Accurate Electromagnetic Transient Simulations of HVDC Cables and Overhead Transmission Lines

H.M.J. De Silva, A.M. Gole and L.M. Wedepohl

Abstract-- This paper introduces three different procedures for improvements in the accuracy at very low frequencies of time domain simulation models for overhead lines and underground cables. In the first two methods, a suitable modification is introduced to the functional form of the rational function approximated in the curve fitting procedure for the phase-domain model, such that the model is perfectly accurate at dc frequency. By comparing with analytical results, the paper also quantifies the errors made in simulation when the fitting at very low frequencies is inaccurate. These procedures are numerically efficient and robust. These two methods are also compared against a third less accurate method which simply adds a series dc resistor in the model to correct the dc response. Simulation results are presented for underground cable systems. The modifications introduced in the paper are expected to be useful in the simulation of High Voltage Direct Current (HVDC) transmission systems.

Keywords: Electromagnetic transients, rational function approximation, direct phase domain model, constrained least squares.

I. Introduction

HIGH Voltage direct current (HVDC) transmission of power over long distances is seeing increasing application with HVDC lines and cables being installed all over the world. Simulation models for such systems are required to be accurate over a very wide frequency range from zero Hertz -which is the nominal frequency on the line; to several tens of kilohertz -for thyristor switching and other transients.

The use of modern phase domain modelling techniques coupled with parameter estimation using Vector Fitting has greatly improved the accuracy of time-domain models for transmission lines and cables [2]. Although the time-domain model simulates the frequency range from a few Hertz to about several kilohertz, it has been difficult to get a good fit in the close neighborhood of 0 Hz (dc). For HVDC lines and cables, it is very important to accurately reproduce the response in the close vicinity of 0 Hz, as that is the nominal frequency on the line. The paper shows that forcibly trying to fit the characteristic at these extremely low frequencies requires a high order for the fitting rational function and sometimes leads to inaccurate fitting.

Time domain models require rational function approximation of entries of the characteristic admittance and propagation matrices. The paper proposes an efficient new approach based on linearization and constrained least squares that achieves a highly accurate fitting over the full frequency range. A modification is introduced to the functional form of the rational function approximated in the curve fitting procedure, such that the rational function approximations of the propagation matrix and the characteristic admittance can be fitted with more accuracy without having to substantially increase the number of poles. In this approach, the dc response is exact. Two possible variants of the functional form are considered. In the first approach, the admittance and propagation transfer functions are reformulated so that the dc response is factored out as an additive constant which can be directly selected. In the second approach, the transfer function is first fitted over the entire frequency range, which typically results in some fitting error at precisely zero Hz. A low frequency first order pole is then added to the resultant fitted function in order to realize the exact response at dc, without significantly affecting the remainder of the frequency response.

Time domain simulations of various transients on HVDC underground cables are presented to verify the validity of the approach. The above proposed methods are also compared with a simplified alternative approach which merely adds a corrective series resistance into each conductor to get the correct dc line resistance. However, this approach is shown to have poorer accuracy compared with the above approaches.

II. Phase domain modeling

In the discussion to follow, the term ‘line’ refers to both the overhead line and the underground cable systems, as the treatment developed is common to both. For an n-phase transmission line having length l, the frequency domain solution of the traveling wave equation can be expressed by the well know matrix-vector equations at each end-of the line given by [1],

$$I_k = Y_c V_k - A(Y_c V_m + I_m) \quad (1)$$

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$$I_m = Y_c V_m - A(Y_c V_k + I_k) \quad (2)$$

In the above equations, V and I are n dimensional voltage and current vectors and subscripts ‘k’ and ‘m’ denote sending-end and receiving-end of the line. Y and Z are (n×n) shunt admittance and series impedance matrices per unit length respectively. The (n×n) Characteristic admittance matrix Yc and the (n×n) Propagation matrix A are calculated as below using matrix functions [1]:

$$Yc = \sqrt{(YZ)^{-1}}Y \quad (3)$$

$$A = e^{-\sqrt{YZ}l} \quad (4)$$

In order to implement the model in the time domain, the elements of Yc and A are approximated with rational functions of suitable orders M and N [1] in the form shown below in (5) and (6). Such forms can easily be converted into differential equations which can be numerically integrated.

$$A_{i,j}(s) = \sum_{p=1}^{N} \frac{c_p e^{-s\tau}}{s - a_p} \quad (5)$$

$$Yc_{i,j}(s) = \sum_{q=1}^{M} \frac{c_q}{s - a_q} + d \quad (6)$$

The unknown coefficients, c_p and a_p (p = 1: N) in equations (5) and a_q, c_q (q = 1: M) and d in (6) are calculated using an efficient robust technique called Vector Fitting [3]. Note that the time delay (τ) in equation (5) is estimated before the fitting procedure. Sometimes for accurate curve fitting additional terms are added to (5) with different time delays (see equation (10)). For most practical transient simulation studies, it is sufficient to consider frequencies from zero Hz to 1 MHz for the fitting procedure. For simplicity, consider a single-conductor case.

A. Issues with Fitting of the Transfer Matrices at Low Frequency

At very low frequencies, the equations (3) and (4) reduce to,

$$Yc(s \to 0) \approx \sqrt{\frac{sC}{R_{dc}}} \quad (7)$$

$$A(s \to 0) \approx 1 - \sqrt{sCR_{dc}}\,l \quad (8)$$

Where,

C = capacitance per unit length (F)
R_{dc} = dc resistance of the line per unit length (Ω)

The square root term of s in equations (7) and (8) does not permit a rational function approximation with a low order; and thus a higher order rational function may be needed for the fitting, if very low frequencies are considered.

Consider the simple three single-core coaxial cable configuration shown in Fig 1, with data as in Table I. The frequency response of a typical entry Yc(1,1) of its characteristic admittance matrix Yc is shown in Fig 2., which also shows a plot of a rational function approximation of the form (6) obtained by limiting the lower bound of fitting frequency f_{min}. to 1 Hz.. Note that the fitting at frequencies lower than the lower bound is poor. Table II shows the order of the fitted transfer function (maintaining the same fitting error), with different lower bounds f_{min}. For the propagation function, decrease in the lower bound form 1 Hz to 1e-3 Hz results an increase in the order by 6 poles. This clearly indicates that the required order of the fitted function rises rapidly as the lower fitting frequency is reduced.

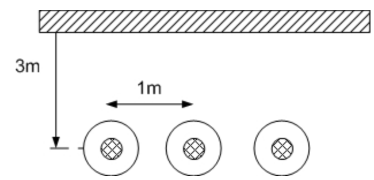

Fig 1: Simple Cable System: 3 single-core coaxial cables.

TABLE I
CABLE DATA

Radius of solid conductor (m)	0.00127
Outer radius of insulation (m)	0.00228
Dc resistance (Ω/ km)	0.034
Relative permittivity of insulation	2.85
Earth resistivity (Ω-m)	100

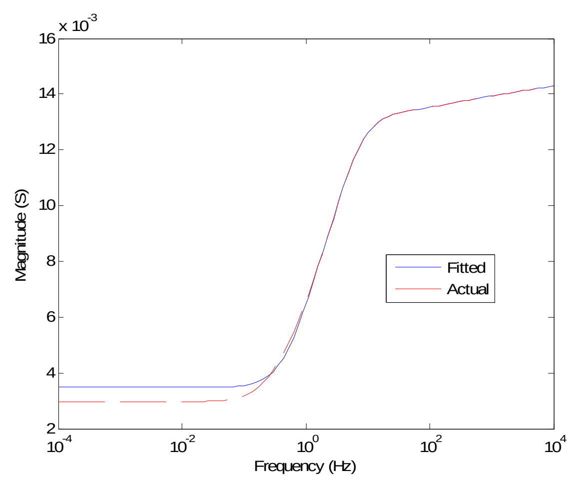

Fig 2: Magnitude of Yc(1,1)

TABLE II
ORDER OF RATIONAL FUNCTION APPROXIMATION

f_{min} (Hz)	f_{max} for A (kHz)	Order for A(1,1)	Order for Yc(1,1)
1e-3	1.63	22	20
1e-1	1.63	20	18
1	**1.63**	**16**	**16**
10	1.63	**16**	**16**

In order to observe the impact of the lower fitting bound, simulations are carried out on the cable system in Fig 1 with the sending end of the first conductor energized with a step voltage and with the receiving end grounded. The two remaining conductors are also grounded at both ends. If a sufficiently small lower bound in fitting is not used, the time domain simulation shows an incorrect long-term (i.e. after 1 s) dynamic response and also shows an incorrect dc solution as demonstrated by the plots of sending end current in Fig 3. The template for comparison is an analytical solution obtained by a direct frequency domain (FD) solution obtained by inverting the Laplace Transform of the exact transfer function. The response with a lower bound of 1 Hz, which is often selected by simulation tool users when studying dc systems, has 12 % steady state error. Reducing the lower bound to 0.1 Hz, reduces the error to 1%, but achieves this with a significant increase in the fitting order as discussed earlier. Note that poor fitting at very low frequencies is a major source of error when modelling dc lines.

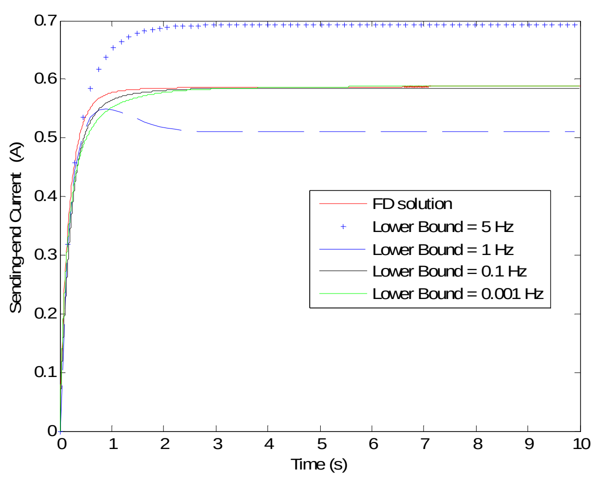

Fig 3: Sending-end current of the first conductor under short circuit conditions with different lower bounds considered for fitting.

Later in the paper, three approaches to overcome the problem are introduced that allow a lower order rational function approximation and the same time give the correct dc response. The proposed methods can be applied to any phase domain model. The Universal Line Model (ULM) [2] is considered in this paper, for which a brief description is provided below.

B. Universal Line Model

The Universal Line Model (ULM) [2] is a numerically efficient, robust phase domain model currently implemented in many electromagnetic transient programs such as PSCAD/EMTDC. This model is directly formulated in the phase domain to avoid difficulties with frequency dependent transformation matrices. In ULM, Yc(i,j) has the same formulation shown in equation (5) and all elements of Yc share the same set of poles derived from fitting the trace of Yc.

Although the propagation matrix is also ultimately represented in the phase domain, an intermediate modal domain analysis is conducted to obtain the travel times and poles for each mode. In the modal domain, this propagation function for the i^{th} mode is:

$$A^m{}_i(s) \approx e^{-s\tau_i} \sum_{n_i=1}^{N_i} \frac{c_{n_i}}{s-a_{n_i}} \tag{9}$$

Here, N_i is the order of approximation and unknown coefficients c_{n_i}, a_{n_i} are calculated using Vector fitting. τ_i is the delay corresponding to the i^{th} mode. When converted back to the phase domain, the elements of the propagation matrix are realized in the form (10) where each of the summation terms contains the poles and the travel times for a particular mode. The constants c'_{n_i}'s are obtained using least squares fitting.

$$A_{i,j}(s) \approx e^{-s\tau_1} \sum_{n_1=1}^{N_1} \frac{c'_{n_1}}{s-a_{n_1}} + e^{-s\tau_2} \sum_{n_2=1}^{N_2} \frac{c'_{n_2}}{s-a_{n_2}} + ... \tag{10}$$

III. METHODS FOR DC CORRECTION FOR THE ULM

This section introduces three approaches for improving the dc and low frequency response for transmission lines without significantly increasing the order of fitting.

A. Dc Correction by Adding Series Dc Resistor in Each Phase

The simplest method for correcting the error at dc frequency is to add a suitable series resistance in each conductor. The value of this series resistance is equal to the correct dc resistance minus the value obtained from the fitted function. Although, the model gives accurate dc response, there is a noticeable error at other frequencies. For an example, consider a cable having an inner conductor and sheath. Series dc resistances (6.21 ohms and -5.79 Ohms) must be added to inner conductor and sheath respectively to ensure correct dc resistance. Fig 4 shows receiving-end voltage for the sheath, if the sending-end is energized with step voltage. The sheath is kept open at both ends. The simulation output is compared with the FD solution indicating a noticeable error (the maximum error of 0.04 A or expressed as a percentage about 10.1% occurs at approximately $t \approx 7ms$). Although such an error may be within acceptable limits, in many instances, the addition of the negative resistance could introduce a net negative resistance at certain frequencies which leads to an unstable simulation.

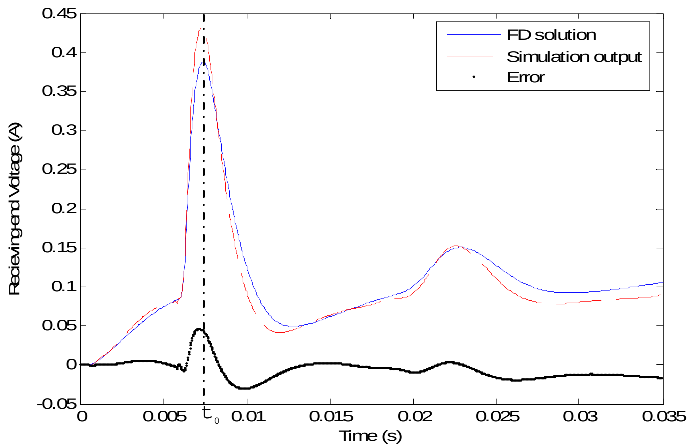

Fig 4: Receiving-end voltage of the sheath

B. Dc Correction by Adding a Pole and Residue

The known characteristic (elements of admittance or propagation matrix) is first fitted with a rational polynomial $f^{fitted}(s)$ as in the conventional phase domain method. A real pole $a_0 \in (0, 2\pi f_{\min} k), k < 1$ with a suitable residue c_0 is added to it, so that the modified function gives the exact dc value at zero frequency. This modification increases the order of the rational function by only one and does not affect the high frequency asymptote. Also, as the cutoff frequency of the additional term is smaller than the lower fitting bound, this correction is achieved with a very small error to the fitted part.

$$f^{\text{mod}}(s) = \frac{c_0}{s + a_0} + f^{fitted}(s) \qquad (11)$$

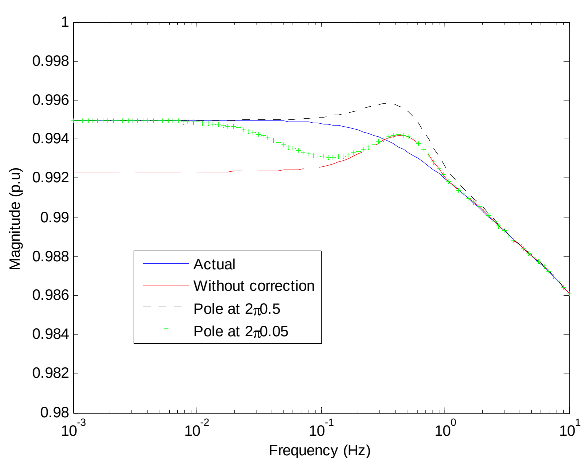

Fig 5: Magnitude of A(1,1) after and before addition of pole and residue

The choice of a_0 (or k in above paragraph) is selected by another optimization process that minimizes the error between the actual frequency response and that of f^{mod}. As seen in the frequency response of one of the propagation matrix elements A(1,1) in Fig 5, the selection of the pole at frequencies $2\pi \times 0.05$ r/s or $2\pi \times 0.5$ r/s , all give the accurate response at frequencies approaching dc, but the pole at $2\pi \times 0.5$ r/s gives the closest fit over the entire low frequency range. Note that without any correction, the frequency response curves of Fig. 5 indicate the presence of a steady state error. The corresponding time domain simulations for the line current are shown in Fig 6 where this steady state error is visible. When no correction is applied, this error is unacceptably large 0.2 A (33.2 %). When the correction is applied, there is no error in the steady state response. The pole frequency must be carefully selected. If this is too small, then the time required to reach steady state becomes large as seen in Fig. 6 for $2\pi \times 0.05$ r/s . This is also seen from Fig. 5, where the frequency curves for $2\pi \times 0.05$ r/s begins to deviate from the analytical result at 10^{-2} Hz. If the pole frequency is too large, it begins to interfere with the original fitted result $f^{fitted}(s)$ as indicated by (11). In the above case, the original function was fitted with a lower frequency bound of 1 Hz (see Table I) so the pole must be at a frequency less than 1 Hz. Selecting a frequency of 0.5 Hz ($2\pi \times 0.5$ r/s) gives a more accurate response that closely matches the analytical result as seen in Fig. 6. There is some error in the initial 2 s, with the maximum error of 0.13 A occurring at about 1 s.

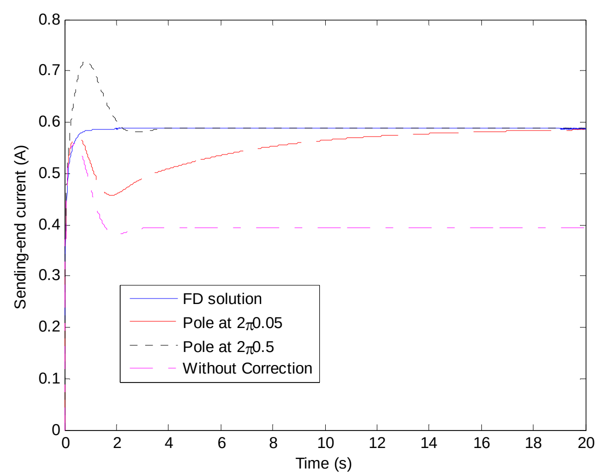

Fig 6: Sending-end current of first conductor with different poles selected.

Comments on Complexity of Fitting Method: The proposed method of adding a pole is carried out after the vector fitting approach described in Section II. The vector fitting is used to fit the frequency response characteristic with a relatively large value of $f_{\min}$ of say, 1 Hz. As seen in Table I, this can be achieved with much reduced number of poles (6 less for A(1,1), 4 less for Yc(1,1)) than a fit to a lower frequency of say 10^{-3} Hz. Hence the order of the modified fitted function $f^{\mathrm{mod}}(s)$ is just increased by one (due to the added pole). Hence the corresponding time domain model has a significantly reduced number of poles and is thus numerically more efficient. In addition, it guarantees the exact value for the dc component.

C. Dc Correction by Changing the Functional Form

If the equations (6) and (10) are written in an equivalent form (12) and (13), it is readily seen that putting s=0 results in a single term which is the response at dc (i.e. the term $d_{dc,theoretical}$). By selecting this term to be precisely the known exact dc value, a perfect fit at dc is guaranteed. This approach is introduced for the ULM in this paper, although an equivalent approach for the Z-Line Model is discussed in paper [6].

$$A_{i,j}^{\mathrm{mod}}(s) \approx \sum_{n_1=1}^{N_1} \frac{c_{n_1} s e^{-s\tau_1}}{s-a_{n_1}} + \sum_{n_2=1}^{N_2} \frac{c_{n_2} s e^{-s\tau_2}}{s-a_{n_2}} + \ldots + d_{dc,theoritical} e^{-s\tau_1} \quad (12)$$

$$Yc_{i,j}^{\mathrm{mod}}(s) = \sum_{m=1}^{M} \frac{c_m s}{s-a_m} + d_{dc,theortical} \quad (13)$$

These modified equations can be re-expressed in the form similar to equation (6) and (10) so as to make the formulation amenable to the vector fitting process. The details are as given in Appendix. A minor drawback of the method is that although the dc error is eliminated, the resultant propagation function at very large frequency deviates marginally from zero, which is contrary to the physical properties of typical propagation functions. The error made is small, however the terms C_n in equation 12 can be slightly perturbed using another least squares fitting to taper the high frequency response to zero. Note that this procedure does not alter the correct dc value.

As seen from the frequency response plots of the three elements of the first column of propagation matrix A in Fig. 7, this approach results in excellent fit over the entire frequency range, without any increase in the order of the fitted function. The corresponding time domain simulation for a short circuit on the cable in Fig 3 is shown in Fig 8, over a 10 s interval and shows accurate reproduction of the response obtained by purely frequency domain calculations. A similar accurate response is obtained for an open circuit termination as seen in Fig 9, which also confirms that the higher frequencies are also accurately simulated.

<u>Comments on Complexity of Fitting Method :</u> For the cable system shown in Fig 1, the proposed change in functional form results in a perfect dc fit of the actual frequency response characteristic. The orders of the fitted transfer functions for typical parameters, say A(1,1) and Yc(1,1) are both 16. This is the same as that for a fitting with a lower frequency of 1 Hz (see Table II), which has been shown to give a poor dc response. Trying to achieve a good dc response by reducing the lower fitting frequency of say 10^{-3} Hz adds 6 additional poles to A(1,1) and 4 to Yc(1,1) and yet may not give the accurate response at exactly 0 Hz..

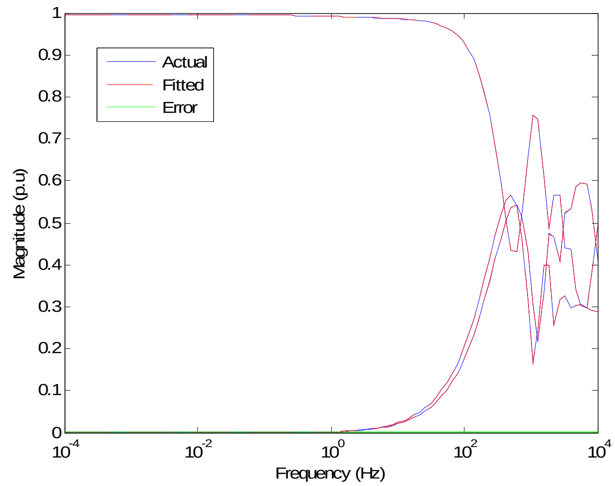

Fig 7: Actual and Fittted Magnitudes of A(1,1), A(2,1) and A(3,1) with Change in Functional Form

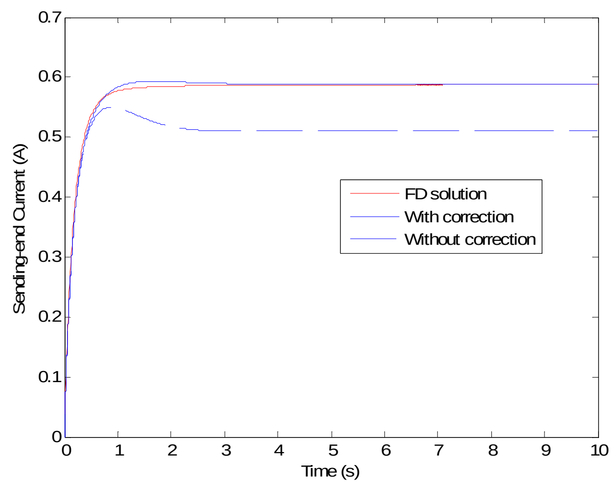

Fig 8: Sending-end Current of First Conductor with and without Correction

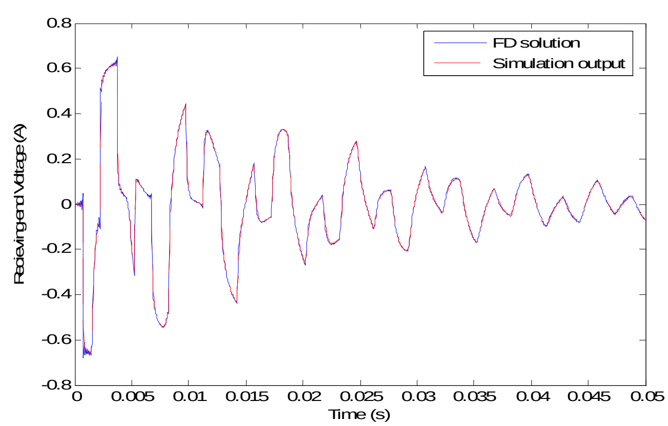

Fig 9: Induced voltage on middle conductor under open circuit conditions, if the left conductor is energized with step voltage.

IV. Conclusions

The paper demonstrates that the traditional approach to modelling cables and transmission lines in the time domain using fitted admittance and propagation characteristics can result in significant errors at frequencies approaching dc. Although some improvements can be made with reducing the lower bound of the fitting frequency to make the dc fitting more accurate, this usually results in an increase in the order of the fitted transfer functions. Alternative approaches which modify the form of the implemented transfer function, either by adding a low order pole or by reformulation in a form that permits direct specification of the dc values, result in accurate simulations over the entire frequency range from dc to higher frequencies. These two methods are more accurate and potentially more stable in comparison with a simplified treatment of adding a corrective series resistance in each phase. In particular, the second of these two method is especially easy to implement and is recommended for the modelling of HVDC transmission systems in which faithful reproduction of the very low frequency behavior is just as important as high frequency behavior.

V. Appendix

Using proper choice of variables, functions in equation (13) can be converted into a form suitable for vector fitting.

$$Yc_{i,j}^{\mathrm{mod}}(s) = \sum_{m=1}^{M} \frac{c_m s}{s - a_m} + d_{dc,theortical} \qquad (A.1)$$

$$Yc_{i,j}^{\mathrm{mod}}(s) = \sum_{m=1}^{M} \frac{c''_m}{s'' - a''_m} + d_{dc,theortical} \qquad (A.2)$$

where,

$$c''_m = -\frac{c_m}{a_m} \qquad s_0 = \frac{1}{s} \qquad a''_m = \frac{1}{a_m}$$

The unknown coefficients (a''_m's and c''_m's can be calculated using Vector fitting). Same procedure can be applied to equation (12) for A(i,j).

VI. Acknowledgment

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VII. Biographies

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