# Transformer Short-Circuit Representation: Investigation of Phase-To-Phase Coupling

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Abstract-- Transformer short-circuit behavior is typically represented in electromagnetic transient simulation programs by the inverse of the leakage inductance matrix, also known as the A-matrix or [A].

The terms of [A] represent the magnetic coupling (flux linkages) between windings, i.e. between coils on the same core leg. In addition to the representation of coupling between windings of the same phase, intra-phase coupling effects can also be included.

This paper investigates the sensitivities of results of EMTPtype simulation to intra-phase entries of [A]. A method for obtaining [A] that includes intra-phase coupling is developed and verified by comparing simulation results with laboratory measurements. The percent error between measured and simulated short-circuit currents that result from implementing a full and simplified [A] was quantified. A correlation of the size of the elements representing intra-phase coupling in [A] with transformer MVA size is investigated and valuable conclusions and recommendations are made.

Keywords: EMTP, Leakage Representation, Short-Circuit, Transformer Models.

# I. INTRODUCTION

IMPORTANT work has been done during recent years to improve traditional transformer models in simulation packages like EMTP. One of the latest [1], [2] is a topologically-correct hybrid transformer model developed for low- and mid-frequency transient simulations. This model uses a matrix description of leakage effects known as the Amatrix or [A], which is the inverse of the leakage inductance matrix. The terms of [A] represent the magnetic coupling (flux linkages) between windings, i.e. between coils on the same core leg and include the effects of both positive and zero sequence behaviors. In addition to the representation of coupling between windings of the same phase, intra-phase

Presented at the International Conference on Power Systems Transients (IPST'07) in Lyon, France on June 4-7, 2007 coupling effects can also be included. However, if measurements of zero-sequence or some other type of unbalanced operation are not available, then it is difficult or impossible to estimate the intra-phase entries of [A], which must then be omitted (left as zero).

In the case of balanced three-phase operation, it may be sufficient to consider only the coupling of windings on the same phase, neglecting phase-to-phase coupling. For unbalanced or transient cases, this may not be the case. This paper investigates the sensitivities of results of EMTP-type simulation to intra-phase entries of [A]. The phase-to-phase coupling that exists between the windings of individual transformers is studied and quantified with the objective of investigating the importance of including these off-diagonal elements in [A] for computer simulations.

Different laboratory tests are performed on transformers of different sizes and configurations to obtain the parameters for [A] through a new method. The measurement techniques, determination of [A], and precision issues are addressed along with a summary of trends or correlation of parameters with size and type of transformer. The "simplified" and "full" matrices generated for each transformer are implemented in an EMTP model and a comparison of the simulation results with laboratory measurements is done in both cases.

# II. DESCRIPTION OF THE SIMPLIFIED [A]

The per-phase representation of the leakage reactances of an N-coil transformer is a fully coupled N-node inductance network. The A-matrix is constructed through an admittance formulation where each of the inductive elements in the network is determined through a binary short-circuit test.

For the case of a single phase, two-winding transformer, the representation shown in Fig. 1 (referred to the low-voltage side) from [3] can be made, where  $a_{H-X}$  is the inverse of the leakage inductance between the high (H) and low (X) voltage windings of the transformer and *c* is the turns ratio.



Fig. 1. Two-port network for a single-phase, two-winding transformer

The corresponding [A] is symmetrical and is given in (1).

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$$[A] = [L]^{-1} = \begin{bmatrix} \frac{a_{H-X}}{c^2} & \frac{-a_{H-X}}{c} \\ \frac{-a_{H-X}}{c} & a_{H-X} \end{bmatrix}$$
(1)

Expanding this formulation for three phases, the A-matrix has the following structure:

As it can be seen in (2), the leakage representation of a transformer can be done neglecting the coupling between coils of a different phase. A method for determining the value of the off-diagonal elements of [A] (for each sub-matrix) that represent this effect is developed in the following section.

# III. PROPOSED METHOD - FULL [A]

In the same manner as admittance matrix can be determined, if an N-coil transformer is tested by applying a voltage on one coil and short-circuiting the rest, repeating this process for every coil gives a relationship between the exciting voltages and short-circuit currents on every coil of the transformer. Consider a three-phase, two-winding transformer that consists of a total of six coils. Fig. 2 shows a test on the Phase-1, high-voltage coil. Note that all of the measured currents are injected currents.



Fig. 2. Single-phase short-circuit test on the high-voltage, phase-1 coil.

For the test of Fig. 2, the network of Fig. 3 can be constructed:



Fig. 3. Six-node network of a three-phase two-winding transformer.

From this test, applying  $V_{1H_0}$  gives us  $I_{1H_0} I_{2H_0} I_{3H_0} I_{1L_0} I_{2L}$ , and  $I_{3L}$  of Fig. 3. With the use of the AC resistance  $R_H$  and  $R_L$ of the high and low-voltage windings, the nodal voltages V' in the network can be calculated. For the case of the excited winding of Fig. 3,

$$V'_{1H} = V_{1H} - I_{1H}R_{H}$$
(3)

and for the rest,

$$V'_i = I_i R_i \tag{4}$$

where  $I_i$  and  $R_i$  refer to the injected current and the AC resistance of each coil of the transformer respectively.

The above parameters can be put in matrix form to show a relationship with [A]:

$$\frac{1}{\omega} [A] [V'_i] = [I_i] \tag{5}$$

From each test, six relationships between voltages and currents are established by exciting a different winding at a time and short-circuiting all of the others. The formulation is now as in (6).

$$\frac{1}{\omega} \begin{bmatrix} a_{11} a_{12} & \dots & a_{16} \\ a_{21} & \dots & & \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ a_{61} & \dots & a_{66} \end{bmatrix} \begin{bmatrix} V'_{1H} \\ V'_{2H} \\ V'_{3H} \\ V'_{1L} \\ V'_{2L} \\ V'_{3L} \end{bmatrix} = \begin{bmatrix} I_{1H} \\ I_{2H} \\ I_{3H} \\ I_{1L} \\ I_{2L} \\ I_{3L} \end{bmatrix}$$
(6)

Expanding this relationship by combining a total of six tests, the final formulation for [A] is given in (7)

$$[A] = \omega \cdot [I_i] [V'_i]^{-1} \tag{7}$$

where

$$[I_{i}] = \begin{bmatrix} I_{1H}^{Test1} & I_{1H}^{Test2} & \cdots & I_{1H}^{Test6} \\ I_{2H}^{Test1} & I_{2H}^{Test2} & \cdots & I_{2H}^{Test6} \\ I_{3H}^{Test1} & I_{3H}^{Test2} & \cdots & I_{3H}^{Test6} \\ I_{1L}^{Test1} & I_{1L}^{Test2} & \cdots & I_{1L}^{Test6} \\ I_{2L}^{Test1} & I_{2L}^{Test2} & \cdots & I_{2L}^{Test6} \\ I_{3L}^{Test1} & I_{3L}^{Test2} & \cdots & I_{3L}^{Test6} \end{bmatrix}$$

$$(8)$$

and

$$[V'_{i}] = \begin{bmatrix} V'_{1H} & V'_{1H} & I_{est2} & \cdots & V'_{1H} \\ V'_{2H} & V'_{2H} & \cdots & V'_{2H} \\ V'_{3H} & V'_{2H} & \cdots & V'_{2H} \\ V'_{3H} & V'_{3H} & V'_{3H} & \cdots & V'_{3H} \\ V'_{1L} & V'_{1L} & \cdots & V'_{1L} \\ V'_{2L} & V'_{1L} & V'_{1L} & \cdots & V'_{1L} \\ V'_{2L} & V'_{2L} & \cdots & V'_{2L} \\ V'_{3L} & V'_{3L} & \cdots & V'_{3L} \end{bmatrix}$$
(9)

#### **IV. MEASUREMENTS**

Measurements were taken from three different transformers to benchmark the model. These were three-legged 15-kVA 240Y:208Y V, five-legged 150-kVA 12470Y:208Y V, and three-legged 500-kVA 12470 $\Delta$ :480Y V transformers. Since the transformers were available for testing in the laboratory, the general approach used to obtain the parameters for the model was to record the voltage and current waveforms and do the corresponding post-processing. The tests are described below.

#### *A. Three-phase short-circuit tests.*

A three-phase voltage was applied to the high-voltage side with the low voltage winding shorted. The phase-to-neutral voltage waveforms in each of the high-voltage coils were acquired along with the phase currents. By implementing the zero-crossing method described in [2], the short circuit impedances, i.e., the winding resistance and leakage reactance of the three phases were obtained. These values were used to generate the simplified A-matrix.

#### B. Single-phase short-circuit tests.

The method described in the last section was applied by opening the delta windings of the transformers when necessary. A MATLAB® program was generated to perform the required calculations starting from the extracted waveforms for this test and the AC resistances obtained from the three-phase, short-circuit tests. These calculations were used to obtain the full A-matrix.

# V. ATP IMPLEMENTATION

After the A-matrices were obtained, an ATP representation of the short-circuit behavior of the transformer was made. Different tests were simulated and the behavior of the model using both the full and simplified [A] was compared to the acquired waveforms. Fig. 4 shows the case of a single-phase, short-circuit test performed on the low-voltage, Phase-1 coil.



Fig. 4. ATP Implementation of the A-matrix.

### VI. ERROR QUANTIFICATION

The difference between the measured and simulated currents using the full and simplified [A] relative to the peak of the measured current will be calculated to determine the error introduced in the model and it is given in (10).

$$Current\% = [(I_{Meas} - I_{Simul}) / Peak(I_{Meas})]x100$$
(10)

# VII. RESULTS

## A. A-Matrices

In order to implement the full A-matrices in ATP, the average of the corresponding pairs of off-diagonal elements was obtained to make the matrix symmetric. The structure of the matrix represents an isotropic behavior between windings of a transformer but due to accuracy constraints in the measurements, the results obtained from tests between windings were not exactly the same from both sides but with a percent error in a range between 1.5-3.3%. The full and simplified matrices obtained for the three transformers are shown below and the values are given in per-unit.

For the 15-kVA transformer,

$$[A]_{15} = \begin{bmatrix} 6.9839 & 0.1004 & 0.0783 & -6.8907 & -0.0308 & -0.0058 \\ 0.1004 & 6.7471 & 0.1218 & -0.0156 & -6.7587 & -0.0200 \\ 0.0783 & 0.1218 & 6.7291 & 0.0160 & -0.0234 & -6.7448 \\ -6.8907 & -0.0156 & 0.0160 & 6.9254 & 0.0503 & 0.0167 \\ -0.0308 & -6.7587 & -0.0234 & 0.0503 & 6.9021 & 0.0296 \\ -0.0058 & -0.0200 & -6.7448 & 0.0167 & 0.0296 & 6.8838 \end{bmatrix} x 10^4$$

$$[A]_{15} = \begin{bmatrix} 6.3898 & 0 & 0 & -6.3898 & 0 & 0 \\ 0 & 6.0970 & 0 & 0 & -6.0970 & 0 \\ 0 & 0 & 6.5838 & 0 & 0 & -6.5838 \\ -6.3898 & 0 & 0 & 6.3898 & 0 & 0 \\ 0 & -6.0970 & 0 & 0 & 6.0970 & 0 \\ 0 & 0 & -6.5838 & 0 & 0 & 6.5838 \end{bmatrix} x 10^4$$

For the 150-kVA transformer,

$$[A]_{150} = \begin{bmatrix} 1.1325 & -0.0338 & -0.0412 & -1.0221 & 0.0414 & 0.0444 \\ -0.0338 & 1.2421 & -0.0326 & 0.0382 & -1.0920 & 0.0389 \\ -0.0412 & -0.0326 & 1.1906 & 0.0388 & 0.0357 & -1.0670 \\ -1.0221 & 0.0382 & 0.0388 & 0.9204 & -0.0396 & -0.0377 \\ 0.0414 & -1.0920 & 0.0357 & -0.0396 & 0.9549 & -0.0398 \\ 0.0444 & 0.0389 & -1.0670 & -0.0377 & -0.0398 & 0.9650 \end{bmatrix} x^{10^4}$$
$$[A]_{150} = \begin{bmatrix} 1.1940 & 0 & 0 & -1.1940 & 0 & 0 \\ 0 & 1.3127 & 0 & 0 & -1.3127 & 0 \\ 0 & 0 & 1.2107 & 0 & 0 & -1.2107 \\ -1.1940 & 0 & 0 & 1.1940 & 0 & 0 \\ 0 & -1.3127 & 0 & 0 & 1.3127 & 0 \\ 0 & 0 & -1.2107 & 0 & 0 & 1.2107 \end{bmatrix} x^{10^4}$$

Finally, for the 500-kVA transformer,

0

$$[A]_{500} = \begin{bmatrix} 7.5763 & 0.1161 & 0.0657 & -7.1104 & 0.1062 & 0.1692 \\ 0.1184 & 7.8665 & 0.0807 & 0.1469 & -7.2146 & 0.1783 \\ 0.0741 & 0.0898 & 7.6596 & 0.1547 & 0.1315 & -7.1558 \\ -7.5029 & 0.1662 & 0.1458 & 7.0929 & -0.0442 & -0.0649 \\ 0.1101 & -7.5765 & 0.1342 & -0.0510 & 7.0610 & -0.0741 \\ 0.1527 & 0.1758 & -7.5339 & -0.0640 & -0.0771 & 7.3212 \end{bmatrix} x 10^3$$
$$[A]_{500} = \begin{bmatrix} 7.4220 & 0 & 0 & -7.4191 & 0 & 0 \\ 0 & 7.4916 & 0 & 0 & -7.4886 & 0 \\ 0 & 0 & 7.7256 & 0 & 0 & -7.7225 \\ -7.4191 & 0 & 0 & 7.4161 & 0 & 0 \\ 0 & -7.4886 & 0 & 0 & 7.4856 & 0 \end{bmatrix} x 10^3$$

To observe a trend (if any) between the ratio of the offdiagonal and main-diagonal elements of each sub-matrix within [A] and transformer size, each off-diagonal element was divided by the average of the ones in the main-diagonal. The minimum, maximum and average values were registered and the results are summarized in Table I.

0 -7.7225

0

0 7.7194

TABLE I Percent Ratio of Off-Diagonal to Main-Diagonal Elements In [A] For Different Size Transformers

Size (kVA)	Min (%)	Max (%)	Average (%)
15	0.04	1.8	0.642
150	2.8	4.4	3.60
500	0.6	2.49	1.51

From the results, a correlation of the size of the offdiagonal elements with transformer MVA size could not be observed.

## B. Waveform Comparisons-15-kVA Transformer

Fig. 5 shows a comparison of the acquired and simulated current waveforms of Phase 1 when exciting the low-voltage coil and short-circuiting the rest in a single-phase, short-circuit test on the 15kVA transformer. The simulation was done implementing the full A-matrix.



Fig. 5. Measured and simulated phase-1 current waveforms from a single-phase, short-circuit test.

After implementing the simplified [A] for the same simulation, the results obtained for the excited phase were in very close agreement with the ones shown in Fig. 5. These can be more easily appreciated in Table II which shows the maximum percent error between the measured and simulated high-voltage currents of the Phase-1 coils using both full and simplified A-matrices.

 TABLE II

 MAXIMUM PERCENT ERROR, 15-KVA TRANSFORMER

 Current
 Full [A] (%)

 III [A] (%)
 Simplified [A] (%)

 I1H
 13.6
 12.8

15.8

 $I_{1L}$ 

Fig. 6 shows the corresponding short-circuit currents measured on the high-voltage coils for the same test along with the simulated currents from the ATP model using the full A-matrix.

16.6



Fig. 6. Phase-2 and Phase-3 current waveforms on the high-voltage side obtained from a single-phase, short-circuit test.

Fig. 7 shows the measured and simulated short-circuit currents on the low-voltage coils for the same test.



Fig. 7. Phase-2 and Phase-3 current waveforms on the low-voltage side obtained from a single-phase, short-circuit test.

By modeling with the simplified [A], the short-circuit currents of Figs. 6 and 7 appear with a value of zero.

# C. Waveform Comparisons-150-kVA Transformer

Figs. 8 thru 11 show the waveforms obtained from a singlephase, short-circuit test performed on the high-voltage, Phase-1 winding along with the simulated currents obtained by implementing the full and simplified [A].



Fig. 8. Measured and simulated short-circuit currents of the excited winding from a single-phase, short-circuit test.



Fig. 9. Measured and simulated short-circuit currents of the low-voltage winding from a single-phase, short-circuit test.



Fig. 10. Short-circuit currents on phases 2 and 3, high-voltage side, from a single-phase, short-circuit test.



Fig. 11. Short-circuit currents on phases 2 and 3, low-voltage side, from a single-phase, short-circuit test.

As in the case of the 15-kVA transformer, the short-circuit currents on the high and low-voltage coils of phases 2 and 3 appear with a value of zero when the simplified [A] was used.

Table III shows the maximum error introduced in the currents of Phase 1 by implementing both A-matrices. Besides differences in magnitude, a phase shift was also observed. It was found that the results were very sensitive to the value of the AC resistance implemented in ATP. Thus, accurate values of winding resistance are important to obtain a minimum error in the simulation results.

TABLE III

MAXIMUM PERCENT ERROR AND PHASE SHIFT ( $\phi$ ), 150-KVA TRANSFORMER						
Current	Full [A]	Full [A]	Simplified	Simplified [A]		
	(%)	$\Phi$ (deg)	[A] (%)	$\Phi$ (deg)		
I <sub>1H</sub>	2.8	-12.52	7.02	-11.01		
I <sub>IL</sub>	9.47	-19.47	5.23	-17.71		

# D. Waveform Comparisons- 500-kVA Transformer

Fig. 12 shows the measured and simulated waveforms of the excited coil when performing a single-phase short-circuit test on the high-voltage coil of Phase 1 and implementing the full and simplified [A] in ATP. For the rest of the phases, the currents obtained with the simplified [A] had a value of zero.



Fig. 12. Measured and simulated short-circuit currents of the excited winding from a single-phase, short-circuit test.

Finally, Table IV shows the maximum error introduced in the currents of Phase 1 when implementing both A-matrices.

MAXIMUM PERCENT ERROR AND PHASE-SHIFT (Ф), 500-KVA TRANSFORMER							
Current	Full [A]	Full [A]	Simplified	Simplified [A]			
	(%)	$\Phi$ (deg)	[A] (%)	$\Phi$ (deg)			
I <sub>1H</sub>	0.6	-5.4	2.59	-5.4			
I <sub>1L</sub>	3.67	-12.52	2.2	-12.52			

TABLE IV Maximum Percent Error and Phase-Shift (d) 500-kVA Transe

# VIII. CONCLUSIONS

A method for obtaining the full [A] for three-phase transformers was developed and verified. By implementing the obtained matrices in an ATP model and comparing the simulated waveforms with laboratory measurements, an acceptable accuracy level was achieved for the excited phase when implementing both the full and simplified [A]. For the remaining phases, the simplified [A] could not replicate the short-circuit behavior observed through measurements. It can be concluded that the off-diagonal elements of [A] are important when simulating cases of unbalanced operation where mutual coupling between phases should be considered. Examples of off-diagonal elements for three transformers were given and can be implemented in a full [A] for future studies.

## IX. FUTURE WORK

Measurements for more transformers of different size are necessary to determine a trend between this characteristic and the size of the off-diagonal elements of [A].

Continued investigation of [A] is necessary to determine if the zero-sequence behavior of the transformer model could be affected by omitting the off-diagonal elements.

### X. References

- B. Mork, F. Gonzalez, D. Ishchenko, D. Stuehm, and J. Mitra, "Hybrid transformer model for transient simulation: Part I: Development and parameters," *IEEE Trans. Power Delivery*, vol. Paper TPWRD-00760-2005.R1, (accepted for publication June 22, 2006), 2006.
- [2] B. Mork, F. Gonzalez, D. Ishchenko, D. Stuehm, and J. Mitra, "Hybrid transformer model for transient simulation: Part II: Laboratory

measurements and benchmarking," *IEEE Trans. Power Delivery*, vol. Paper TPWRD-00761-2005.R2, (accepted for publication June 22, 2006), 2006.

- [3] B.A. Mork, F. Gonzalez, D. Ishchenko, "Leakage Inductance Model for Autotransformer Transient Simulation," Presented at the International Conference on Power System Transients, Montreal, Canada, 2005. [Online]. Available: http://www.ipst.org/IPST05Papers.html
- [4] H. W. Dommel and et.al., *Electromagnetic Transients Program Reference Manual (EMTP Theory Book)*. Portland, OR: Prepared for BPA, Aug. 1986.
- [5] *Alternative Transients Program Rule Book.* Leuven EMTP Center, Jul 1987.

#### XI. BIOGRAPHIES

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