Phase-Domain Network Equivalents for Electromagnetic Transient Studies

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Abstract—This work presents a methodology for deriving a sparse network equivalent representation in the phase-domain based on time-domain fitting. The proposed formulation corresponds to a Norton Equivalent (admittance in parallel with a steady state current source) for three-phase system representations. The equivalent admittance is a polynomial matrix in the *z*-domain. Imbedding a set of constraints in the fitting equations, which are solved using quadratic programming, attains the robustness of the representation, i.e. stability and passivity. Results demonstrating the features of the derived representation are presented for the case of a three-phase circuit comprised by an alternating voltage source and a transmission line.

Keywords: Electromagnetic transients, network equivalent, phase-domain, time-domain fitting, constrained least squares.

I. INTRODUCTION

For electromagnetic transient studies, a detailed transmission system representation can become unacceptable in digital simulations, due the complexity and size of modern power systems. Besides, it requires a heavy computational effort, especially in applications for real-time transient calculations. The adopted solution for large system is to have a detailed representation of the region of great interest, usually called study system, and to represent the remaining system, called external system, by network equivalents. To analyze the transient phenomena with high accuracy, it is necessary to obtain equivalents that are accurate and also stables. For network equivalent representations, the derived impedance or admittance can be obtained in modal domain, as addressed in several works [1]-[5] or in the phase-domain as shown here.

The goal of this work is to obtain network dynamic equivalents for transient studies considering: a phase-domain representation, the internal sources of the study system, the accuracy. This work presents network equivalents described by a technique based on time-domain fitting, which was used, initially, for transmission line representation in the phase-domain [6]. Here, sparse network equivalents in phase-domain

for the three-port[1] case are obtained to represent a three-phase system taking internal sources into account. The proposed formulation corresponds to the multiphase Norton Equivalent Circuit (admittance matrix in parallel with a steady state current source vector) shown in Fig.1, which is suitable to represent traveling wave effects in transmission lines.
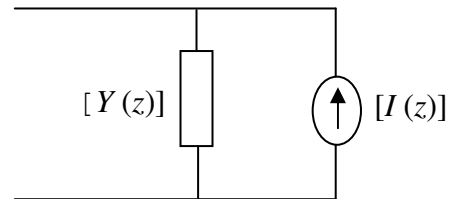

Fig. 1. Network equivalent in the frequency domain.

Each element of the admittance matrix is a rational polynomial function in the *z*-domain. Also, to ensure robustness, stability and passivity constraints are included in the fitting process to determine the equivalent network parameters by forcing the poles of the fitted functions to be inside the *z*-plane unity circle and by forcing the eigenvalues of the real part of the admittance matrix $Y(\omega)$ to be positive.

The fitting equations and constraints are solved simultaneously using quadratic programming [7], [8] and the degree of sparsity of the parameters is adjusted to control the accuracy of the system representation. The chosen equivalent form can be easily integrated into transient calculation routines. It is also closely related to the topology of digital filters. This facilitates the implementation of the derived model for the purpose of real-time transient calculation in computers with limited architecture, as in the case Digital Signal Processing (DSP) cards.

The methodology to obtain the network equivalent is presented as follows: section II presents how to calculate the equivalent admittance matrix including the topics: parameter identification, determination of the parameters order, sparsity, stability and passivity requirements. In section III it is shown how to obtain the current source. Results are discussed in section IV and Conclusions are presented in section V.

II. EQUIVALENT ADMITTANCE

The admittance matrix parameters are obtained from time-domain fitting of the computed transient response of the system in phase-domain. For the sake of clarity, the theory will be explained for single-port and, then, three-port networks.

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[1]Each terminal (interface) between study and external systems is called port.

A. Fitting Equations

Consider the single-port linear network of Fig. 2. In the z-domain, the current and voltage are related as

$$I_1(z) = Y_1(z).V_1(z) \qquad (1)$$

in which $Y_1(z)$ is the rational function:

$$Y_1(z) = \frac{b(z)}{a(z)} = \frac{b_0 + b_1 z^{-1} + \cdots + b_p z^{-p}}{1 + a_1 z^{-1} + \cdots + a_p z^{-p}} \qquad (2)$$

The order p of the polynomials $a(z)$ and $b(z)$ is assumed to be the same. Taking the inverse z transform of (1), the current in the time-domain is:

$$i_1(t) = b_0 v_1(t) + \sum_{k=1}^{p} \left\{ b_k v_1(t - k\Delta t) - a_k i_1(t - k\Delta t) \right\} \qquad (3)$$

The right hand side of (3) represents a conductance b_0 in parallel with a current source, accounting for the summation term, which is computed only from past values. This form facilitates the integration of the line representation into the EMTP (Electromagnetic Transients Program). Once the current and voltage time-domain waveforms are available, the identification of the coefficients of $a(z)$ and $b(z)$ can be done rewriting (3) as:

$$\sum_{k=1}^{p} a_k i_1(t - k\Delta t) - \sum_{k=0}^{p} b_k v_1(t - k\Delta t) = -i_1(t) \qquad (4)$$

Assuming zero initial conditions and that the current and the voltage at each time step Δt are available totaling N data points, the above equation can be written in matrix compact form:

$$[I \quad -V] \begin{bmatrix} a_1 \\ \vdots \\ a_p \\ b_0 \\ \vdots \\ b_p \end{bmatrix} = \begin{bmatrix} -i_1(0) \\ -i_1(\Delta t) \\ -i_1(2\Delta t) \\ \vdots \\ -i_1((N-2)\Delta t) \\ -i_1((N-1)\Delta t) \end{bmatrix} \qquad (5)$$

Where the submatrices I and V, in (5), are given as:

$$V = \begin{bmatrix} v(0) & 0 & \cdots & 0 \\ v(\Delta t) & v(0) & \cdots & 0 \\ \vdots & \vdots & \cdots & 0 \\ v((p-1)\Delta t) & v((p-2)\Delta t) & \cdots & 0 \\ \hline v(p\Delta t) & v((p-1)\Delta t) & \cdots & v(0) \\ \vdots & \vdots & & \vdots \\ v((N-1)\Delta t) & v((N-2)\Delta t) & \cdots & v((N-p-1)\Delta t) \end{bmatrix} \qquad (6)$$
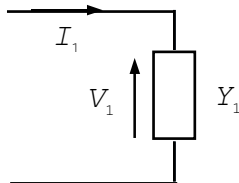
and

Fig. 2. Single-port network.

$$I = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ i(0) & 0 & \cdots & 0 \\ i(\Delta t) & i(0) & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ i((p-2)\Delta t) & i((p-3)\Delta t) & \cdots & 0 \\ \hline i((p-1)\Delta t) & i((p-2)\Delta t) & \cdots & i(0) \\ \vdots & \vdots & & \vdots \\ i((N-2)\Delta t) & i((N-3)\Delta t) & \cdots & i((N-p-1)\Delta t) \end{bmatrix} \qquad (7)$$

Here, it is assumed that $N \gg p$ leading to an overdetermined system for which a solution in the least squares sense is obtained. A constrained optimization method is used in order to achieve stability and passivity criteria. The system is stable if all the roots of $a(z)$ in (2) are inside the unit circle. If unstable, the coefficients of $a(z)$ are modified until all roots are within the unit circle. The system is passive, i.e. a dissipative network, if the real part of the admittance is positive for all frequencies (for the multi-port case the eigenvalues of the corresponding matrix must be larger than zero). If not passive, modify the coefficients of $b(z)$ until a passive representation is reached. The fitting equations and constraints are solved simultaneously by means of an optimization technique based on quadratic programming.

The above procedure is extended and applied to multi-port network equivalents as in the case of the three-phase system representation shown in Fig. 3. In the z-plane, the current and voltage are related according to:

$$\begin{bmatrix} Y_{11}(z) & \cdots & \cdots \\ Y_{21}(z) & Y_{22}(z) & \cdots \\ Y_{31}(z) & Y_{32}(z) & Y_{33}(z) \end{bmatrix} \begin{bmatrix} V_1(z) \\ V_2(z) \\ V_3(z) \end{bmatrix} = \begin{bmatrix} I_1(z) \\ I_2(z) \\ I_3(z) \end{bmatrix} \qquad (8)$$

Here, all elements of the admittance matrix $Y(z)$ are rational functions in the z-plane sharing the same poles, i.e. the same denominator $a(z)$:

$$Y_{ij}(z) = \frac{b_{ij}^k(z)}{a(z)} = \frac{b_{ij}^0 + b_{ij}^1 z^{-1} + \cdots + b_{ij}^p z^{-p}}{1 + a_1 z^{-1} + \cdots + a_p z^{-p}} \qquad (9)$$

Note that k in b_{ij}^k is not an exponent. It identifies the coefficient of z^{-k}. The first term of $a(z)$ is equal to 1.

The admittance matrix, in (8), is symmetric. Then, only the parameters for the six elements shown in the lower partition need to be determined. The parameter identification in (9) is computed from short-circuit tests at the three-phase system ports. The tests are performed directly in the phase-domain with all voltages and current sources of the external system in Fig. 3 set to zero. A unit-step voltage is applied to ports 1, 2 and 3 at a time.
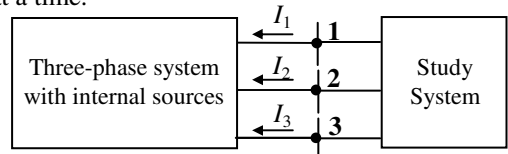

Fig. 3. Three-port network equivalent.

The voltages and currents waveform data from the short-circuit tests are put into matrix form similar to (6) and (7), respectively, and used to build up the general set of equations for the determination of the parameters in the least squares sense:

$$R\,x = s \qquad (10)$$

where, $x = \begin{bmatrix} a & b_{11} & b_{21} & b_{31} & b_{22} & b_{32} & b_{33} \end{bmatrix}^T$;

$$s = \begin{bmatrix} -i_1^1 & -i_2^1 & -i_3^1 & -i_2^2 & -i_3^2 & -i_3^3 \end{bmatrix}^T$$

and

$$R = \begin{bmatrix} I_1^1 & -V_1 & 0 & 0 & 0 & 0 & 0 \\ I_2^1 & 0 & -V_1 & 0 & 0 & 0 & 0 \\ I_3^1 & 0 & 0 & -V_1 & 0 & 0 & 0 \\ I_2^2 & 0 & 0 & 0 & -V_2 & 0 & 0 \\ I_3^2 & 0 & 0 & 0 & 0 & -V_2 & 0 \\ I_3^3 & 0 & 0 & 0 & 0 & 0 & -V_3 \end{bmatrix}$$

In vector x, a represents $a(z)$, the set of p coefficients of the denominator, and each b_{ij} is the set of $p+1$ coefficients of the numerator for each element $Y_{ij}(z)$ of the admittance matrix (see (9)); the total number of variables is $p+6(p+1)$; i_m^n in vector s represents the current into port m when the voltage source is connected to port n during the short-circuit test. The dimension of s is $6N \times 1$; in matrix R, V_n is a submatrix of voltage as in (6) and I_m^n is a submatrix built using the data from i_m^n as in (7). The dimension of R is $6N \times (p+6(p+1))$. For the parameter estimation, one needs to determine the order p and the sparsity and to ensure stability and passivity. These issues are described next.

B. Determination of the Order p

Following the approach used in [5], [6], the Singular Value Decomposition (SVD) method is used to determine p. The submatrices I_m^n in matrix R are initially built using a value for p much larger than the presumed order for the network representation. As $N \gg p$, the initial value used can be half of the data points number, N. SVD is applied to the lower partitions of each submatrix I_m^n (matrix portion below dashed line of (7)). This procedure results in six different values for the first estimate for p. Here, the chosen value in each SVD analysis is equal to the number of singular values that are around 10^8 times larger than the smallest singular value. Since the admittance matrix elements share the same denominator $a(z)$, only one value of p is required, which is the largest one. Considering modal domain representation, the SVD procedure for the ground mode would indicate a value of p smaller than a suitable one in this case (due to the high losses in the ground mode, the traveling waves corresponding to the reflections at the farthest nodes are attenuated). Thus, p for the ground mode is determined for multiplying the obtained value by a factor greater than one which accounts for the differences in the travel times of the modes. Here, based on an investigation, the calculated p is multiplied by a factor equal to 1.5 for obtaining a high degree of sparsity in the parameter calculations.

C. Sparsity in the Identification of the Parameters

The described method leads to an accurate representation with respect to the overall fitting but it is non-sparse. The order of the representation, p, can be large, depending on the time step used and the travel time of the line, leading to a large number of the parameters (equal to $7p+6$ for stability routine) which should be submitted to quadratic programming routine (Matlab Optimization Toolbox), posing difficulties to its convergence. To overcome this limitation and to reduce the computational effort in transient calculations, a sparse representation is then obtained. Sparsity is considered in the parameter calculations with the elimination of some of the parameters setting them to zero, as follows:

1) Estimate the parameter order p (explained in section II.B);

2) Obtain a basic solution for a and b_{ij} using (10);

3) For the user-defined N_S (number of nonzero coefficients for each set a and b_{ij}), only the positions of the N_S largest coefficients of each set of parameters are identified: a and b_{ij} are recalculated;

4) Modify the matrix R in (10), keeping only the columns corresponding to the positions of the N_S largest coefficients of each set. The other columns are, obviously, eliminated once its corresponding coefficients of each set will be zero;

5) Solve (10) with the reduced matrix R. Each calculated parameter should be put to the correspondent position in solution vector x. The remaining coefficients are set to zero, resulting in the sparse basic solution for each set a and b_{ij};

6) Check for stability and passivity of the network representation using the requirements presented in section II.D. Check for the desired overall fitting error. If the stability, passivity and fitting requirements are not satisfied in a predetermined number of iterations, N_S is increased and the steps 2 to 6 are repeated.

D. Stability and Passivity Requirements

Stability and passivity criteria must be satisfied for a passive network. From of the analysis of the admittance matrix in (8), the stability and passivity constraints are obtained from linearization and the set of equations is solved by means of constrained optimization using Quadratic Programming.

- Stability

Each element of the admittance matrix in (8) is a rational function in the z-domain, $b_{ij}(z)/a(z)$, with the same denominator. The representation is stable if each root of the scalar polynomial $a(z)$ have absolute value smaller than 1. If an unstable solution from (10) is obtained, increments Δa are calculated to be added to a in the solution vector x, in order to move unstable roots to within the unit circle. The set of

stability constraints is formulated as

$$J_S \ \Delta a + |z_U| \leq 1 \qquad \Rightarrow \qquad J_S \ \Delta a \ \leq 1 - |z_U| \quad (11)$$

where z_U is the set of unstable roots of $a(z)$ and J_S is the jacobian matrix where each element is [5]:

$$\frac{\partial |z_j|}{\partial a_i} = |z_j| \operatorname{Re} \frac{1}{\sum\limits_{k=1}^{p} k a_k z_j^{-k+i}} \qquad (12)$$

where z_j is each unstable root in the set z_U and a_i is each coefficient of the set a.

- *Passivity*

For any exciting complex voltage, the passive behavior of the network representation is ensured if the active power towards the network is always positive. Then, the real part of the admittance matrix in (8), matrix $G(\omega)$, must be positive definite [9], [10]. Thus, all eigenvalues of $G(\omega)$ must be larger than zero. The passivity constraints are expressed as a function of the set of coefficients b_{ij} only. The first step is to identify if there is at least one negative eigenvalue of $G(\omega)$ in each frequency considered[2]. If this is the case, the passivity criterion is violated and increments Δb_{ij} for all set of coefficients b_{ij} are calculated in such a way that each eigenvalue becomes positive. The second step is to identify each frequency ω_i in which there is any eigenvalue λ_i of $G(\omega)$ smaller or equal to 10^{-6}. Since the approach is based on linearization, one needs the partial derivative that relates each eigenvalue λ_i to each element (denoted by k) of the coefficient set b_{ij}. This can be written as the product of two partial derivatives:

$$\frac{\partial \lambda_i}{\partial b_{ij}^k} = \frac{\partial \lambda_i}{\partial G_{ij}(\omega_i)} \frac{\partial G_{ij}(\omega_i)}{\partial b_{ij}^k} \qquad (13)$$

The first partial derivative is an indication on how a small perturbation in each element $G_{ij}(\omega_i)$ would affect each eigenvalue λ_i. This term is calculated numerically. In fact, since $G(\omega)$ is a real symmetric matrix, its eigenvalues are real and the perturbation, for the off diagonal elements, has to be applied to two elements at a time. This is done multiplying these elements by a number very close to 1 to allow the numerical calculation of the derivative of $G(\omega)$.[3] The results are used to approximate the partial derivative as $\Delta \lambda_i / \Delta G_{ij}(\omega_i)$. The second term is an indication on how a small perturbation in each element of the coefficient set b_{ij}

would affect its corresponding element $G_{ij}(\omega_i)$. It is calculated analytically as [5]:

$$\frac{\partial G_{ij}(\omega_i)}{\partial b_{ij}^k} = \operatorname{Re}\left[\frac{e^{-j\omega_i k}}{a(z)} \right] \qquad (14)$$

The passivity constraint to be solved simultaneously to the fitting equations is then

$$J_{Pb} \ \Delta b_{ij}^k + \lambda_i > 0 \Rightarrow - J_{Pb} \ \Delta b_{ij}^k < \lambda_i \qquad (15)$$

where, J_{Pb} represents the jacobian matrix in which each element is calculated using (13) and Δb_{ij}^k is each increment to be added to the non-passive solution for each b_{ij}^k.

E. Final Remarks

Due to non-linearity in the relations between constraints and increments Δa and Δb_{ij}^k, the procedure to obtain a robust sparse phase-domain network representation is iterative and has two main steps: the stability criterion is addressed first and then the passivity. It can be stated as follows: From the basic solution for x in (10) (with the modifications for the sparsity), the roots of $a(z)$ are calculated and the set of roots with absolute values greater or equal to .99 is identified and then J_S is calculated. Equation (10) is modified to include the stability constraint:

$$\min_{\Delta x} \ \|R\Delta x - [s - Rx]\| \qquad (16)$$

subject to $\begin{bmatrix} J_S & 0 \end{bmatrix} \begin{bmatrix} \Delta a \\ \Delta b_{ij} \end{bmatrix} \leq \begin{bmatrix} 1 - |z_U| \\ 0 \end{bmatrix}$.

This problem is solved using quadratic programming (Matlab Optimization Toolbox), as

$$\frac{1}{2} \Delta x^T H \ \Delta x - f^T \Delta x \qquad (17)$$

subject to $D\Delta x \leq e$, where $H = R^T R$ and $f = R^T [s - Rx]$.

A correction Δx is calculated in each iteration resulting in the updated solution x_u as $x_u = x + \Delta x$. This follows until the stability requirement is satisfied. In the second step, the passivity enforcement, the calculated stable set a remain the same and (16) is modified accordingly:

$$\min_{\Delta b_{ij}^k} \ \|R_b \Delta b_{ij}^k - [(s - R_a a) - R_b b_{ij}^k]\| \qquad (18)$$

subject to $- J_{Pb} \ \Delta b_{ij}^k < \lambda_i$, where R_b is the part of matrix R related to b_{ij}^k; R_a is the part of matrix R related to a. A correction Δb_{ij}^k is calculated in each iteration, resulting in the updated set $b_u = b_{ij}^k + \Delta b_{ij}^k$. Once passivity is satisfied, the iterative process is finished. For each iteration, in this passivity routine, the entire eigenvalues are not submitted to quadratic programming routine. The passivity requirement is reached gradually, thus the eigenvalues have a small variation in each iteration.

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[2] $G(\omega)$ is calculated for N_f equally spaced frequency points in the normalized frequency range $0 \leq \omega < \pi$. For the first loop in the passivity routine N_f is assumed to be 100. For the subsequent loops, N_f is assumed to be 1000 and 10,000 respectively.

[3] The number chosen here is 1.0005 which is a suitable number to be used in connection with the optimization technique described in Section II.A.

Fig.4. Untransposed three-phase transmission line.

III. CURRENT SOURCE

The consideration of the internal sources is possible for transient calculations from the steady state. Thus, the initial conditions must be calculated. This process is especially facilitated for steady state for a given frequency ω_0 (here, corresponding to 60 Hz). The first step is to obtain the 60 Hz steady state opened-circuit voltage waveforms at ports 1, 2 and 3 using the phase-domain three-phase system with the internal sources in Fig. 3 from simulations carried out with PSCAD™/EMTDC® [11]. The steady state voltage in each port (denoted for i), Vss_i, is specified as phasor in a routine developed in Matlab® [12] to perform electromagnetic transient calculations using the proposed equivalent, as:

$$Vss_i = Vmod_i \left[\cos(Vangle_i) + j\sin(Vangle_i) \right] \qquad (19)$$

where $Vmod_i$ and $Vangle_i$ are the absolute value and the angle of opened-circuit voltage in each port, respectively.

In this situation, the voltage and current are related by the equivalent admittance value at the digital frequency ω_{0d} (corresponding to 60 Hz), $Y\left(e^{j\omega_{0d}}\right)$, which is calculated using (2), considering the calculated parameters in section II. Thus, the corresponding current phasor in each port, Iss_i, is calculated using:

$$\begin{bmatrix} Iss_1 \\ Iss_2 \\ Iss_3 \end{bmatrix} = Y\left(e^{j\omega_0 d}\right) \begin{bmatrix} Vss_1 \\ Vss_2 \\ Vss_3 \end{bmatrix} \qquad (20)$$

To account for initial conditions, it is shown in (3) that an appropriate set of past voltage and current values must be calculated, which are included in Matlab transient routine. The past values for voltage, in each port, are given using the counterpart of (19) in time-domain:

$$v_i(n+p+2) = V \bmod_i \cos(\omega_0 n \Delta t + Vangle_i) \qquad (21)$$

and from (20), the corresponding current values are:

$$i_i(n+p+2) = \operatorname{Im} od_i \cos(\omega_0 n \Delta t + Iangle_i) \qquad (22)$$

where n = -1, -2,…,-p ; $\operatorname{Im} od_i$ and $Iangle_i$ are the absolute value and the angle of current in each port, respectively. The current source for the Norton equivalent in each port, Isc_i, is calculated using:

$$Isc_i(k) = \operatorname{Im} od_i \cos(\omega_0 (k-1)\Delta t + Iangle_i) \qquad (23)$$

where k = 1,…,N. N is a number of points from the transient beginning until the final simulation time.

IV. RESULTS

An 69 kV three-phase circuit composed by a ideal sinusoidal voltage source of 1 p.u. and an untransposed transmission line in Fig. 4 is used to demonstrate the technique.

A. Parameters Identification

PSCAD™/EMTDC® was used to produce the short-circuit waveform data required (voltage and current sequences), described in section II.A, as input data to a Matlab routine developed to identify the parameters of the equivalent. A 20
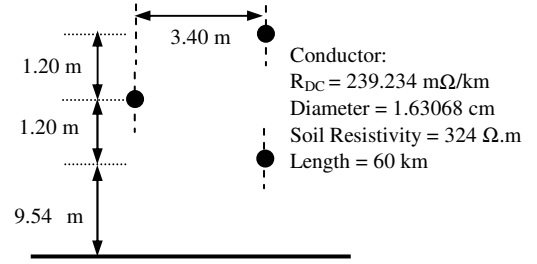
µs time step and a simulation time of 5 ms are used. As a measure of accuracy (using voltage curve, as an example), the overall fitting error, F_{error}, is calculated as

$$F_{error} = \|V_{SDTNE} - V_{EMTDC}\| / \|V_{EMTDC}\| \qquad (24)$$

where SDTNE and EMTDC subscripts refer to sequences using the Sparse Discrete-Time phase-domain Network Equivalent calculated with Matlab® and the EMTDC® phase-domain model, respectively. The order of the equivalent is determined applying SVD to current matrices. Singular values for the current matrices I_1^1, I_2^1, and I_3^1 are shown in Fig. 5, assuming, as a first estimate, the value of p to be equal to 100.
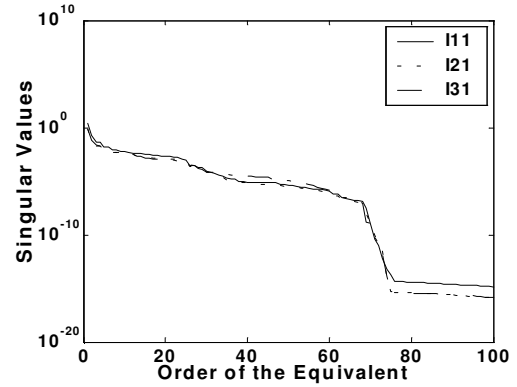

Fig. 5. Singular values.

The obtained value is 66. The value of the order used here is 100 obtaining a value for N_s equal to 13. Thus, the number of calculated parameters is 91 (7N_s) instead of 468 (7p+6), reducing the complexity of the problem and the computational effort. The largest absolute value among the roots of $a(z)$ is 0.9966, which means that the representation is stable and the stability routine is not necessary for this case. The matrix $G(\omega)$ has a total of 660 negative eigenvalues for 10,000 frequency points, which means that the passivity routine is required. Using the passivity routine, all eingenvalues become positives, leading to a passive representation with the lowest eigenvalue being equal to +5.4360x10⁻⁶. Fitting errors are calculated comparing the calculated current sequences with the current sequences obtained from PSCAD™/EMTDC® simulations (short-circuit tests). The largest error, based on (24), is 2.5013x10⁻². Therefore, the resulting parameters are quite satisfactory. If the time step changes it is necessary to do a refitting.

B. Electromagnetic Transient Calculations

To further check the equivalent performance, transient simulations are carried out according to Fig. 6. Resistances equals to 1.2 Ω are connected at 1, 2 and 3 ports when the

switches are closed at t = 54,18 ms. First of all, the steady state opened-circuit voltage peak value at three ports obtained from simulations and the closed time are passed to Matlab transient routine. Thus, the initial conditions and the steady state current source in time-domain can be calculated according to section III. Voltage and current curves at three ports are obtained using EMTDC® and SDTNE representation, using the same time step and the simulation time equal to 200 ms. The voltage and current curves at port 3, shown in Fig. 7, present the largest fitting errors calculated, using (24), equals to 2.1561×10^{-2} and 1.8205×10^{-2}, respectively.

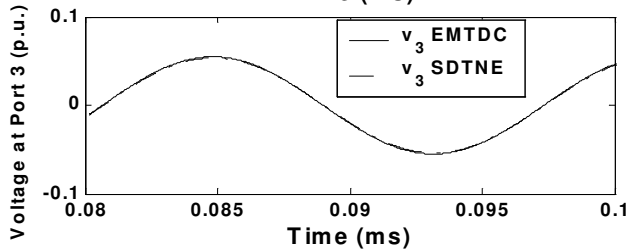
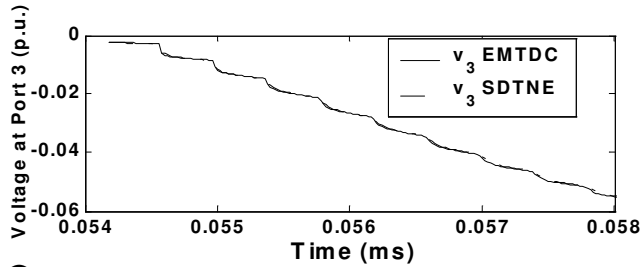Fig. 6. Electromagnetic transient calculations.

Fig. 7. Voltage curves at port 3 (EMTDC® and SDTNE).
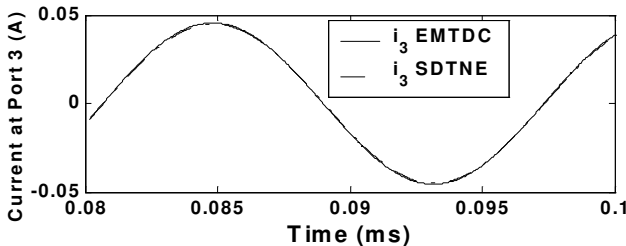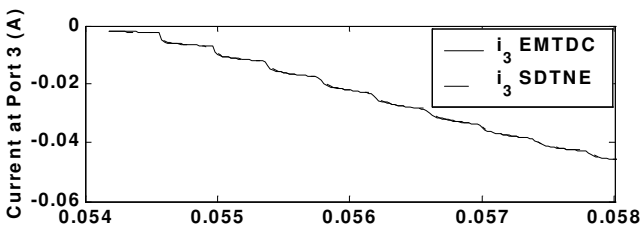

Fig. 8. Current curves at port 3 (EMTDC® and SDTNE).

V. Conclusions

A methodology for deriving a sparse network phase-domain equivalent based on time-domain fitting has been presented. The main contributions are: a phase-domain representation,

considering general network configurations; routines to enforce stability and passivity in the equivalent and sparsity in the parameter calculations. To demonstrate the performance of the derived representation, transient calculations were carried out for a three-phase system with internal sources. Voltage and current curves at the system ports were compared to transient simulations using the SDTNE implemented in Matlab and phase-domain model of EMTDC® with noticeable agreement.

VI. Acknowledgment

VII. References

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VIII. Biographies

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