

# Transformer mechanical stress caused by external short-circuit: a time domain approach

A. C. de Azevedo, A. C. Delaiba, J. C. de Oliveira, B. C. Carvalho, H. de S. Bronzeado

**Abstract**— This paper presents the results of an investigation carried out on the field of short circuit effects upon transformers electromechanical forces calculation. A time domain transformer model based on magnetomotive forces and reluctances is used. This model allows for simulating the transformer transient and steady state behaviors regarding electric, magnetic and mechanical variables. The methodology is applied to a typical power transformer operating under nominal and short circuit conditions. For comparison purposes a due to the lack of accepted performance values, a finite element program is utilized for similar studies and the results are compared to the time domain ones.

**Keywords:** Electromagnetic force, finite element method, short-circuit currents, transformer modeling, time domain.

## I. INTRODUCTION

**D**URING normal lifetime, transformers are submitted to a variety of electrical, mechanical and thermal stresses. One of the most critical situations is that caused by external short circuits, which produces high currents in the transformer windings and hence high internal forces in the windings. These forces are potential sources for damaging transformers and techniques to mitigate the impacts of these are considered in [1, 2].

For this paper purposes, a time domain transformer model based on magnetomotive forces and reluctances is proposed to evaluate the electromechanical forces occurring in the transformers. This method gives a good insight to achieve a comprehensive view of the overall transformer magnetic and electrical behaviors under distinct conditions. Parameters of a three-phase 100 MVA transformer are used to illustrate the methodology potentiality.

Due to difficulties in finding reference values to validate the calculations, the time domain analytical results are compared to corresponding results extracted from a traditional and well-accepted finite element approach, known as Finite Element Method Magnetics program (FEMM). This software is dedicated to the resolution of electromagnetic problems

using 2D domain [3]. This methods works well when the equipment has a core of permeability significantly greater than 1, what is the present case.

## II. ELECTROMAGNETIC FORCES WITHIN TRANSFORMERS

It is known that the electromagnetic forces in transformer windings are generated by the interaction between current density and leakage flux density. These forces can be calculated by (1). The expression is given in [4].

$$\vec{f} = \vec{J} \times \vec{B} \quad (1)$$

Where:  $\vec{f}$  is the force density vector,  $\vec{J}$  is the current density vector and  $\vec{B}$  is the leakage flux density vector.

Fig. 1 illustrates a typical leakage flux distribution within the transformer. Near to the winding ends, the leakage flux bends towards the core, making shorter its return path. It can be seen that, at the top and bottom ends of the windings, the main leakage flux has both axial and radial components and in the throughout the winding length it is practically axial.

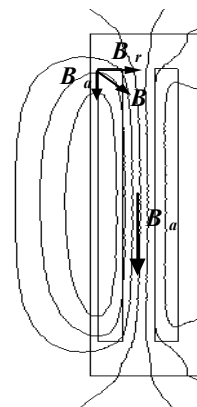


Fig. 1. Typical flux distribution in a transformer with concentric windings.

In a transformer with concentric windings, the axial component of leakage flux density ( $B_a$ ) interacts with the current in the windings, producing a radial force ( $F_r$ ). This is a well known phenomenon responsible for the mutual axial repulsion between the inner and outer windings. The radial flux component ( $B_r$ ) interacts with the windings currents, producing an axial force ( $F_a$ ) which acts in such a way to produce an axial compression or expansion of the winding coils [2].

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With the transformer operating under normal conditions, the forces are small. However, during external fault situations, the currents and fluxes reach high values, producing extreme electromagnetic axial and radial forces. As the leakage flux can be expressed as a function of the current, the resultant force will be proportional to the squared current, independently of the type of transformer windings arrangement [5].

In general, transformers are designed to withstand the maximum current peak of three-phase short circuits calculated as if the transformers were connected to an infinite busbar [5].

The equation to determine the short circuit current level ( $I_{sc}$ ) is given by expression (2). This is in accordance with [6]. The variable  $k$  is related to the asymmetry factor,  $S_n$  is the transformer rated power in MVA,  $V$  is the rated voltage and  $Z$  the transformer impedance in pu.

$$i_{sc} = \frac{k\sqrt{2}S_n 10^6}{\sqrt{3}VZ} A \quad (2)$$

#### A. Radial Forces in Concentric Windings

The radial forces within a transformer with concentric windings are calculated as given in [6]. Fig. 2 shows the resultant forces at the inner and outer windings. In addition to the forces, it is also shown the axial field density ( $B_a$ ). This field is considered as being constant all over the space between the windings. Hence, the force, in per unit of length, throughout the length of the coils, remains practically constant.

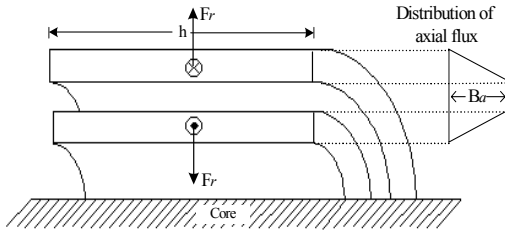


Fig. 2. Cross section of a transformer with concentric windings - axial field density ( $B_a$ ) and radial force ( $F_r$ ).

By neglecting the flux spreading out effect at the winding ends, the instantaneous ampere-turns ( $ni$ ) of each winding is responsible for producing the leakage field density ( $B_a$ ) given by:

$$B_a = \frac{4\pi(ni)}{10^4} T \quad (3)$$

This flux density will interact with the current, producing the mean radial force ( $F_r$ ) given by:

$$F_r = \frac{2\pi(ni)^2 D_m}{h} 10^{-7} N \quad (4)$$

Where:  $i$ ,  $n$  and  $h$  refer, respectively, to the current, the number of turns and the length of the transformer winding. The variable  $D_m$  is the mean winding diameter to which the radial force is calculated.

This force will act in such a way to produce a hoop stress

upon the outer windings and a compressive stress on the inner winding pushing it towards the limb. These forces are illustrated in Fig. 3.

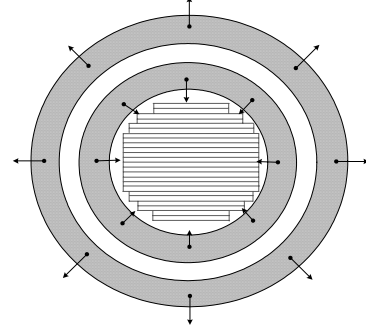


Fig. 3. Forces producing hoop stress and compressive stress in concentric windings.

The mean hoop and compressive stress in the concentric windings are calculated considering that the winding has  $n$  turns with a cross section  $a_c$  [6], i. e.:

$$\sigma_{mean} = \frac{F_r}{2na_c} N/m^2 \quad (5)$$

#### B. Axial Forces in Concentric Windings

The analytical calculation of the radial leakage field, and therefore, the axial force, is not as simple as the axial calculations [6]. Nevertheless, a well-accepted approach to achieve the above uses the Residual Ampere-turns Method. This is based on the principle that any arrangement of concentric windings can be split into two groups, each one having balanced ampere-turns relationship. The first group produces the axial field and the other the radial one [5].

Fig. 4 shows the radial field distribution and the axial forces in an arrangement with asymmetrical windings, with the height of the outer winding shorter than that of the inner winding. This winding asymmetry causes a large radial flux density in the region where the imbalance of ampere-turns occurs [7].

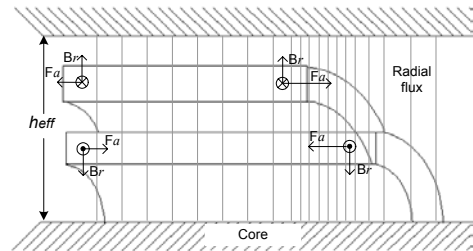


Fig. 4. Radial field density and axial force [6].

To calculate the axial force by the mentioned method, it is necessary to know the effective path length of the radial flux  $h_{eff}$ ; the average radial flux density  $B_r$  and the average value of ampere-turns  $(1/2)a(ni)$ . The variable  $a$  is the length of the section that causes the asymmetry expressed as a fraction of the total length of the winding. This section could be considered as being a group of short-circuited coils in the

windings.

The average radial flux density is given by:

$$B_r = \frac{4\pi a(ni)}{10^4 2h_{eff}} T \quad (6)$$

To determine the axial force ( $F_a$ ) for a transformer with asymmetry in one end of the external winding, the following equation is used:

$$F_a = \frac{2\pi a(ni)^2 \pi D_m N}{10^7 h_{eff}} N \quad (7)$$

Reference [6] provides expressions to calculate electromagnetic forces for various asymmetric conditions, including specific tap arrangements.

The axial forces cause bending between radial spacers [7] and, in this case, the axial stress related to axial forces is given by:

$$\sigma_{mean} = \frac{F_a L^2}{2tb^2} N/m^2 \quad (8)$$

Where:  $F_{ax}$  is distributed axial force (N/m);  $L$  is distance between stampings (m);  $b$  conductor axial dimension (m) and  $t$  is the conductor radial dimension (m).

### III. TIME DOMAIN MODEL

Different approaches have been traditionally used to obtain time domain transformer models. These methods are mainly based on electric equations (duality, equivalent electric circuit), electric/magnetic equations (equivalent electric-magnetic circuit) and, also, on magnetomotive force /reluctance models [8, 9]. In this paper, the representation of three-phase core type transformer will be carried out through mmf/reluctance model used in a time domain simulator. The choice of the mmf/reluctance model is particularly advantageous as it allows the interactions between the magnetic fluxes of different phases and the distinct connections of the windings can be done easily. Using this simulator, a 100 MVA transformer having four concentric windings per phase was implemented. Fig. 5 shows the equivalent magnetic circuit for this transformer.

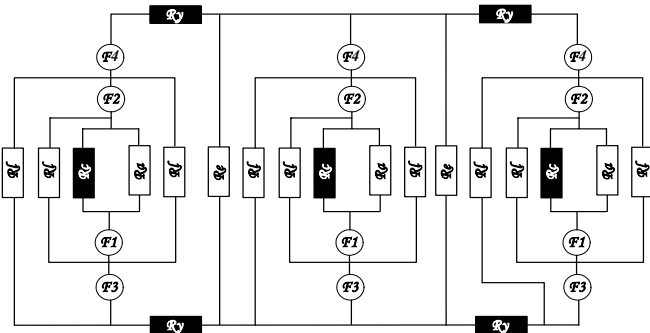


Fig. 5. Transformer equivalent electromagnetic model.

In Fig. 5:  $\mathfrak{R}_c$  and  $\mathfrak{R}_y$  (box in black) are the non linear reluctances corresponding to the core legs and yokes;  $\mathfrak{R}_a$ ,  $\mathfrak{R}_l$

and  $\mathfrak{R}_e$  are the linear reluctances corresponding to the air paths between the core and internal winding, between the inner and outer windings and that due to the air space outside the windings; and  $\mathcal{F}_1$ ,  $\mathcal{F}_2$ ,  $\mathcal{F}_3$  and  $\mathcal{F}_4$  are the magnetomotive forces produced by the windings.

Fig. 6 illustrates the physical parameters required to evaluate the area ( $A_{Leak}$ ) used to calculate the leakage fluxes and, consequently, the axial and radial electromagnetic forces [10].

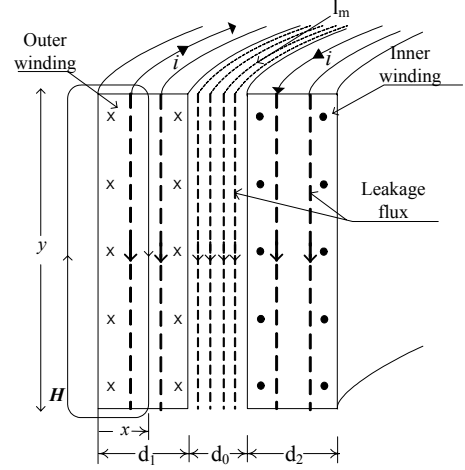


Fig. 6. Concentric windings physical parameters.

The required area ( $A_{Leak}$ ) can be calculated by (9).

$$A_{Leak} = l_m \left( \frac{d_1}{3} + d_0 + \frac{d_2}{3} \right) m^2 \quad (9)$$

Where:  $l_m$  is the average length of the winding circumference;  $d_1$  and  $d_2$  are the thickness of the outer and the inner windings, respectively, and  $d_0$  is the space thickness between the windings.

### IV. TIME DOMAIN AND FEMM RESULTS

The study results to be presented in this paper consist in two situations concerning transformer operation: normal operation (rated load) and short circuit conditions. The results obtained using the time domain model is compared to those obtained by the Finite Element Method Magnetics (FEMM) approach.

#### Case A. Time-Domain Results

The results achieved with the normal (rated load) operating conditions are given in Fig 8 (only those variables at the outer winding are illustrated).

Fig. 7(a) gives the load currents and Fig. 7(b) shows the leakage magnetic flux in the space between the windings. The values shown are relatively low in comparison to the flux density peak. The peak flux is around 0.2 T.

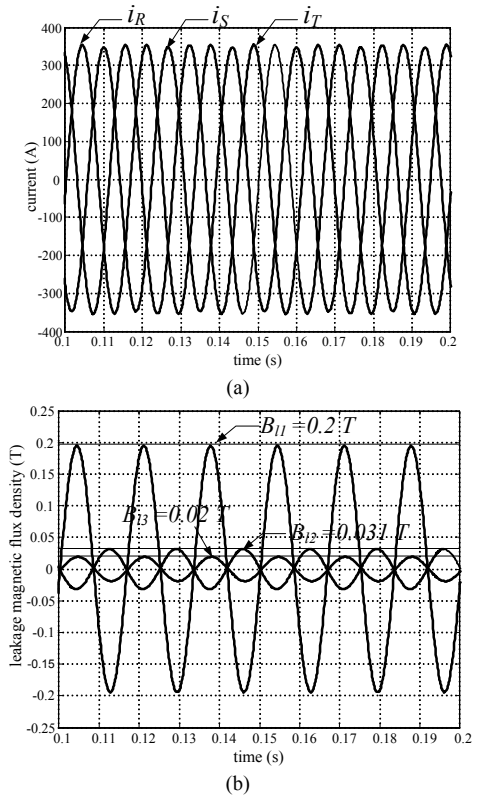


Fig.7. Transformer normal operation: (a) Current waveforms (b) Leakage flux density.

Once the short circuit is established a new set of results are obtained. Fig. 8(a) provides the current waveforms and shows that the phenomenon occurs at 200 ms. The related leakage flux is given in Fig. 8(b). It can be noticed that the peak value for this variable is about 3.6 T. This is within standard values during short-circuit conditions.

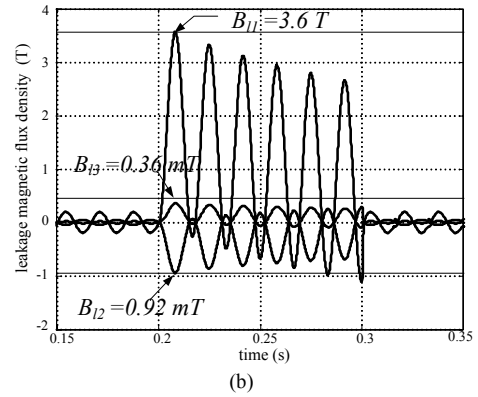
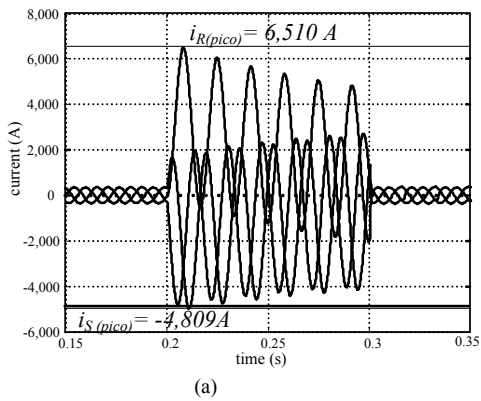


Fig. 8. Short circuit condition: (a) currents waveforms; (b) leakage flux density.

By knowing the time domain current and leakage magnetic flux density, the radial force can be calculated by expression (4). The results associated to the highest current, for both the normal and short-circuit conditions are given in Figs. 9 and 10, respectively, for the external and internal winding.

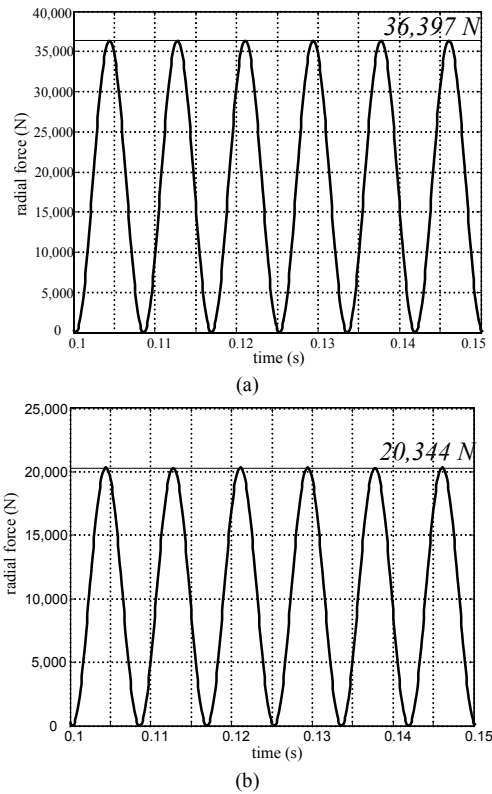


Fig.9. Radial force at normal operation. (a) external winding; (b) internal winding.

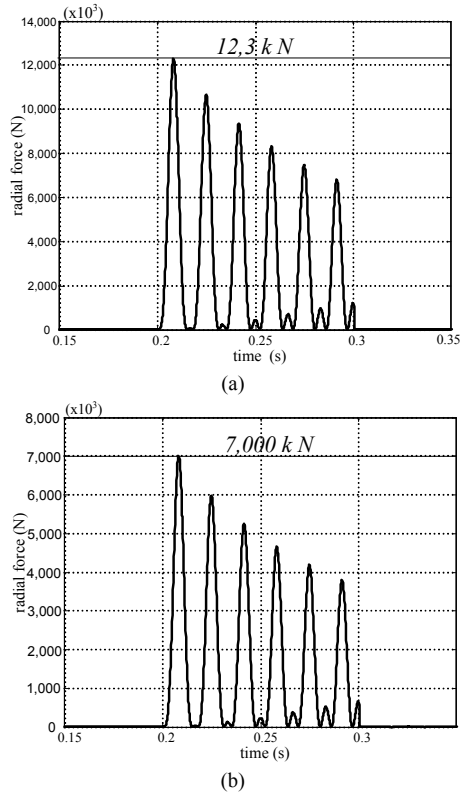


Fig.10. Radial force in short-circuit: (a) external winding; (b) internal winding.

By comparing the results it can be seen that the radial force during the short circuit increases dramatically, being about 338 times the peak force during normal operation for the external winding.

The mechanical stresses associated to the radial force are shown in Fig. 11, as given by equation (5). Due to explained reason only the hoop and compressive stress are considered.

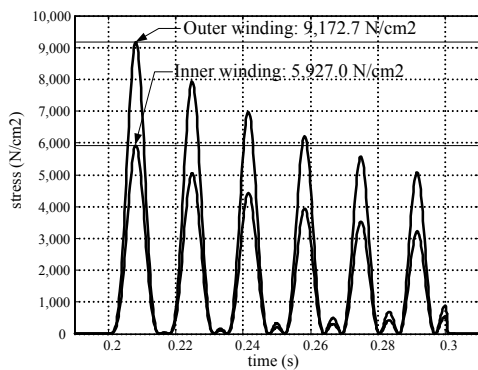


Fig.11. Mechanical stresses during short-circuit.

The maximum value found for the mechanical stress was 9,173 N/cm<sup>2</sup>.

### Case B. FEMM Results

The results here discussed are associated to the use of the well accepted FEMM (Finite Element method Magnetics) finite element software. Further considerations about this

software can be found in [3].

Following standard procedures, the effect of applying an external short-circuit was represented by an injection of 6,510 A at the HV and 11,061 A at the LV windings. The results are shown in Fig. 12. This figure provides the flux density pattern. In this condition, the leakage flux is relatively large in comparison to the rated flux leakage. The short circuit current level was determined in accordance with (2) using an asymmetry factor of 1.6.

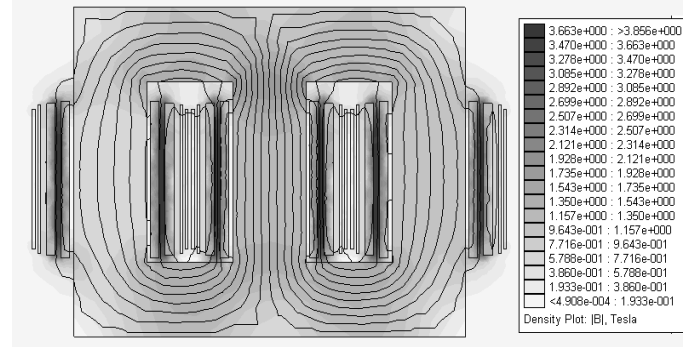


Fig.12. Magnetic flux density with short circuit conditions.

Fig. 13 shows the leakage flux distribution along the winding extension. It can be observed that, during the short circuit, the leakage flux density reaches values of about 3.6 T. With normal conditions the value was 0.2 T.

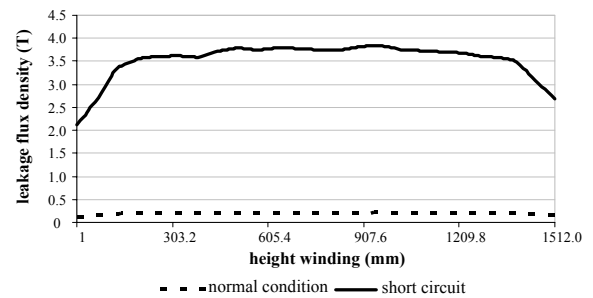


Fig.13. Leakage flux density along the winding height: normal and short circuit conditions.

Table I synthesizes the main results obtained from the Time Domain and the FEMM approach.

RESULTS SUMMARY WITH NORMAL AND SHORT-CIRCUIT CONDITIONS		
Normal Operation		
	Time domain	FEMM
Leakage magnetic flux density [T]	$200 \times 10^{-3}$	$200 \times 10^{-3}$
Magnetic flux density [T]	1.6	1.6
Radial force [N]		
Outer winding	$36.4 \times 10^3$	$36.9 \times 10^3$
Inner winding	$20.3 \times 10^3$	$20.6 \times 10^3$
Short Circuit Condition		
Leakage magnetic flux density [T]	3.5	3.6
Magnetic flux density [T]	1.23	1.32
Radial force [N]		
Outer winding	$12.3 \times 10^6$	$13.5 \times 10^6$
Inner winding	$7.0 \times 10^6$	$7.5 \times 10^6$

It can be clearly seen that the results derived from the two methods are in close agreement. By taking the FEMM calculations as reference values, the time domain software performance has shown to be appropriated to calculate internal forces and to obtain equipment mechanical stress information.

## V. CONCLUSIONS

This paper was concentrated on the use of a time domain program using reluctance and fmm models to calculate mechanical stresses occurring in transformer windings during short-circuit conditions. Using typical data for a 100 MVA, 230 kV transformer studies were carried out to highlight the method potentiality.

Due to the lack of reference values for validation purposes, the time domain results were then compared to corresponding ones obtained from a well established software based on finite elements. The values obtained from both strategies have demonstrated the methodology here proposed provided similar performance with both normal and short-circuit conditions. This indicates that the time domain approach can be assumed as a good procedure to evaluate transformer internal forces and mechanical stresses.

The investigations were carried out only for radial forces. The software use to achieve the stresses caused by axial force is to be developed in the sequence.

## VI. REFERENCES

- [1] Kulkarni, S. V., Khaparde, S. A., "Transformer Engineering - Design and Practice", Marcel Dekker, Inc, New York, 2004.
- [2] IEEE Guide for Failure Investigation, Documentation and Analysis for Power Transformers and Shunt Reactors, IEEE Standard C57.125, 1991.
- [3] Meeker, D., (September 2006) "Finite Element Method Magnetics - User's Manual Version 3.4" [online]. Available: <http://femm.berlios.de>.
- [4] Yun-Qiu, T., Jing-Qiu, Q., Zi-Hong, X., "Numerical Calculation of Short Circuit Electromagnetic Forces on the Transformer Winding", IEEE Transaction on Magnetic vol. 26, No.2, March, 1990.
- [5] Heathcote, J. Martin, "J&P Transformer Book", 12<sup>th</sup> ed., Oxford, Elsevier Science Ltd, 1998.
- [6] Waters, M., "The Short-Circuit Strength of Power Transformers", McDonald & Co. Ltd, London, 1966.
- [7] The Short Circuit Performance of Power Transformers, Brochure CIGRE WG 12.19, 2002.
- [8] Apolônio, R., Oliveira, J. C., Bronzeado, H. S., Vasconcellos, A. B., "The Use of Saber Simulator for Three-Phase Non-Linear Magnetic devices Simulations: Steady-State Analysis", The 7<sup>th</sup> Brazilian Power Electronic Conference, Fortaleza, Ceará, Brazil, September, 2003, pp 524-529.
- [9] Yacamini, R., Bronzeado, H. S., "Transformer Inrush Calculations using a Coupled Electromagnetic Model", IEE Proc. Sci Meas. Technol. Vol. 141, no. 6, November, 1994, pp 491-498.
- [10] Slemon, G. R., "Magnetolectric Devices: Transducers, Transformers and Machines", 5<sup>th</sup> ed, John Wiley and Sons, 1966.

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