Evaluating the Importance of Properly Representing Actual Transmission Line Transposition for Electromagnetic Transient Studies

A. V. Elguera, M. C. Tavares and C.M. Portela

Abstract-- In this paper transmission line transposition is analyzed focusing electromagnetic transient phenomena in the range up to 10 kHz. Line transposition is implemented to decrease phase unbalances at fundamental frequency. In the present paper it will be analyzed the error of treating transmission line as ideally transposed for all frequency ranges, specifically when dealing with switching transients up to 10 kHz. A theoretical analysis is performed identifying the unbalances between the phases considering the transmission line ideally transposed and considering the line with actual transposition sections. The difference between both line representations under transient was analyzed. The frequency dependence of transmission line parameters was properly represented.

Keywords: Electrical Parameters, Electromagnetic Transients, Frequency Dependence, Line Transposition.

I. INTRODUCTION

THE transmission system should not introduce any unbalance to the energy transported. However, the geometry of the transmission towers do create unbalances because the distances between phases and between the phases to earth are not equal; consequently, it can generate unbalance to the power flows at fundamental frequency.

The geometries of the towers of high voltage transmission lines produce impedance asymmetry, which in turn causes corresponding voltage and current unbalance at the line end. The effect of line asymmetry can be eliminated at fundamental frequency by the use of phase transposition, dividing the line into three, or multiples of three sections. Accordingly, transpositions are often used in long transmission lines as a mean of balancing fundamental frequency impedance / admittance of the line. The transposition in transmission line

Presented at the International Conference on Power Systems Transients (IPST'07) in Lyon, France on June 4-7, 2007 consists in the change of phases positions.

In electromagnetic transients studies it is usual to represent the transmission lines as an ideally transposed line (ITL). A line with actual transposition section (TL) can be accounted as an ITL in steady state, but not for all frequency range [1][2]. The correct representation of the transmission line is of big concern to electromagnetic transients studies.

In this paper it will be shown that the representation of ITL for all frequencies is not totally correct. This is achieved through the analysis of the line response in frequency domain. The line is represented through transfer function for all frequency range both for: a line with its actual transposition sections and for an ideally transposed line. Finally a transient simulation was performed for both line representation.

It was possible to observe the error of considering an actual transposed line as an ideally transposed line for all frequencies important during the occurrence of electromagnetic transient phenomena.

II. FUNDAMENTAL THEORY

A. Transmission Line Electrical Parameters Calculated for Single Circuit

The electrical parameters of a transmission line are expressed in matrices form, whose dimensions correspond to the number of line conductors (sub conductors that compose the phase and the ground-wires). The matrices of longitudinal and transversal parameters are reduced to equivalent matrices, whose dimensions correspond to the number of phases of the line.

1) Matrix of Longitudinal Parameters

$$Z_{ij} = (Rc_{ij} + Re_{ij}) + j \cdot (Xc_{ij} + Xe_{ij} + Xext_{ij})$$

i, j = 1,2,3,..., n (1)

Where:

n: number of conductors.

Rc+jXc : internal impedance of conductor per unit length.

jXext : external reactance between conductors per unit length (supposing ideal ground).

Re+ jXe : correction of the external impedance per unit length (because the ground is not an ideal conductor).

The units of all elements are in Ω/m .

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2) Matrix of Transversal Parameters per unit length

$$Y_{ij} = j \cdot 2 \cdot \pi \cdot \omega \cdot \varepsilon_0 \cdot [A]^{-1}$$
⁽²⁾

Where:

- ω : angular frequency (*rad/s*).
- ϵ_0 : permittivity of air (8.85 pF/m)
- A : matrix defined by elements $\ln(D_{ij}/d_{jj})$,

$$D_{ii}, d_{ii}$$
: distance between conductor *i* and its image *j*

and distance between conductors i and j, respectively.

In the Fig. 1 it is presented the tower of the transmission line analyzed.





Fig. 2. Self resistance per unit of length.

The longitudinal parameters were calculated in frequency domain for the transmission line analyzed, supposing a ground resistivity of 1000 Ω .m. In Fig. 2 and Fig. 3 the self resistance and self inductance, per unit of length, in frequency domain are presented.

The electrical parameters presented are calculated

supposing a non-transposed line. It can be observed in Figs. 2 and 3 that the self values for phases "a" and "b" are not equal, while the values for phase "a" and "c" are equal, which is coherent with the line tower depicted in Fig. 1.



Fig. 3. Self inductance per unit of length.

B. Multiphase Transmission Lines

The fundamental equations that describe wave propagation for a multiphase transmission line in frequency domain can be presented as:

$$\frac{d[V_p]}{dx} = -[Z][I_p]; \quad \frac{d[I_p]}{dx} = -[Y][V_p]$$
(3)

The second order differential equations, in matrix form, involving state variables (voltage and current) are defined as the following:

$$\frac{d^{2}[V_{p}]}{dx^{2}} = [Z][Y][V_{p}]; \quad \frac{d^{2}[I_{p}]}{dx^{2}} = [Y][Z][I_{p}]$$
(4)

where:

x : longitudinal distance.

[Z] : impedance matrix per unit of length

[Y] : admittance matrix per unit of length

 $[V_p]$ and $[I_p]$: column phase-to-earth voltages and longitudinal phase current vectors, respectively.

The great difficulty in the solution of differential equations for multiphase transmission line is due to coupling between the phases, as the variation in each phase depends on the others.

With the aid of linear transformations to voltage and current (eigenvalues and eigenvectors) it is possible to represent a multiphase system eliminating the coupling.

This procedure makes it possible to manipulate "n" uncoupled circuits, where "n" is the number of phases in original circuit. The relation between voltage and current in natural propagation modes "m" and phase "p" is defined by:

$$[\mathbf{V}_{\mathbf{n}}] = [\mathbf{T}_{\mathbf{v}}] \cdot [\mathbf{V}_{\mathbf{m}}] \tag{5}$$

$$[\mathbf{I}_p] = [\mathbf{T}_i] \cdot [\mathbf{I}_m] \tag{6}$$

Where $[T_v]$ and $[T_i]$ are square matrices related to the voltage and current, respectively.

Applying the equations (5) and (6) in (4), the equations to represent the natural propagation modes are:

$$\frac{d^{2}[V_{m}]}{dx^{2}} = [T_{v}^{-1}] \cdot [Z][Y] \cdot [T_{v}] \cdot [V_{m}] = \Gamma^{2} \cdot [V_{m}]$$
(7)

$$\frac{d^{2}[\mathbf{I}_{m}]}{dx^{2}} = [\mathbf{T}_{i}^{-1}] \cdot [\mathbf{Y}][\mathbf{Z}] \cdot [\mathbf{T}_{i}] \cdot [\mathbf{I}_{m}] = \Gamma^{2} \cdot [\mathbf{I}_{m}]$$
(8)

The matrices $[T_v]$ and $[T_i]$ are chosen so that through linear transformation, the matrices Γ^2 of (7) and (8) becomes diagonal matrices. The matrices Γ^2 are formed by the eigenvalues and are related to the line wave propagation [4].

In (9) the modal propagation coefficient matrix is presented. The matrix $[Z_c]$ also has diagonal form in modal domain and is defined by $[Z_c] = \sqrt{[Z] \cdot [Y]^{-1}}$.

$$[\gamma] = \sqrt{[Y][Z]} = \begin{bmatrix} \gamma_1 & 0 & 0 \\ 0 & \gamma_2 & 0 \\ 0 & 0 & \gamma_3 \end{bmatrix}$$
(9)

Each transformation matrix, described in (7) and (8) may be solved separately like a set of single-phase wave equations.

The transmission line can be represented as a two-port elements (ABCD constants), as depicted in (10) and (11), in mode domain where $[\gamma]$ and $[Z_c]$ are diagonal matrices

$$\begin{bmatrix} \begin{bmatrix} V_{2-abc} \\ I_{2-abc} \end{bmatrix}_{mode} = \begin{bmatrix} \begin{bmatrix} A \end{bmatrix} & \begin{bmatrix} B \\ \begin{bmatrix} C \end{bmatrix} & \begin{bmatrix} D \end{bmatrix}_{m} \cdot \begin{bmatrix} \begin{bmatrix} V_{1-abc} \\ I_{1-abc} \end{bmatrix}_{mode}$$
(10)

being:

$$\begin{bmatrix} [A] = \cosh([\gamma] x) & [B] = -[Zc] \sinh([\gamma] x) \\ [C] = -[Z_c]^{-1} \sinh([\gamma] x) & [D] = [A] \end{bmatrix}$$
(11)

C. Ideally Transposed Line (ITL)

A transmission line is called ideally transposed line (ITL) if the transposition is supposed equivalent to ideal transposition for the entire frequency range of the study. The transposition results in an almost perfect balanced line parameters if the line is divided in small transposition sections when compared with ¹/₄ of wave-length (λ) of the involved frequencies. For fundamental frequency this is achieved with section around 100 km, and total line parameter can be considered as the mean value of each line section (12).



Fig. 4. Line scheme transposition with three sections (TL3). \Box

$$[Z_{\text{IT}}] = \frac{[Z_1]}{3} + \frac{[Z_2]}{3} + \frac{[Z_3]}{3} = \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix}$$
(12)

This equivalent matrix of ITL has all diagonal elements equal and all off-diagonal elements also equal.

The Clarke's transformation, T_{CL} , matrix can be used as the transformation matrix for ITL:

$$[T_{CL}] = \begin{bmatrix} 1/\sqrt{3} & -1/\sqrt{6} & 1/\sqrt{2} \\ 1/\sqrt{3} & 2/\sqrt{6} & 0 \\ 1/\sqrt{3} & -1/\sqrt{6} & -1/\sqrt{2} \end{bmatrix}$$
(13)

It is usual to consider the line as ITL for the entire frequency range in transient studies, which is not valid for higher frequencies where $\frac{1}{4}$ of the wave length is not much higher than the line transposition cycle (300 km). For instance for 5th harmonic, 300 Hz, supposing fundamental frequency equal 60 Hz, the wave length is a little less than 1000 km, and the line cycle of 300 km cannot be considered much lower than $\frac{1}{4}$.

D. Line with Actual Transposition (TL4)

A line with actual transposition can be represented with two or three structures of transposition per transposition cycle. A line with two structures of transposition (TL3) has the same scheme of Fig. 4, where each section is considered as a small non-transposed line. A line with three structures of transposition or four sections has the particularity of equal phase positions at both transposition cycle extremities (Fig. 5).



Fig. 5. Line scheme transposition with four sections (TL4).

To represent an actual transposed line each section should be considered as a non-transposed line (NTL) and the transposition itself should be included as an additional element.

The modal transformation of a non-transposed line has to be obtained through eigenvectors of Γ^2 [3][4][5].

E. Analyzing Wave Propagation in Phase Domain

The solution of the wave propagation (3) using modal transformation is an elegant process. For ITL the analysis of the sequence components or homopolar/non-homopolar modes behavior is interesting for power systems studies.

The propagation equation can be solved straight in phase domain, avoiding any modal transformation.

Consider a uniform line with length section ℓ . Supposing a very short finite section $\Delta \ell$, considering the derivative approximation at point x applied to the interval x and $x + \Delta \ell$ (what results in an error of the order of $\Delta \ell$). Between the two points identified by 1 and 2 the distance $\Delta \ell = \ell / 2^n$:

$$\mathbf{U}_2 = \mathbf{U}_1 - \mathbf{Z} \cdot \Delta \ell \cdot \mathbf{I}_1 \tag{14}$$

$$I_2 = -Y \cdot \Delta \ell \cdot U_1 + I_1 \tag{15}$$

or

$$\begin{vmatrix} \mathbf{U}_2 \\ \mathbf{I}_2 \end{vmatrix} = \mathbf{W}_{\Delta \ell} \cdot \begin{vmatrix} \mathbf{U}_1 \\ \mathbf{I}_1 \end{vmatrix}, \qquad \mathbf{W}_{\Delta \ell} = \begin{vmatrix} \mathbf{I} & -\mathbf{Z} \cdot \Delta \ell \\ -\mathbf{Y} \cdot \Delta \ell & \mathbf{I} \end{vmatrix}$$
(16)

being $W_{\Delta \ell}$ the voltage and phase current transfer function between the two terminals of section length $\Delta \ell$ and **I** the identity matrix. Supposing a very short finite section $\Delta \ell$, considering the derivative approximation varying linearly with x, applied at the interval x and $x + \Delta \ell$ (what results in an error of the order of $\Delta \ell^2$). Between the two points identified by 1 and 2 the distance $\Delta \ell = \ell / 2^n$:

$$U_2 + Z \cdot \frac{\Delta \ell}{2} \cdot I_2 = U_1 - Z \cdot \frac{\Delta \ell}{2} \cdot I_1$$
 (17)

$$Y \cdot \frac{\Delta \ell}{2} \cdot U_2 + I_2 = -Y \cdot \frac{\Delta \ell}{2} \cdot U_1 + I_1$$
 (18)

or

$$\begin{vmatrix} \mathbf{U}_{2} \\ \mathbf{I}_{2} \end{vmatrix} = \mathbf{W}_{\Delta \ell} \cdot \begin{vmatrix} \mathbf{U}_{1} \\ \mathbf{I}_{1} \end{vmatrix} ,$$

$$\mathbf{W}_{\Delta \ell} = \begin{vmatrix} \mathbf{I} & \mathbf{Z} \frac{\Delta \ell}{2} \\ \mathbf{Y} \frac{\Delta \ell}{2} & \mathbf{I} \end{vmatrix}^{-1} \cdot \begin{vmatrix} \mathbf{I} & -\mathbf{Z} \frac{\Delta \ell}{2} \\ -\mathbf{Y} \frac{\Delta \ell}{2} & \mathbf{I} \end{vmatrix}$$
(19)

being $W_{\Delta \ell}$ the voltage and phase current transfer function between the two terminals of section length $\Delta \ell$ and **I** the identity matrix.

The formulation (19) allows a $\Delta \ell$ value higher than the one in equation (16), avoiding the use of an extremely small " $\Delta \ell$ " section length, which may originate a significant numerical error due to the valid digits of numerical operations.

The matrix W representing the transfer function of line length ℓ is obtained from the cascade of 2^n identical sections of the line, each one with length $\Delta \ell$. The solution of the wave propagation is obtained by *n* successive squaring of two-port representations of the π -section (Fig. 6) [2].



Fig. 6. Transmission line represented by cascade of π -sections

By example, the matrix applicable to a 300-km line can be obtained from the transfer matrix applicable to a "line" of 300 km / 1024, squared 10 times. for a "line" of 300 km / 1024, the hyperbolic functions **tanh** and **sinh** coincide with their arguments, and **cosh** is practically 1, with high numerical accuracy the error of (19) is very small. The ¹/₄ wave length for 10 kHz is around 7.5 km, which is much higher than 300 km / 1024. So, the very simple indicated procedure, based in (19) and (as an example), in 10 successive squaring, is equivalent to consider hyperbolic functions (e.g. **sinh**, **cosh**, **tanh**) applied to matrices and to manipulate the obtained matrices.



Fig. 7. Transmission line scheme with actual transposition (TL4).

III. LINE TRANSFER FUNCTION

Analyzing (11) it can be seen that matrices [A], [B], [C] and [D] represent the line transfer function, being all frequency dependent.

A. Transfer Function Matrix of an Actual Transposed Line

The transfer function matrix of the actual transposed line (TL4) depicted in Fig. 7 considers sections of non-transposed line and the transposition itself. The transformation matrices of the non-transposed line sections were obtained using the Squaring Method described above.

The matrix representation of transfer function in phase domain of that non-transposed section is:

V _{pa-2}		A ₁₁	A ₁₂	A ₁₃	B ₁₁	B ₁₂	B ₁₃	V _{pa-1}	
V _{pb-2}		A ₂₁	A_{22}	A 23	B_{21}	B_{22}	B ₂₃	V _{pb-1}	
V _{pc-2}	_	A ₃₁	A ₃₂	A 33	B_{31}	B_{32}	B ₃₃	V _{pc-1}	(20)
I _{pa-2}	-	C ₁₁	C ₁₂	C ₁₃	D ₁₁	D ₁₂	D ₁₃	I _{pa-1}	
I _{pb-2}		C ₂₁	C_{22}	C_{23}	D_{21}	D_{22}	D ₂₃	I _{pb-1}	
I _{pc-2}		C ₃₁	C ₃₂	C ₃₃	D ₃₁	D ₃₂	D ₃₃	I _{pc-1}	

The representation of the others sections has the same form of (20) but these representations change when they are associated with the transposition matrix.

The transposition transfer function is expressed by:

$$[T1] = [T2] = [T3] = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$
(21)

The scheme of TL4 representation is presented in Fig.6.

In the Fig. 7 the transfer function matrix represents each section. To obtain the complete line representation it is necessary to multiply the line sections transfer function matrices by the transposition transfer function in sequence. Finally, the total transference matrix that represents the complete line TL4 is obtained.



Fig. 8. A22 modulus for ITL and LT4.

IV. RESULTS

In Fig. 8 to Fig. 11 some results are presented for elements A22 and B13 of two-port element (17) considering the actual

line transposition (TL4) and ideal transposition (ITL). The sub-matrix A represents the voltage ratio supposing the line opened at the sending end ($I_1 = 0$), while the sub-matrix B represents the ratio between the voltage at the receiving end and the current at the sending end, supposing the line short-circuited at sending end ($V_1 = 0$).

The others elements of transfer function matrix have similar behavior. For frequencies above 180 Hz the discrepancy between the two line representations increases.



Fig. 11. B13 arguments for ITL and TL4.

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In Fig. 12 to Fig. 15 the modulus error considering the ideal transposition representation and the actual transposed line representation are presented.

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The error calculated was the ratio between the difference between the two representations and the LT4 result. The discrepancy between the two representations is significant and should be considered for quality studies and electromagnetic transient studies as well. If it is simulated a phenomenon with dominant frequency in the range where the line representation is inadequate, the results obtained will be incorrect.







Fig. 13. Error between ITL and TL4 B13 modulus.



Fig. 14. Error between ITL and TL4 C12 modulus.

V. TRANSMISSION LINE ENERGIZATION

In order to observe the influence of the line representation in a transient study a simple case of line energization was simulated in EMTDC. The system modeled consisted of an ideal source (fundamental frequency 60 Hz) behind a system equivalent modeled as a 50-km long line considered ideally transposed. The line was modeled both as ITL and TL4 with 312.5 km length. A pre-insertion resistor of 300 Ω was used. The frequency dependence of the longitudinal parameters was modeled through the phase domain model available in PSCAD/EMTDC. The system voltage was 440 kV and the nominal base voltage used was 359.26 kV. The parameters of the equivalent 50 km line considered to model the equivalent system, at 60 Hz, are:





 TABLE I

 VOLTAGE AT RECEIVING END – PHASE TO GROUND PEEK VALUES

order	nominal voltage			
	ITL	TL4		
-	1.25	1.28		
10	1.86	1.52		
17	1.64	1.80		
30	1.53	1.46		
38	1.45	1.40		
53	1.37	1.33		



Fig. 16. Voltage at receiving end terminal for LT4 and LIT -10^{th} harmonic currentnjection.

Some energization cases were simulated as depicted in Table I. A 50 A_{rms} harmonic current source (positive sequence) was included in some cases to highlight the difference among the line representations. For instance, for the case with the 10th harmonic current injection the difference was rather high, around 22 %. For the case with 17th harmonic current injection the difference overvoltage difference was 10 %. In the former case the highest overvoltage occurred for the ideally transposed line while in the second case the highest

overvoltage occurred when the transposition was properly represented.

The voltages at receiving end for both line representations during the energization simulation are presented in Fig. 16.

The cases were simulated to point out the discrepancy between the models. No effort was applied to identify worst cases, but if a series of statistical switching were simulated the difference between the representations would be stressed.

VI. CONCLUSIONS

The correct representation of a transmission line is very important to perform accurate studies. This paper presents an analysis pointing out the inaccuracies of representing a line with actual transposition sections as an ideally transposed line for all frequency range in an electromagnetic transient study.

The discrepancy between the actual line representation and the approximate representations as ITL can produce very different results when the line is submitted to electromagnetic transients due to usual maneuvers or during the occurrence of faults. Specifically for the line in example, the presence of the 10th or the 17th harmonics can result in very different perturbation in terms of voltage and current in the extreme of the line. As presented, the worst case does not always occur when the line has a simplified or a more accurate representation. It is important, therefore, to properly represent the system being analyzed.

When a specific case is studied, the interaction between the line and rest of the system should also be analyzed, which can even increase the differences found.

The transposed line must be correctly represented, mainly if loads with high harmonic generation exist in the system.

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