assumed that the inductance computation is performed after convergence of the nonlinear magnetic field problem has been achieved, for a given set of input currents. Therefore, in the inductance computation, the Jacobian matrix is available from the preceding nonlinear iteration. For each winding, the solution of a linear equation is required, and the inductance matrix is obtained column-wise. If a factorization of the Jacobian matrix is available, the computation of the inductance matrix is very efficient.

#### IV. COUPLING WITH THE TRANSIENTS PROGRAM

Once calculated, the inductance coefficients are fed into the transients program. The form in which this is done depends on the particular implementation being used: there must be a way of stopping the transients program time stepping, calling an external program (which in this case is the finite element program), and read the updated values of the parameters. Also, the inductance values are one step behind because the current levels used in its calculation are those from the previous time step. In the literature referring to coupled problems, this approach is called weak coupling, because the circuit and field simulations use parameters separated one time step. A strong coupling can only be achieved by writing and solving the extended equation. The consequences of this time lag are discussed below. The transients program we have used in this implementation is ATP. The control of an external program can be implemented through the facilities provided by the TACS (Transient Analysis of Control Systems) and MODELS components. For the implementation discussed here, the TACS facilities have been selected in the form of a device type 69 for the control of the external field solver; this facility allows the incorporation of arbitrary user code in any programming language as long as the object can be linked with the ATP, as reported in [5]. As for the updating of the inductance coefficients, two possibilities arise: the first is to use RLC branches to specify the inductance coefficients as calculated by the field solver, and the second to resort to an associated circuit (equivalent resistance and current source) for each inductance.

##### A. TACS-controlled RLC branches

TACS-controlled resistances and voltage or current sources are well known. Recently, support for TACS-controlled RLC branches has been added [5], opening the possibility of supplying a set of inductance values. Initial tests, however, have convinced us not to follow this path for the following reasons: only un-coupled branches are supported at the moment, and there seems to be some kind of checking of the arguments, since the program stops with a run-time error when one of the inductance coefficients is negative; also, for constant or nearly constant inductance values the results do not seem correct. As a result, this approach can not be used at the moment; however, it is our impression that making the necessary adaptations to the program source code could make this route the right path to follow.

##### B. Associated circuits

Since the direct specification of inductance coefficients (coupled or not) is not satisfactory, we have had to use the associated circuit of an inductance after integration by the trapezoidal rule, just as the constant inductances are discretized by the transients program logic [6]. This is possible since TACS allows the specification of resistance and current sources to be connected to the electrical network; one possible scheme for including the system of coupled inductances is to synthesize an equivalent network of TACS-controlled un-coupled resistances. For instance, a two-winding transformer

has a four-entry inductance matrix given by

$$[L] = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}. \quad (16)$$

Assuming that both windings are grounded, then an equivalent  $\pi$  or  $T$  network of un-coupled inductances can be found [7]. The inductances are converted to a parallel connection of a resistance and a current source following the trapezoidal rule. The transient simulation is started by ATP using initial values for the inductances and calculating a set of winding currents. At the end of the time step, the TACS part is executed and a call the external finite element program is made, which supplies updated inductance values. The TACS-controlled resistance network is updated and the process repeated for the remaining time steps.

##### C. Time lag

It is well known that inaccuracies result from the time lag introduced by the TACS computation of nonlinear elements [8]. To illustrate the effects of this time lag in the inductance values, we present the solution of the simple network shown in Fig.1, where the nonlinear inductance is given by

$$\lambda = \tanh(i) + 0.01i \quad (17)$$

(for simplicity, this analytic function is specified as an external Fortran function, and is not the result of a field model). The circuit parameters are:  $V_s = 400\cos(2\pi ft - \phi)$  V,  $t \geq 0$ s,  $R = 0.05\Omega$ ,  $f = 60$ Hz,  $\phi = 90^\circ$ .

Figs. 2–4 compare the nonlinear ATP solution with the ATP-TACS solution for various time step sizes; the differential inductance was specified, as discussed above. It is seen that for a large time step, the weak coupling solution is inaccurate, although stability is preserved. For smaller time steps, the accuracy is improved; satisfactory results, however, are only obtained with a very small time step (0.00001)s, which is about 10 times smaller than the step size required for the ATP nonlinear solution. Applied to a coupled field-circuit model, however, this solution to the time delay problem can be impractical since the number of field equations far exceeds the number of circuit equations and at each time step several iterations of the nonlinear magnetic field problem have to be calculated.

Compensation has been used to alleviate the time delay problem [8]. However, the authors are unsure whether this approach is available in ATP. Another possible solution would be the use of a prediction formula to try to approximate the true value of the currents over the next time step, but this does not really solve the problem of the coupling. Clearly, a sort of iteration is needed in order to achieve a more tightly coupled solution and therefore allow the use of a larger time step. In ATP, this could be implemented in the form of an iterated type 94 device in Models, or the issuing of SPY commands to force a number of ATP-TACS iterations for each time step. Incidentally, the compensation approach requires iteration inside TACS [8], so that repeated solution of the finite element equations would be required at each time step. These

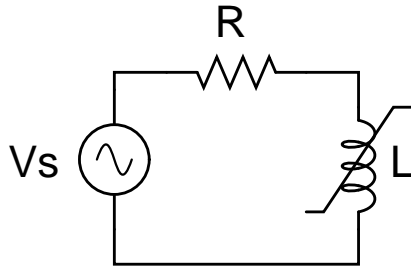


Fig. 1. Test circuit with nonlinear inductor (17).

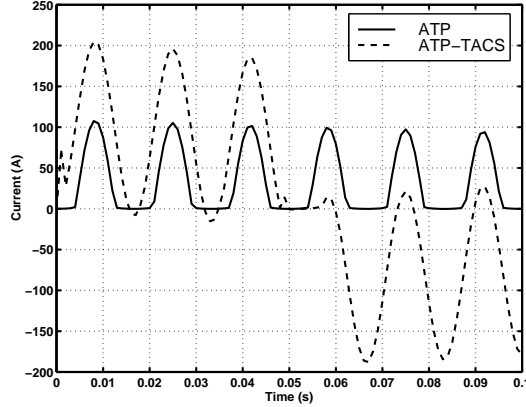


Fig. 2. Current in nonlinear inductor (17) solved by nonlinear (ATP) data case and ATP-TACS (time step 0.001s).

approaches are being investigated, but no conclusions can be offered at this stage. In the following, the basic scheme with TACS and a reduced time step is used.

### V. COUPLED ATP-FEM SOLUTION

For the transformer geometry shown in Fig. 5, the coupled solution of the energization transient has been obtained. The secondary of the transformer was left open, so that the circuit reduces to the case of a single nonlinear inductance, as shown in Fig. 1. The circuit parameters are:  $V_s = 50\cos(2\pi f - \phi)$  V,  $t \geq 0$ s,  $R = 0.05\Omega$ ,  $f = 60$  Hz,  $\phi = 90^\circ$ .

In order to compare the ATP-only solution (obtained by modeling a nonlinear inductor, type 93) against the ATP-FEM coupled solution, a table of current-flux linkage was calculated using the finite element program, resulting in the values shown in Table I. The corresponding profiles of apparent and differential inductance are shown in Fig. 6. It is interesting to note how, although the flux linkage curve is monotonically increasing, the apparent and differential inductance profiles show peaks and valleys in the low-current region. This is due to the way the flux distributes inside the core and in the surrounding air region and the shape of the BH curve of the magnetic material.

Figs. 7 and 8 show the current and voltage in the primary winding of the transformer for a time step of 0.001s. The values calculated by ATP alone (with the winding modeled as a nonlinear inductance), are compared against those calculated by the ATP-FEM coupled scheme. The accuracy is not good,

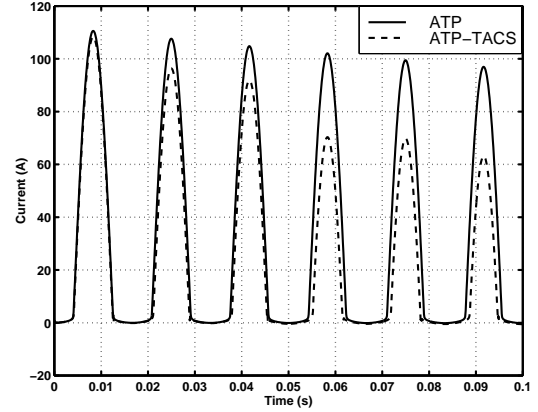


Fig. 3. Current in nonlinear inductor (17) solved by nonlinear (ATP) data case and ATP-TACS (time step 0.0001s).

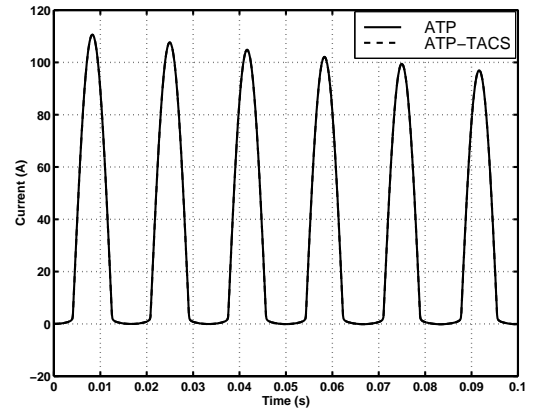


Fig. 4. Current in nonlinear inductor (17) solved by nonlinear (ATP) data case and ATP-TACS (time step 0.00001s).

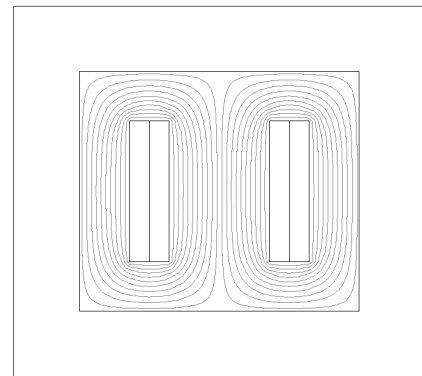


Fig. 5. Flux contours from finite element solution of the magnetic vector potential. An air region surrounding the magnetic core is shown.

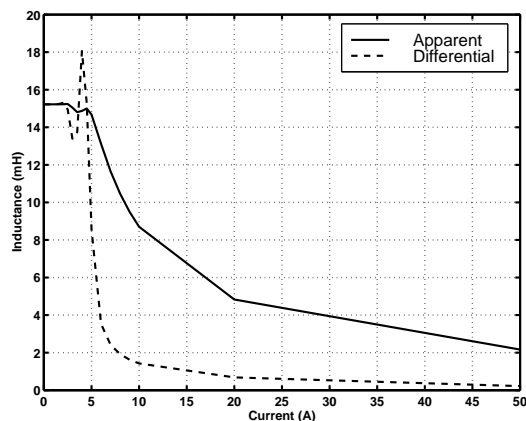


Fig. 6. Apparent and differential inductance as a function of current, for transformer primary winding.

TABLE I  
FLUX LINKAGE (TRANSFORMER PRIMARY WINDING).

Current (A)	Linkage (mWb)
0.5	7.611042
1.0	15.222563
1.5	22.840930
2.0	30.458964
2.5	38.099410
3.0	45.136547
3.5	51.813802
4.0	59.480589
4.5	67.505614
5.0	73.390009
6.0	78.707155
7.0	81.589619
8.0	83.736003
9.0	85.498725
10.0	87.016969
20.0	96.585016
50.0	108.205983
100.0	116.358387
1000.0	194.508814

although stability is not lost; this confirms what had been observed for the analytic inductance expression above. The accuracy of the current is the worst affected, while the voltage values are closer to the true values.

A smaller time step yields better results, as shown in Figs. 9 and 10, where the time step is 0.0001s. The current values are considerably closer to the reference value, and the voltage values are practically the same. In this case, the time step sizes of both the ATP-only and of the ATP-FEM solutions were 0.0001s, so that the accuracy is better than expected.

Finally, Fig. 11 compares the calculated primary currents when a time step of 0.00001s. The accuracy is only slightly improved and the added computational cost is not justified in this case.

The time required to simulate the whole transient is obviously larger for the ATP-FEM coupled solution, because the solution of a finite element nonlinear problem is required at each time step, which involves the repeated solution of a set of equations whose number is around 1000 for the test case, or

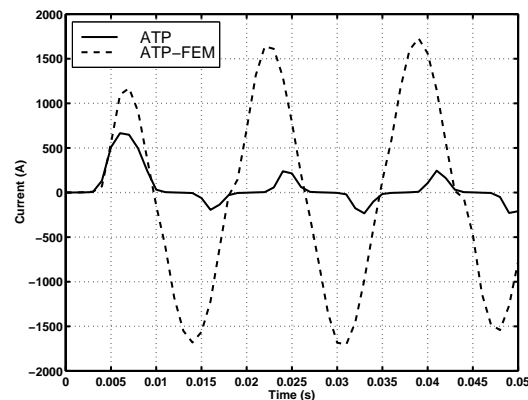


Fig. 7. Current in the primary winding of the transformer during energization. Time step is 0.001s.

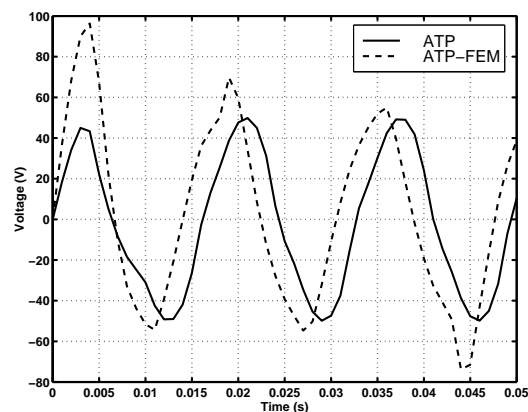


Fig. 8. Voltage in the primary winding of the transformer during energization. Time step is 0.001s.

larger for more detailed field models. Each time step requires an iteration process, where the number of iterations depends on the current level: for low current, 3 iterations are generally enough, but for the cases with current of hundreds of amperes, around 10 iterations are needed. For the cases tested, a time step of 0.0001s results in a simulation time of a few minutes for the ATP-FEM scheme, compared with a few seconds for the ATP-only solution.

## VI. CONCLUSIONS

A method for obtaining coupled solutions for circuit-field cases has been presented. The method relies on the computation of differential inductance coefficients for the magnetic structure, and the communication of those inductance values to the transients program, where the required discretization is used to update the electrical network parameters. The fact that current values are used to calculate the inductances used in the next time step introduces a time lag which results in accuracy problems, forcing a small time step if accuracy is to be maintained. Remedies for this accuracy problem have been proposed in the context of other nonlinear TACS elements, and further work is being carried out for this purpose.

Another area to improve is the mechanism to feed back

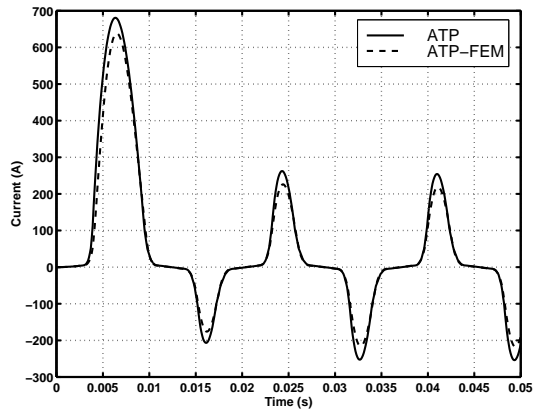


Fig. 9. Current in the primary winding of the transformer during energization. Time step is 0.0001s.

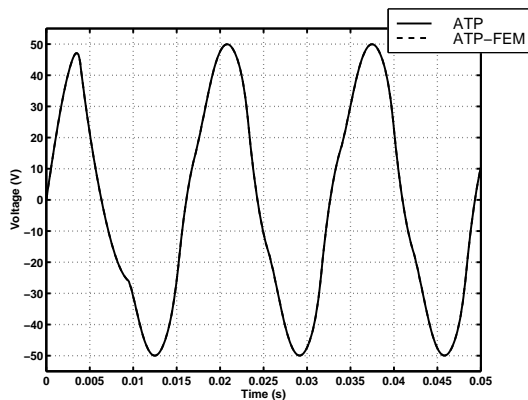


Fig. 10. Voltage in the primary winding of the transformer during energization. Time step is 0.0001s.

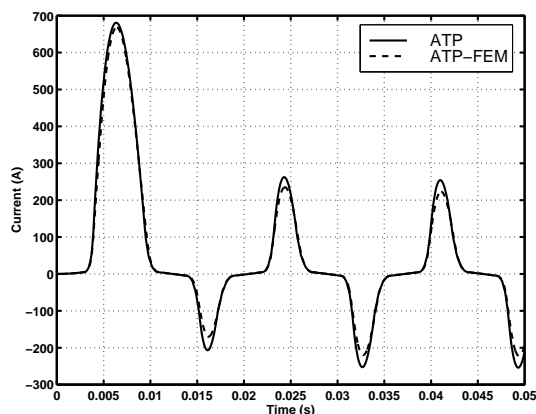


Fig. 11. Current in the primary winding of the transformer during energization. Time step is 0.0001s.

the inductance coefficients into the transients program. Uncoupled TACS-controlled resistances and current sources had to be used in the present implementation, because initial tests using the more direct updating of un-coupled RLC branches yielded mixed results. Also, the addition of TACS-controlled coupled RLC branches would be very convenient for the kind of work discussed here, provided the a general implementation (and not only one where an earth path is assumed) is contemplated. Nonetheless, the coupling of a transients program and a finite element solver has been demonstrated, opening the way for future developments, for instance field modeling of transmission lines, cables and electrical machines.

## REFERENCES

- [1] E. Melgoza, D. Rodger, "Comparison of table models of electromagnetic actuators", *IEEE Transactions on Magnetics*, Vol. 38, No. 2, pp.953–956, March 2002.
- [2] C. M. Arturi, "Transient simulation of a three-phase five-limb step-up transformer following an out-of-phase synchronization", *IEEE Transactions on Power Delivery*, Vol.6, No.1, pp.196–207, January 1991.
- [3] N. A. Demerdash, T. W. Nehl, "Electric machine parameters and torques by current and energy perturbations from field computations", *IEEE Transactions on Energy Conversion*, Vol. 14, No. 4, pp.1507–1522, December 1999.
- [4] E. Melgoza, J. L. Guardado, V. Venegas, J. H. Tovar, "Computation of the differential inductance matrix of magnetic devices by a sensitivity approach", *11th International IGTE Symposium on Numerical Field Computation in Electrical Engineering*, Graz, Austria, September 2004.
- [5] Can/Am EMTP News, Vol. 3, No. 2, April 2003.
- [6] H. W. Dommel, "Digital computer solution of electromagnetic transients in single- and multiphase networks", *IEEE Transactions on Power Apparatus and Systems*, Vol.88, No.2, pp.734–741, April 1969.
- [7] Bonneville Power Administration, "Electromagnetic Transients Program (EMTP) Theory Book", Portland, 1986.
- [8] S. Lefebvre, J. Mahseredjian, "Improved control systems simulation in the EMTP through compensation", *IEEE Transactions on Power Delivery*, Vol.9, No.3, pp.1654–1662, July 1994.

## BIOGRAPHIES

**Enrique Melgoza** (M<sup>1993</sup>) was born in Morelia, Mexico in 1967. He received the BSc (1994) and MSc (1996) degrees in Electrical Engineering from Instituto Tecnológico de Morelia (ITM) and the PhD from the University of Bath, U.K. Since January 1996 he is a member of the faculty at ITM's Programa de Graduados e Investigación en Ingeniería Eléctrica. His interests include finite element analysis of electromagnetic fields, dynamic performance of electric machinery and electromagnetic transients.

**Leonardo Guardado** was born in Los Mochis, Mexico, in 1959. He received the BSc degree from UMSNH in 1983, the MSc (1986) and PhD (1989) degrees in electrical engineering from UMIST, Manchester, UK. He joined IIE (Instituto de Investigaciones Eléctricas) in 1983 working in the modeling of switching transients in transmission lines and electrical equipment. He is currently with Instituto Tecnológico de Morelia.

**Vicente Venegas** was born in Morelia, Mexico. He received the BSc (1992) and MSc (1996) degrees in electrical engineering from Instituto Tecnológico de Morelia, and the PhD (2004) from Instituto Politécnico Nacional. His interests include electromagnetic transients and finite element analysis of electrical machines. He is currently with Instituto Tecnológico de Morelia.