Leakage Inductance Model for Autotransformer Transient Simulation

B. A. Mork, *Member, IEEE*, F. Gonzalez, *Member, IEEE*, D. Ishchenko, *Member, IEEE*

Abstract—This paper provides a thorough analysis of the leakage inductance effects of an autotransformer, reconciling the differences between the 3-winding "black box" assumption made in factory short-circuit tests and the actual series, common, and delta coils. An important new contribution is inclusion of the leakage effects between coils and core and creating a topologically correct point of connection for the core equivalent.

Keywords: Autotransformers, EMTP, Inductance, Transformer Models, Transient Simulations.

I. Introduction

THIS methodology is developed to calculate autotransformer coil reactances and formulate the inverse leakage inductance matrix. The formulation is based on short-circuit impedance data typically provided in standard factory test reports. Ultimately this leakage representation is being incorporated into a new "hybrid" transformer model for simulation of low- and mid-frequency transient behaviors [4].

The key advancement is to establish a topologically correct point of attachement for the core. The resulting "N+1" winding leakage representation cannot be directly produced by the commonly used BCTRAN supporting routine of EMTP.

Therefore, it is useful to document the development of this "N+1" leakage inductance representation, which also is in the form of the [A] matrix (inverse inductance matrix $[L]^{-1}$) [1]. The elements that form this matrix include the effect of the respective turns ratios between coils. The representation is of the actual series, common, and delta coils, and not of the three-winding "black box" equivalent typically assumed.

II. Model Development

A. Short-circuit Test Data

In general, for an N-winding transformer, the per-phase representation of the leakage reactances of the windings is a fully-coupled N-node inductance network, as shown in Fig. 1. [A] can be topologically constructed just as any nodal admittance matrix. The individual L^{-1} leakage values can be determined from binary short-circuit tests. In the special case of an autotransformer, or for a three-winding transformer in general, [A] reduces to a simple three-node delta-connected circuit. Common practice has been to convert this delta to a wye or "star" equivalent for steady-state short-circuit or load-flow calculations. This star equivalent often contains a negative inductance at the medium voltage terminal, which can be of concern for some transient simulations [2], [3].

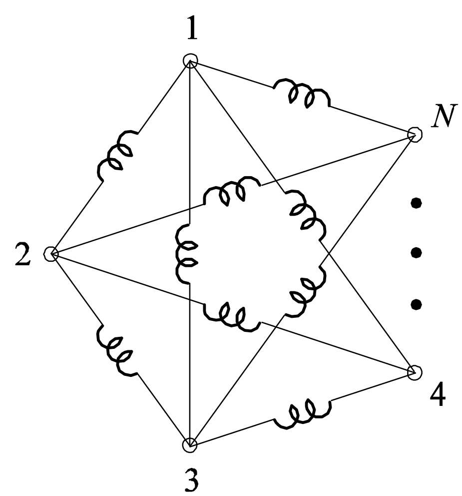

Fig. 1. Short circuit representation for N-winding transformer.

Data typically available from factory short-circuit tests are: Short-circuit impedances in %, MVA base of each winding, and short-circuit losses in kW.

In the leakage representation developed here as part of the hybrid model [4], [5], short-circuit reactances and coil resistances are separately represented, making it convenient to work directly with [A] and its purely inductive effects. The turns ratios of the coils can also be directly incorporated into [A].

Leakage effects also exist between the core and the coils. This is conceptually dealt with by assuming a fictitious infinitely-thin $N+1^{th}$ "coil" at the surface of the core. Fig. 2 shows the conceptual implementation of the N+1 winding flux leakage model for the reduced case of a two winding transformer. The reader is directed to Appendix B for definitions of terminal labels and subscript notations. Cylindrical coils are assumed, but this approach is generally applicable for other coil configurations [9]. These core-to-coil leakage effects are important for detailed models but are not considered (or measured) in factory tests. The $N+1^{th}$ winding serves as an attachment point for the core equivalent [4].

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B. A. Mork, F. Gonzalez and D. Ishchenko are with the Dept. of Electrical Engineering, Michigan Technological University, Houghton, MI 49931, USA.

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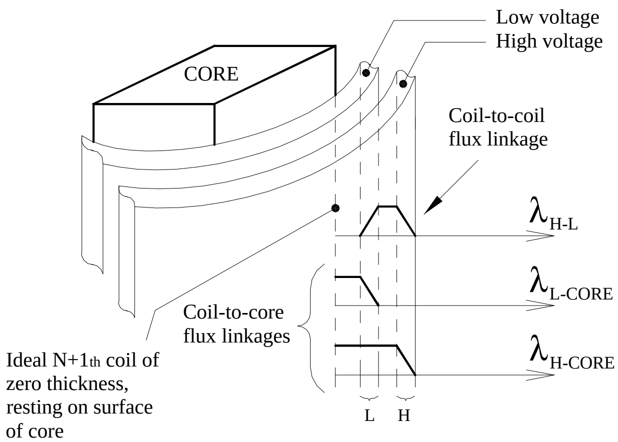

Fig. 2. Conceptual implementation of N+1th winding flux leakage model.

The generic black box equivalent does not represent the actual series, common and delta coils of the autotransformer, but rather assumes that the three windings are rated according to respective terminal voltages and currents. However, for a detailed model, the actual coil topology must be represented. This is illustrated on a per-phase basis in Fig. 3.

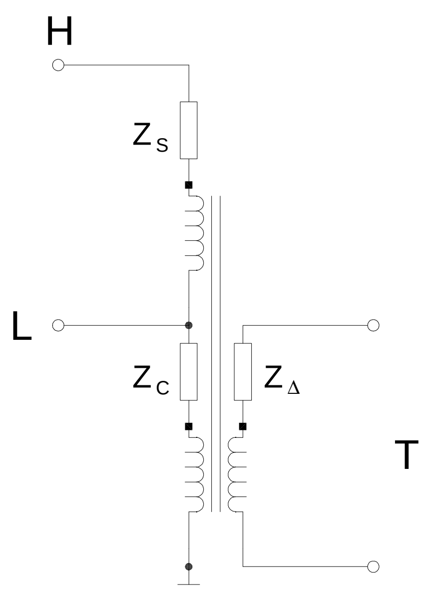

Fig. 3.Autotransformer configuration and impedances of each coil.

The leakage reactances between the coils can be calculated according to the flux linkages of Fig. 2 [9]. Since the fictitious N+1th coil is interior to all other coils, leakage flux between the core and coils will typically be more than that between the innermost coil and other coils. Hence, (1)-(3) can be used to describe the leakage reactances between core and the series, common and delta coils respectively, where K represents the additional effect of leakage between innermost coil and core. Note that the delta coil is assumed to be innermost, i.e. having the least coil-to-core leakage of any coil. In general, (3) is associated with the coil having minimum coil-to-core leakage.

$$X_{S\ CORE} = (K+1)\cdot X_{CD} + X_{SC} \tag{1}$$

$$X_{C\ CORE} = (K+1)\cdot X_{CD} \tag{2}$$

$$X_{D\ CORE} = K\cdot X_{CD} \tag{3}$$

X_{SC} usually is quite small since the series and common coils are really one coil with a tap point. X_{SD} is the largest since the series coil will have the highest voltage and insulation build-up. X_{CD} is not quite as large, but significant, since there can be extra insulation and barriers and oil duct space between the delta and the medium-voltage winding. Leakages between coils depend on the type of core (shell or core) and the coil configuration (cylindrical or pancake). As a first approximation, K might be estimated as 0.5, if one assumes that the innermost coil is also the lowest voltage coil, with modest coil-to-core insulation requirements. This rationale is based on [9] and the flux linkage distributions of Fig. 2.

B. Coil reactances

The methodology to calculate the autotransformer coil reactances is presented here. This formulation is based on the short-circuit reactive power that can be obtained from the short-circuit losses. Results obtained are equal to another approach by Dommel [1], but the derivation deals directly with actual impedance values thus avoiding the complexities of transforming per unit values according to the voltage and MVA bases of the series, common and delta coils.

Three “binary” short-circuit tests are usually performed for the autotransformer (Figs. 4-6).

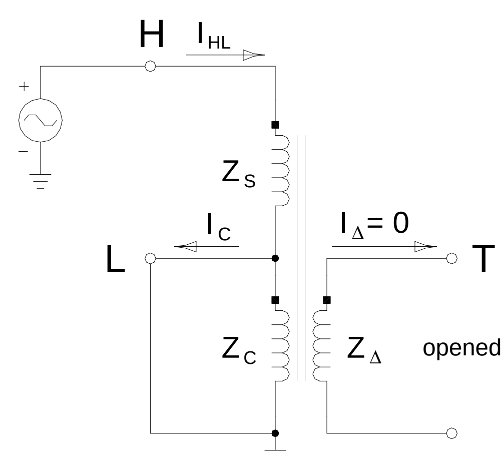

Fig. 4. Per-phase short-circuit test H-L.

Short-circuit H-L (Fig. 4):

$$I_{HL} = \frac{S_{HL}/3}{V_{H,L-L}/\sqrt{3}} = \frac{S_{HL}}{\sqrt{3}\cdot V_{H,L-L}} \tag{4}$$

$$I_C = \left[\frac{S_{HL}/3}{V_{L,L-L}/\sqrt{3}} - I_{HL}\right] = \frac{S_{HL}}{\sqrt{3}\cdot V_{L,L-L}} - I_{HL} \tag{5}$$

$$Q_{HL}/3 = Q_S + Q_C = X_S\cdot I_{HL}^2 + X_C\cdot I_C^2 \tag{6}$$

with Q_{HL} being the reactive power in this test, Q_S the reactive power corresponding to the series coil, and Q_C the reactive power corresponding to the common coil.

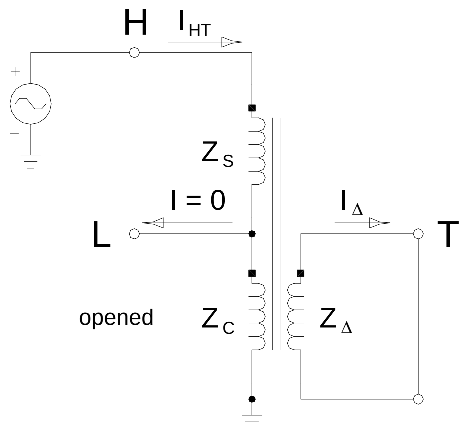

Fig. 5. Per-phase short-circuit test H-T.

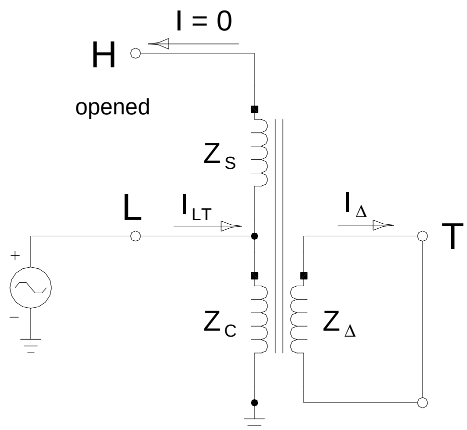

Fig. 6. Per-phase short-circuit test L-T.

Short-circuit H-T (Fig. 5):

$$I_{HT} = \frac{S_{HT}/3}{V_{H,L-L}/\sqrt{3}} = \frac{S_{HT}}{\sqrt{3} \cdot V_{H,L-L}} \tag{7}$$

$$I_\Delta = \frac{S_{HT}/3}{V_{T,L-L}} = \frac{S_{HT}}{3 \cdot V_{T,L-L}} \tag{8}$$

$$Q_{HT}\ /\ 3 = Q_S + Q_C + Q_\Delta = \left(X_S + X_C\right) \cdot I_{HT}^2 + X_\Delta \cdot I_\Delta^2 \tag{9}$$

Short-circuit L-T (Fig. 6):

$$I_{LT} = \frac{S_{LT}/3}{V_{L,L-L}/\sqrt{3}} = \frac{S_{LT}}{\sqrt{3} \cdot V_{L,L-L}} \tag{10}$$

$$I_\Delta = \frac{S_{LT}/3}{V_{T,L-L}} = \frac{S_{LT}}{3 \cdot V_{T,L-L}} \tag{11}$$

$$Q_{LT}\ /\ 3 = Q_C + Q_\Delta = X_C \cdot I_{LT}^2 + X_\Delta \cdot I_\Delta^2 \tag{12}$$

The three equations with three unknowns (6), (9), and (12), can be solved to find coil reactances as a function of reactive powers and rated currents. Results are shown as follows:

$$X_C = \frac{I_{HL}^2 \cdot \left(Q_{HT}\ /\ 3 - Q_{LT}\ /\ 3\right) - I_{HT}^2 \cdot Q_{HL}\ /\ 3}{I_{HT}^2 \cdot I_{HL}^2 - I_{LT}^2 \cdot I_{HL}^2 - I_C^2 \cdot I_{HT}^2} \tag{13}$$

$$X_S = \frac{Q_{HL}\ /\ 3 - X_C \cdot I_C^2}{I_{HL}^2} \tag{14}$$

$$X_\Delta = \frac{Q_{LT}\ /\ 3 - X_C \cdot I_{LT}^2}{I_\Delta^2} \tag{15}$$

C. Short-circuit winding reactances

The star-delta transformation [1] is then applied to obtain the delta equivalent for the three coils. Fig. 7 illustrates the relationship between binary short-circuit test measurements and the individual elements of the delta equivalent.

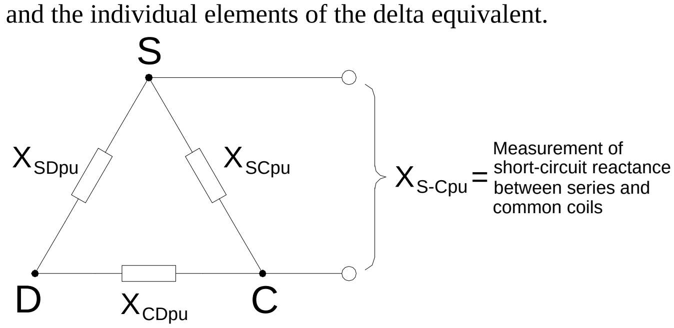

Fig. 7. Calculation of short-circuit winding reactances.

These relationships for all three binary short-circuit tests are given in (16)-(18). Reactances are in per unit using a common MVA base and the voltage base of each coil.

$$X_{S-C\ pu} = \left[\frac{1}{X_{SC\ pu}} + \frac{1}{X_{SD\ pu} + X_{CD\ pu}}\right]^{-1} \tag{16}$$

$$X_{S-D\ pu} = \left[\frac{1}{X_{SD\ pu}} + \frac{1}{X_{SC\ pu} + X_{CD\ pu}}\right]^{-1} \tag{17}$$

$$X_{C-D\ pu} = \left[\frac{1}{X_{CD\ pu}} + \frac{1}{X_{SC\ pu} + X_{SD\ pu}}\right]^{-1} \tag{18}$$

D. Admittance formulation

An admittance-type formulation is used to obtain [A] directly from individual inverse inductances [6]. These inverse inductances are analogous to transfer admittances. The circuit in Fig. 8 represents the overall leakage inductance effects, including the N+1$^{\text{th}}$ coil. Ideal transformers are used to represent the turns ratios of the coils. Inverse inductance values are referred to the lower voltage side in each case.

The methodologies of [1], [8], and [10] can be adapted to obtain the [A] matrix, using the reactances X_{SC}, X_{SD}, X_{CD} solved for from (16)-(18) and converted from per unit to actual values. Inverse inductance values are simply $w\,/X$. Fig. 9 illustrates the contribution of turns ratios and individual inverse inductances to [A]. Conceptually this is a 2×2 submatrix whose elements are added into the appropriate row-column positions of [A]. Parameters are calculated according to (19), which can be derived via the same short-circuit method used to obtain admittance matrix values. This submatrix can also be visualized as a Pi-equivalent. The resulting [A] is symmetric.

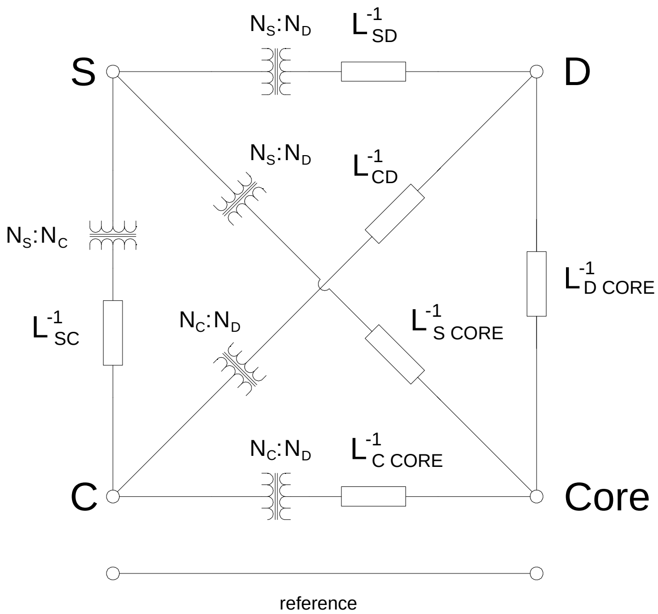

Fig. 8. Admittance formulation for an autotransformer (fictitious winding is included).

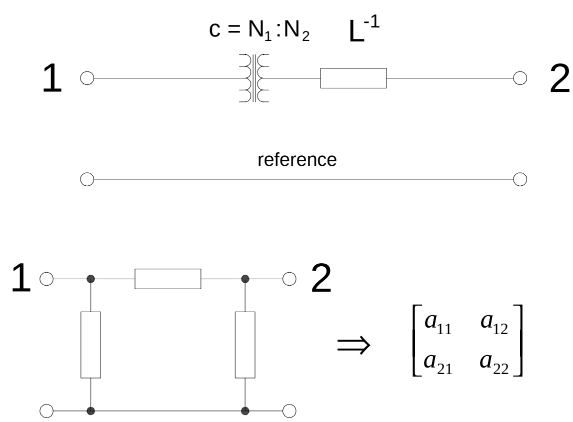

Fig. 9. Incorporation of turns ratio, resulting Pi-equivalent, and contribution each inverse inductance to [A].

$$a_{11} = \frac{L^{-1}}{c^2} \qquad a_{12} = -\frac{L^{-1}}{c} \qquad a_{21} = -\frac{L^{-1}}{c} \qquad a_{22} = -L^{-1} \quad (19)$$

Algorithmically, [A] can be constructed using the methods of [1]. For a general M×M case, the first step is to calculate the reduced M-1×M-1 matrix $[\,L_{pu}^{red}\,]$, whose diagonal elements can be obtained as

$$L_{ii\ pu}^{red} = L_{SC,iM\ pu} \qquad (20)$$

and whose off-diagonal elements are

$$L_{ik\ pu}^{red} = \frac{1}{2}\cdot\left[L_{SC,iM\ pu} + L_{SC,kM\ pu} - L_{SC,ik\ pu}\right] \ . \qquad (21)$$

This matrix is symmetric and its elements can be obtained directly from those calculated by means of (16)-(18).

To include all coils, the admittance matrix formulation will be used. $[\,L_{pu}^{red}\,]$ is first inverted

$$[\,A_{pu}^{red}\,] = [\,L_{pu}^{red}\,]^{-1} \qquad (22)$$

Then, the Mth row and column are added to obtain the full [A] matrix

$$a_{iM\ pu} = -\sum_{k=1}^{M-1} a_{ik\ pu} \qquad (23)$$

$$a_{MM\ pu} = -\sum_{i=1}^{M-1} a_{iM\ pu} \qquad (24)$$

To convert from per unit to the actual values required for EMTP simulation, all elements of [A] are multiplied by the common VA base, and each row and column *i* is multiplied by $1/V_i$.

III. ATP Model

The following autotransformer is implemented here as an example:

- 240/240/63 MVA autotransformer
- Wye-wye-delta autotransformer configuration
- 345GRY/199.2:118GRY/68.2:13.8 kV.

Table I summarizes the intermediate steps in calculating [A] for this transformer. The full three-phase [A] is given in ATP format in Appendix A.

Figs. 10 and 11 show how the individual series, common and delta coils are connected. EMTP simulations of the binary short-circuit tests match with values reported in the factory test report.

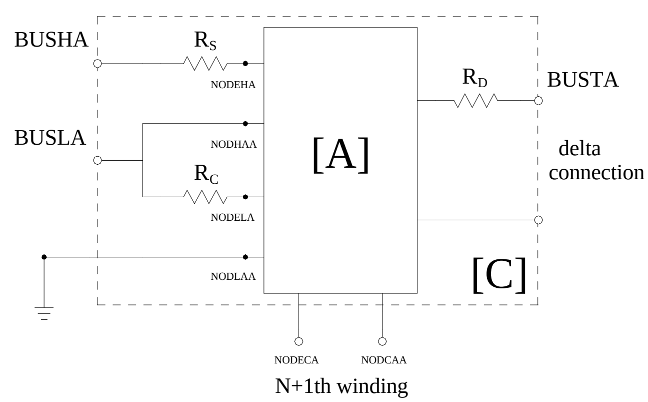

Fig. 10. Autotransformer. Single-phase representation.

IV. Conclusions

The N+1 leakage representation developed here includes the leakage inductances between core and coils, which are not considered in typical EMTP implementations, such as BCTRAN. The actual coils of the transformer are represented, as opposed to a black box N-winding equivalent. Parameters can be obtained from design information or from factory

TABLE I
[A] MATRIX CALCULATIONS

Data (ratings)	Data (short-circuit)	Coil reactances [from (13)-(15)]	Short-circuit reactances [from (16)-(18)]	Assumptions	[A] matrix [from (23)-(24)]
$V_{L\text{-}L,H}$ = 345 kV $V_{L\text{-}L,L}$ = 118 kV $V_{L\text{-}L,T}$ = 13.8 kV S_{base} = 100 MVA	X_{HLpu} = 0.0584 X_{HTpu} = 0.1089 X_{LTpu} = 0.0878	X_S = 32.5475 Ω X_C = -0.9700 Ω X_D = 0.8359 Ω	$X_{S\text{-}Cpu}$ = 0.0562 $X_{S\text{-}Dpu}$ = 0.2095 $X_{C\text{-}Dpu}$ = 0.1393	$V_{CORE} = V_D$ $X_{CORE\text{-}S}=(K+1)\cdot X_{CD} + X_{SC}$ $X_{CORE\text{-}C} = (K+1)\cdot X_{CD}$ $X_{CORE\text{-}D} = K\cdot X_{CD}$ K = 0.5	a_{11} = 13.3 (1/H) a_{22} = 73.6 (1/H) a_{33} = 1448.3 (1/H) a_{44} = 2878.4 (1/H) a_{12} = -26.8 (1/H) a_{13} = 18.9 (1/H) a_{14} = -21.9 (1/H) a_{23} = -134.3 (1/H) a_{24} = 44.3 (1/H) a_{34} =-1672.3 (1/H)

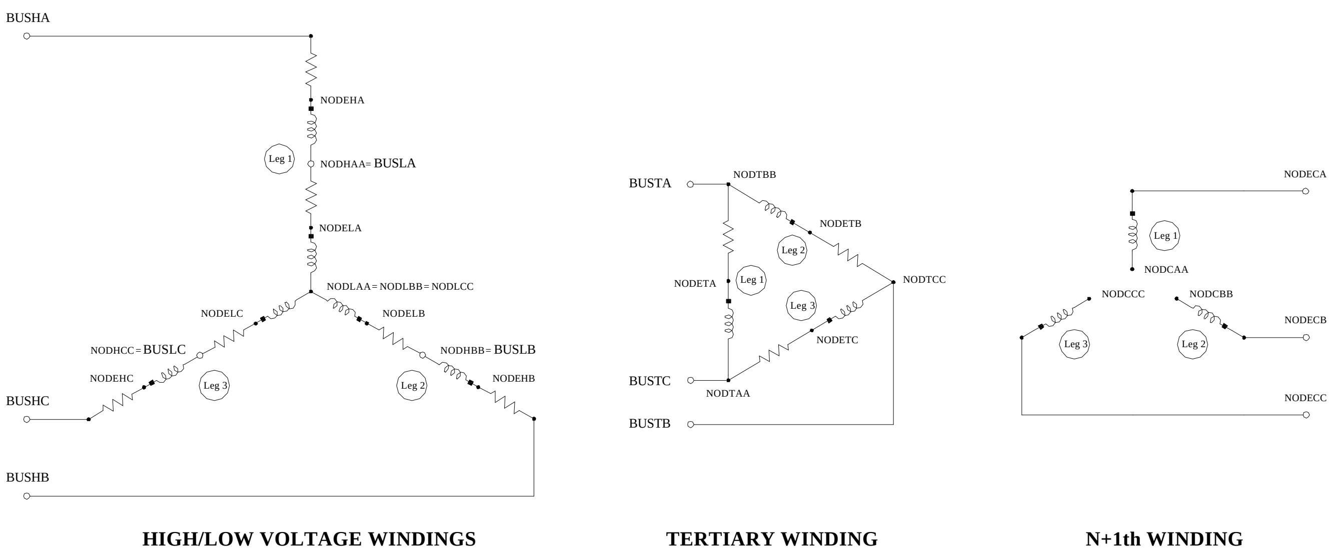

Fig. 11. Autotransformer. Connections.

measurements. Implementation is intuitive and based on the [A] matrix that is commonly used in EMTP programs. Results of the model constructed and tested here match with factory test reports. This leakage model can be used as the basis of low- and mid-frequency topologically-correct transformer models, with representations for the core, capacitive effects, and coil resistances built around it [4].

V. Appendix

A. [A] matrix parameters

```
C --------------------------------------------------
C [A] MATRIX                   [A]                [R]
C --------------------------------------------------
  USE AR
$VINTAGE, 1,
 1NODEHANODHAA              13.2671               0.0
 2NODELANODLAA              -26.805               0.0
                            73.5928               0.0
 3NODETANODTAA              18.9946               0.0
                            -134.33               0.0
                            1448.30               0.0
 4NODECANODCAA              -21.933               0.0
                            44.3135               0.0
                            -1672.3               0.0
                            2878.40               0.0
 5NODEHBNODHBB                  0.0               0.0
                                0.0               0.0
                                0.0               0.0
                                0.0               0.0
                            13.2671               0.0
 6NODELBNODLBB                  0.0               0.0
                                0.0               0.0
                                0.0               0.0
                                0.0               0.0
                            -26.804               0.0
                            73.5928               0.0
 7NODETBNODTBB                  0.0               0.0
                                0.0               0.0
                                0.0               0.0
                                0.0               0.0
                            18.9946               0.0
                            -134.33               0.0
                            1448.30               0.0
 8NODECBNODCBB                  0.0               0.0
                                0.0               0.0
                                0.0               0.0
                                0.0               0.0
                            -21.933               0.0
                            44.3135               0.0
                            -1672.3               0.0
```

```
                                  2878.40                 0.0
 9NODEHCNODHCC                        0.0                 0.0
                                      0.0                 0.0
                                      0.0                 0.0
                                      0.0                 0.0
                                      0.0                 0.0
                                      0.0                 0.0
                                      0.0                 0.0
                                      0.0                 0.0
                                  13.2671                 0.0
10NODELCNODLCC                        0.0                 0.0
                                      0.0                 0.0
                                      0.0                 0.0
                                      0.0                 0.0
                                      0.0                 0.0
                                      0.0                 0.0
                                      0.0                 0.0
                                      0.0                 0.0
                                  -26.805                 0.0
                                  73.5928                 0.0
11NODETCNODTCC                        0.0                 0.0
                                      0.0                 0.0
                                      0.0                 0.0
                                      0.0                 0.0
                                      0.0                 0.0
                                      0.0                 0.0
                                      0.0                 0.0
                                      0.0                 0.0
                                  18.9946                 0.0
                                  -134.33                 0.0
                                  1448.30                 0.0
12NODECCNODCCC                        0.0                 0.0
                                      0.0                 0.0
                                      0.0                 0.0
                                      0.0                 0.0
                                      0.0                 0.0
                                      0.0                 0.0
                                      0.0                 0.0
                                      0.0                 0.0
                                  -21.933                 0.0
                                  44.3135                 0.0
                                  -1672.3                 0.0
                                  2878.40                 0.0
  USE RL
$VINTAGE, 0,
```

B. *Terminal and subscript definitions*

Notation	Definition
H	High voltage terminal
L	Low voltage terminal
T	Tertiary voltage terminal
S	Series coil
C	Common coil
D or Δ	Delta (tertiary) coil
CORE	Fictitious coil on surface of core
L-L	Line-to-line voltages

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VII. Biographies

Bruce A. Mork (M'82) was born in Bismarck, ND, on June 4, 1957. He received the BSME, MSEE, and Ph.D. (Electrical Engineering) from North Dakota State University in 1979, 1981 and 1992 respectively.

From 1982 through 1986 he worked as design engineer for Burns and McDonnell Engineering in Kansas City, MO, in the areas of substation design, protective relaying, and communications. He has spent 3 years in Norway: 1989-90 as research engineer for the Norwegian State Power Board in Oslo; 1990-91 as visiting researcher at the Norwegian Institute of Technology in Trondheim; 2001-02 as visiting Senior Scientist at SINTEF Energy Research, Trondheim. He joined the faculty of Michigan Technological University in 1992, where he is now Associate Professor of Electrical Engineering, and Director of the Power & Energy Research Center.

Dr. Mork is a member of IEEE, ASEE, NSPE, and Sigma Xi. He is a registered Professional Engineer in the states of Missouri and North Dakota.

Francisco Gonzalez was born in Barcelona, Spain. He received the M.S. and Ph.D. from Universitat Politècnica de Catalunya (Spain) in 1996 and 2001 respectively. As visiting researcher, he has been working at Michigan Technological University, at Tennessee Technological University, at North Dakota State University, and at the Norwegian Institute of Science and Technology (Norway). His experience includes eight years as researcher involved in projects related to Power.

In October 2002, Dr. Gonzalez was awarded a Postdoctoral Fellowship from the Spanish Government. Since then, he has been working as Postdoc at Michigan Technological University. His research interests include transient analysis of power systems, lightning performance of transmission and distribution lines, FACTS, power quality, and renewable energy.

Dr. Gonzalez is a member of the IEEE Power Engineering Society.

Dmitry Ishchenko was born in Krasnodar, Russia. He received his M.S. and Ph.D. degrees in Electrical Engineering from Kuban State Technological University, Russia in 1997 and 2002 respectively. In September 2000 he was awarded with the Norwegian Government Research Scholarship and worked as a visiting researcher at the Norwegian Institute of Science and Technology (Norway). His experience includes 5 years as Power Systems Engineer at the Southern Division of the Unified Energy System of Russia.

In February 2003 he joined the Electrical and Computer Engineering Department of Michigan Technological University as a postdoctoral researcher. His research interests include computer modeling of power systems, power electronics, and power system protection.

Dr. Ishchenko is a member of the IEEE Power Engineering Society.