Symmetrical Components for Transient Regime Applications in MV Systems

Wan-Ying Huang and Robert Kaczmarek

***Abstract*-- In order to apply advantageously the symmetrical components theory to Single Line to Ground (SLG) fault transients in distribution systems we take profit of similitude of transient components in faulted phase current and in zero sequence current on faulted feeder. Then we are able to take into account the load current contribution within voltage drop account of zero sequence equivalent circuit. The result is an Extended Zero Sequence circuit, of better accuracy comparing to Traditional Zero Sequence circuit or Full Sequence 0-p-n circuit. It reproduces the overall response of distribution system to an SLG fault inception, whereas traditional equivalent circuits assume only one frequency analysis. Consequently, the new zero sequence circuit is more adapted for evaluation of amplitudes of actual transients or for fault location tasks.**

***Index Terms*—power distribution lines, power cables, fault diagnosis, fault location, equivalent circuits**

I. Introduction

Symmetrical component theory has been for decades a successful approach in power system fault analysis [1], with negative sequence and zero sequence quantities used for relaying. A special challenge is its application to the still unresolved problem of an SLG fault location in distribution networks. The problem is fundamental, as an analysis based on symmetrical components assumes only steady state and single frequency phenomena, either for fundamental frequency or an extracted salient component of charging currents called "main frequency" component. This orientation, denying access to useful fault information contained in transient waveforms, limits efficiency of equivalent circuits in evaluation of transients and in fault location procedures.

For almost two decades two Single Frequency approaches in full sequence 0-p-n equivalent circuit have been studied for fault location task: the method exploiting Main Frequency of charging components [2-5] and the one which uses Rated Frequency component [2, 4, 6]. The simulation results were reported satisfactory for small value of fault resistance.

Not more successful were other location procedures, like the method based on iterative identification of faulted section followed by application of current pattern rules [7], another one using recursive least squares approach [8], an artificial neural networks [9], wavelet, ANN and differential equation [10] or least squares fitting [11]. None of these methods reached beyond 50Ω fault resistance value.

One of reasons may have been lack of truly equivalent circuit capable to reproduce adequate response to fault inception voltage both in transient and in steady state. The principal problem here is with supply voltage, which in case of the zero sequence circuit should be the pre-fault voltage in the fault occurrence point. This voltage, that we call inception one, often cited in literature [12] has never been given an analytical form and usually is replaced by secondary transformer voltage, with intention to assure zero sequence voltage over the parallel branch carrying zero sequence currents (Fig. 2). The proper calculation of this voltage is a topological challenge, because it depends on taking into account the faulted phase current contribution in zero sequence overall voltage drop account, what seems rather contradictory.

We have solved this difficulty by integrating this contribution not explicitly, but rather as a correction factor of input voltage. This voltage is then given an analytical form without altering the zero sequence current paths.

The new circuit is an extended Zero Sequence Circuit. It presents better performances than any of equivalent networks in use, both in terms of waveforms equivalence and the fault parameter's range in location procedures.

The following development and applications are presented within distribution system grounded with Petersen Coil. However, their principle is still valid in systems grounded with high impedance as well as ungrounded systems, but not in solidly grounded ones.

II. Development

We consider a radial network (**Fig. 1**) supplied by a generator through a delta – star transformer grounded with Petersen coil (PC). A single line-to-ground (SLG) fault through a resistance R_f is installed on one of the feeders at the distance l_f from busbar on the phase 3.

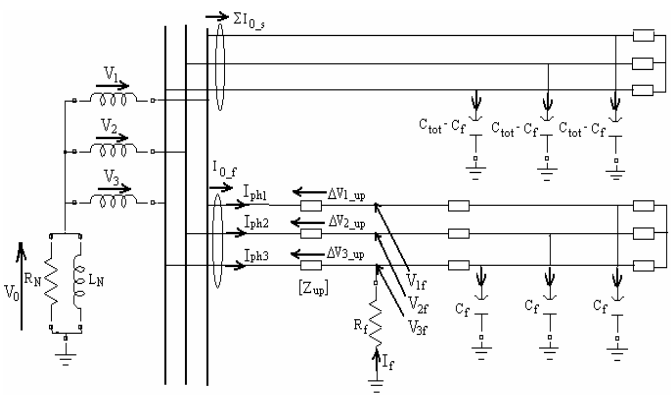

Fig. 1. A phase-to-ground fault in a radial network, all the sound feeders aggregated in one

The traditional zero sequence representation comprises the zero sequence branch in series with the faulted current branch supplied by the secondary transformer voltage, most of the times with line impedances neglected.

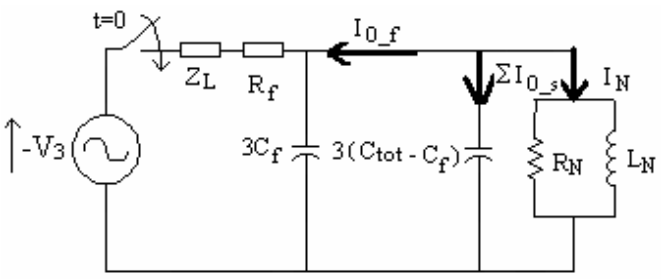

Fig. 2 Traditional equivalent circuit

It can produce good transient response for weak fault resistance values and with the fault located near busbar. Otherwise the equivalence is always poor even if the faulted segment from the fault to busbar is included in series with R_f (Fig. 2).

A. Extended equivalent circuit

With fault on phase 3 we take this phase as reference (1)

$$\left|V_3, V_1, V_2\right|_T = V\left|1, a^2 a\right|_T \tag{1}$$

and the equation system for the network of the Fig. 1 has the form (2)

$$\begin{vmatrix} V_{f3} \\ V_{f1} \\ V_{f2} \end{vmatrix} = \begin{vmatrix} -R_f I_f \\ V_{f1} \\ V_{f2} \end{vmatrix} = V_0 \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix} + V \begin{vmatrix} 1 \\ a^2 \\ a \end{vmatrix} - \left[Z_{up}\right] \begin{vmatrix} I_{ph3} \\ I_{ph1} \\ I_{ph2} \end{vmatrix} \tag{2}$$

where the symbols are: V_{fk} phase to ground voltage on fault emplacement for the k-th phase, V_0 – on Petersen coil, V_k – on the k-th transformer secondary winding, Z_{up} is the matrix of the up-stream line impedances and I_{phk} is the k-th phase steady state current in system with fault.

The voltages V_k are considered the same before and after fault occurrence, in other words the influence of the fault on the internal voltage drops in sources is neglected.

The phase currents on phase 1 and 2 are composed of load and capacitive current, whereas I_{ph3} is the sum of load, capacitive and fault currents. The latter can be decomposed to get explicitly the fault current I_f :

$$I_{ph3} = (I_{ph3} - I_{o\_f}) + I_{0\_f} = (I_{ph3} - I_{0\_f}) + j\omega 3C_f V_0 - I_f \tag{3}$$

We note

$$I'_{ph3} = I_{ph3} - I_{0\_f} \tag{4}$$

and get in the symmetric components domain

$$\begin{vmatrix} V_{fp} \\ V_{fn} \\ V_{f0} \end{vmatrix} = V_0 \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} + V \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} - \begin{vmatrix} Z_{up\_p} & & \\ & Z_{up\_n} & \\ & & Z_{up\_0} \end{vmatrix} \left\{ T^{-1} \begin{vmatrix} I'_{ph3} + j\omega 3C_f V_0 \\ I_{ph1} \\ I_{ph2} \end{vmatrix} - \frac{I_f}{3} \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix} \right\} \tag{5}$$

where the Fortescue transformation matrix T is

$$T = \begin{vmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{vmatrix} \tag{6}$$

When getting back to the phase referential by left-multiplying the equation (5) by T, we obtain (7)

$$V_{f3} = V_3 - E + Z_{up\_s} I_f + kV_0 \tag{7}$$

with the correction component of the supply voltage

$$E = \begin{vmatrix} Z_{up\_p} \\ Z_{up\_n} \\ Z_{up\_0} \end{vmatrix}_T \begin{vmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{vmatrix} \begin{vmatrix} I'_{ph3} \\ I_{ph1} \\ I_{ph2} \end{vmatrix} \tag{8}$$

the capacitive correction factor

$$k = 1 - Z_{up\_s} j\omega 3C_f \tag{9}$$

where the up-stream self impedance is

$$Z_{up\_s} = \frac{1}{3}\left(Z_{p\_up} + Z_{n\_up} + Z_{0\_up}\right) \tag{10}$$

The voltage V_{f3} in (7) can be expressed (Fig. 1) by $V_{f3} = R_f\left(-I_f\right)$ and then the (7) becomes

$$V_0 = \frac{-V_3 + E}{k} - \left[\frac{R_f}{k} + \frac{Z_{s\_up}}{k}\right] I_f \tag{11}$$

This equation is structurally the same as the one representing the traditional zero sequence equivalent circuit (Fig. 2) :

$$V_0 = -V_3 - R_f I_f \tag{12}$$

with the zero sequence voltage V_0 and the fault current I_f supplying the zero sequence branch of Petersen coil and the zero sequence capacitances in parallel. In the new equivalent circuit the fault position is taken into account twice: through the correction component *E/k* of the supply voltage (accounting for the role of the symmetric up-stream impedances) and explicitly through the upstream self impedance in the fault current branch.

In usual conditions the coefficient k can be associated to 1, as the up-stream self impedance is always very weak comparing to that of the capacitive reactance of the faulted feeder. For a network e.g. of 8 line feeders, 30km each, fault location near loads and distributed line parameters C_p=0.14μF and Z_p=(0.198+j0.325)Ω/km the value is $1-(3\cdot 10^{-3}+j10^{-3})\cong 1$. This leads to simple form of the extended zero sequence circuit (Fig. 3) with inception voltage

$$V_{inc} = -V_3 + E \tag{13}$$

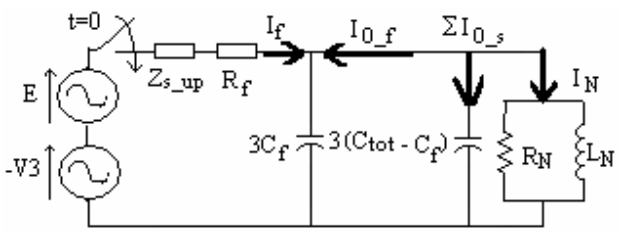

Fig. 3 The new extended zero sequence equivalent circuit

In order to construct the new equivalent circuit we need to know the transformer secondary voltage before the fault occurrence, three phase currents on the faulted feeder, actual tuning and symmetric distributed line impedances. The calculated inception voltage ($-V_3$+E) is very near the simulated value on EMTP complete system model (Tab.1).

TABLE I
INCEPTION VOLTAGE SIMULATED IN FULL EMTP MODEL VS THE EQUATION-BASED VALUES (THE CAPACITIVE COMPONENT NOT CORRECTED)

Fault position	0	0.5	1
Inception voltage [V] as in equation (13)	5507	5183	4874
Inception voltage [V] simulated	5507	5209	4932
Error [%]	0	+0.5	+1

Once the inception voltage properly calculated, the circuit will reproduce transient regimes upon its switching-on at inception moment. The equivalence is much better than that on traditional circuit (Fig. 4), permitting good evaluation of charging currents' amplitudes.

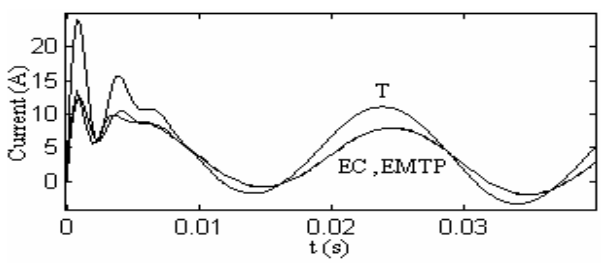

Fig. 4 Faulted residual current from the full EMTP circuit and from its equivalent circuits: the traditional zero sequence circuit (T) and the extended one (EC). in a 10kV network of 8 feeders (22.5+24+26+28+32+34+36+37.5)km, tuning 100%, total loads 10MVA and R_f = 50Ω at 0.8 of the 37.5km feeder's length.

B. Quasi sinusoidal regime of transient state

Under certain conditions (low capacitive lines and heavy loads…) a quasi sinusoidal regime can isolated out of the transient state. This is because zero sequence current on the faulted feeder I_{0_f} is dominated by faulted current I_f (Fig. 5).

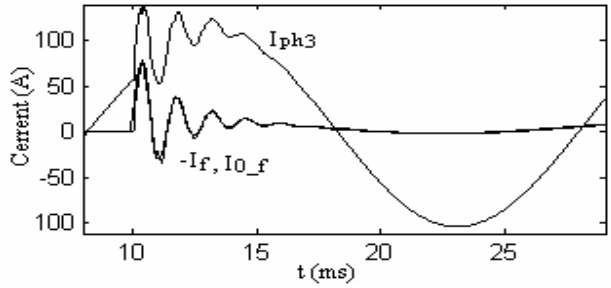

Fig. 5 *(-I_f)* , I_{ph3} and I_{0_f} in a low resistance phase-to-ground fault in a radial network

The zero sequence current being easily measurable, we use it to get a quasi-sinusoidal component I'_{ph3} (4) out of the faulted phase transient current. This component is very close to the pre-fault phase current on the faulted phase (Fig. 6).

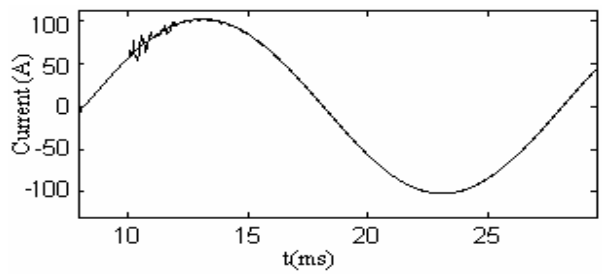

Fig. 6 The pre-fault current on the faulted phase reconstructed out of the after-fault faulted phase current :(Iph_3-I_{0_f}) vs. the original pre-fault current

With zero sequence capacitance the corresponding time constants are very short and the capacitive currents can be rapidly taken as sinusoidal. If on the other hand the fault resistance is not too high, then the zero sequence voltage also takes rapidly sinusoidal form.

Then the right side of the equation system (14) is quasi-sinusoidal and can be reasonably treated with Fortescue transformation, with its right side transformed explicitly when multiplied by T^{-1}:

$$T^{-1}\begin{vmatrix} -R_f I_f - \Delta V_f \\ V_{f1} \\ V_{f2} \end{vmatrix} = T^{-1}\left(V_0 \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix} + V \begin{vmatrix} 1 \\ a^2 \\ a \end{vmatrix} - \left[Z_{up}\right] \begin{vmatrix} I'_{ph3} + j\omega 3C_f V_0 \\ I_{ph1} \\ I_{ph2} \end{vmatrix} \right) \tag{14}$$

with both voltage drops of the fault current associated, the first over fault resistance $-R_f I_f$ and the second over upstream impedance ΔV_f.

When getting back to phase coordinates, the left side, without being transformed explicitly, presents voltage drop over the faulted current branch $(R_f + Z_{up})$ carrying the faulted current I_f, whereas the rignt side completes the extended zero sequence circuit by the inception voltage $-(V_3 - E)$ and the zero sequence branch kV_0.

The assumption of quasi - sinusoidal nature of (14) is pertinent only under conditions mentioned above. As a consequence, the equivalence of circuit issued out of this analysis will be assured in shorter range of fault resistance in cable systems (up to 200Ω) comparing to that in overhead lines (3kΩ).

III. New circuit to fault location

The extended zero sequence circuit of distribution systems grounded with Petersen Coil and an SLG fault (Fig. 1) has been applied in a three parameters least squares problem, where the actual fault resistance R_f , fault position l_f, and inception angle θ are given by the equivalent circuit's best fitting curve, with EMTP currents taken for reference data. The fitting uses data of transient fault regime: either zero sequence current on faulted feeder I_{0_f} or zero sequence current on any of sound feeders I_{0_s}, as well as the neutral point current I_N, always with comparable results. The fitting currents are issued from matlab simulation of the equivalent circuit as on the Fig. 3. The EMTP currents, simulated field data, are calculated with frequency dependant parameters. For robustness reasons the Levenberg – Marquart algorithms is adopted with a modifiable positive term $\mu_k I$ added to diagonal Hessien matrix.

At each iteration the output parameters are under a fulfilment test, satisfied in proximity of the requested minimum of minimization function at confirmation that the next calculation step involves less than 10% variation of R_f, 1% variation of l_f and 0.2° variation of inception angle. As for optimization constraints we have $R_f > 0$ and $0 < l_f < 1$(relative values).

The algorithm has been tested on overhead and cable line radial networks. In an eight feeders line system of lengths (22.5+24+26+28+32+34+36+37.5)km, at 95…105% tuning, inception angles from 0° to 90°, 10MVA total load and the fault resistance up to 3kΩ, with zero sequence current acquired at frequency 600Hz in the acquisition window [5…25]ms from inception moment we have got the fault position with less than 13% mean error relative to fault position. In Table1 and 2 we present EMTP declared values of R_f, l_f and θ_f compared to the fitting R_f, l_f and θ_f values of matlab calculated Equivalent Circuit currents.

In cables high fault resistances are not relevant, the insulation breakdown being generally definitive, quickly developing to permanent solid fault. The cable version of the extended zero sequence circuit [13] can be easily developed on the basis presented here. In tests on a 3 feeders cable network of feeders' length (3.63+4.84+6.05)km, 10MVA total load, at 95…105% tuning and inception angles from 0° to 90°, we have got the fault position with average error less than 12% up to R_f=200Ω.

TABLE II
LINE SYSTEM, EMTP ("ACTUAL") VALUES VS FITTING VALUES CALCULATED FROM EQUIVALENT CIRCUIT

Tuning [%]	EMTP [R_f, l_f, θ_f]	EC [R_f, l_f, θ_f]	Err l_f
95	1, 0.2, 30	2.9, 0.17, 28	-15
100	1, 0.5, 60	4, 0.51, 56	+2
105	1, 0.8, 90	16, 0.69, 84	-13.75
95	300, 0.2, 60	307, 0.16, 58	-20
100	300, 0.5, 90	288, 0.58, 86	+16
105	300, 0.8, 30	300, 0.83, 24	+3.75
95	600, 0.2, 90	584, 0.27, 88	+35
100	600, 0.5, 30	602, 0.52, 26	+4
105	600, 0.8, 60	606, 0.80, 54	0
95	700, 0.2, 90	682, 0.28, 88	+40
100	700, 0.5, 30	701, 0.54, 26	+8
105	700, 0.8, 60	705, 0.80, 54	0
95	1000, 0.2, 30	1003, 0.20, 28	0
100	1000, 0.5, 60	997, 0.55, 56	+10
105	1000, 0.8, 90	1028, 0.70, 84	-12.5
95	3000, 0.2, 60	2979, 0.26, 58	+30
100	3000, 0.5, 90	3019, 0.49, 86	-2
105	3000, 0.8, 30	2981, 0.87, 24	+8.75

TABLE II
CABLE SYSTEM, EMTP ("ACTUAL") VALUES VS FITTING VALUES CALCULATED FROM EQUIVALENT CIRCUIT

Tuning [%]	EMTP [R_f, l_f, θ]	CE [R_f, l_f, θ]	Err l_f
95	1, 0.2, 30	1.0, 0.22, 30	+10
100	1, 0.5, 60	1.2, 0.50, 60	0
105	1, 0.8, 90	2.8, 0.75, 90	-6.25
95	20, 0.2, 60	20, 0.20, 60	0
100	20, 0.5, 90	23, 0.42, 90	-16
105	20, 0.8, 30	20, 0.80, 30	0
95	50, 0.2, 90	52, 0.12, 90	-40
100	50, 0.5, 30	50, 0.51, 30	+2
105	50, 0.8, 60	50, 0.79, 60	-1.25
95	200, 0.2, 30	201, 0.17, 30	-15
100	200, 0.5, 60	203, 0.41, 90	-18
105	200, 0.8, 90	203, 0.67, 59	-16.25

IV. Conclusion

Correct calculation of inception voltage assures good equivalence of new extended zero sequence circuit. This voltage takes into account the contributions of load currents on the fault-to-busbar segment of the faulted line. Applied in curve fitting procedures the new circuit permitted to push forward the parametrical limits in SLG fault location procedures.

V. References

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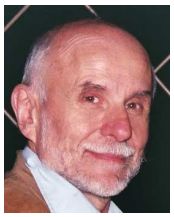
VI. BIOGRAPHIES

Wan-Ying HUANG has graduated from Tsing Hua University in Taiwan with Master Degree in 2002 and in 2006 has got her PHD in the Ecole Supérieur d'Electricité SUPELEC and University Paris XI in France. She is now on a post doctoral research program in SUPELEC, working on voltage stability in systems and on optimal design of electric machines.



Robert KACZMAREK is professor in the Ecole Supérieure d'Electricité (Supelec) in France. His research interest is with power networks (fault detection, network protection, and parameters identification), magnetism (domain observation, loss prediction and measurements in soft and hard magnetic materials) and the design of electric machines