

# Real-time estimation of a delta winding current in a wye-wye-delta transformer

Yong-Cheol Kang, Byung-Eun Lee, Mi-Sun Lee, Sung-Il Jang and Yong-Gyun Kim

**Abstract**—This paper proposes an algorithm for estimating a delta winding current for a wye-wye-delta transformer in real-time. The primary induced voltages of the three phases are calculated from the primary currents and voltages. The induced voltages are referred into the tertiary side i.e. the delta-winding considering the turns ratio of the transformer. Applying the Kirchhoff's voltage law on the delta winding gives a first-order differential equation in terms of the delta winding currents. The delta winding currents is estimated by solving the equation. The performance of the proposed algorithm is investigated under various conditions including magnetic inrush and over-excitation from the EMTP generated data. The algorithm can estimate the winding currents very accurately even under magnetic inrush and over-excitation. This paper concludes by implementing the algorithm into the digital signal processor based prototype test bed.

**Keywords:** Circulating component, Delta winding current, Estimation, wye-wye-delta transformer.

## I. NOMENCLATURE

| Symbol                         | Definition                     |
|--------------------------------|--------------------------------|
| $v_{1A}, v_{1B}, v_{1C}$       | voltages of primary windings   |
| $i_{1A}, i_{1B}, i_{1C}$       | currents of primary windings   |
| $v_{2A}, v_{2B}, v_{2C}$       | voltages of secondary windings |
| $i_{2A}, i_{2B}, i_{2C}$       | currents of secondary windings |
| $v_{3AB}, v_{3BC}, v_{3CA}$    | voltages of tertiary windings  |
| $i_{3AB}, i_{3BC}, i_{3CA}$    | currents of tertiary windings  |
| $i_{3A}, i_{3B}, i_{3C}$       | line currents                  |
| $R_{1A}, R_{1B}, R_{1C}$       | primary winding resistances    |
| $L_{11A}, L_{11B}, L_{11C}$    | primary leakage inductances    |
| $e_{1A}, e_{1B}, e_{1C}$       | primary induced voltages       |
| $R_{2A}, R_{2B}, R_{2C}$       | secondary winding resistances  |
| $L_{12A}, L_{12B}, L_{12C}$    | secondary leakage inductances  |
| $e_{2A}, e_{2B}, e_{2C}$       | secondary induced voltages     |
| $R_{3AB}, R_{3BC}, R_{3CA}$    | tertiary winding resistances   |
| $L_{13AB}, L_{13BC}, L_{13CA}$ | tertiary leakage inductances   |
| $e_{3AB}, e_{3BC}, e_{3CA}$    | tertiary induced voltages      |
| $i_{p3A}, i_{p3B}, i_{p3C}$    | Non-circulating components     |

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|                 |  |
|-----------------|--|
| $i_A$           | Circulating component                              |
| $N_1, N_2, N_3$ | Number of primary, secondary and tertiary windings |

## II. INTRODUCTION

Transformer protection relays must be able to discriminate the internal faults from normal operating conditions such as magnetic inrush and over-excitation faults. Current differential relays have been widely used for transformer protection [1]–[2]. These relays show the satisfactory performance for transformers that includes wye windings only.

However, in the case of the transformers including the delta winding such as a three-phase wye-wye-delta (Y-Y- $\Delta$ ) transformer, a circulating component flows in the delta winding. The circulating component jeopardizes the sensitivity of the differential relays. For a transformer used in the Korean 765 kV substation, the current transformers for measuring the delta winding currents are installed inside the transformers for more reliable protection. This results in increases in the size of the transformers and consequently cost of the transformers. Thus, estimation of the delta winding currents can not only increase the sensitivity of the transformer protection relays but also reduce the cost of the transformers in the ultra-high-voltage system.

The delta winding currents can be decomposed into the two components i.e. a non-circulating component and a circulating component [3]–[4]. The former can be directly estimated using the line currents, which are available. However, the latter can not be estimated directly using the line currents. The circulating component becomes significant during magnetic inrush and over-excitation.

This paper proposes an algorithm for estimating the delta winding currents for a Y-Y- $\Delta$  transformer in real-time. The induced voltages of the primary and secondary windings are referred into the tertiary side considering the turns ratio of the transformer. Applying the Kirchhoff's voltage law on the delta winding gives a first-order differential equation in terms of the delta winding currents. The delta winding currents are estimated by solving the equation. The performance of the proposed algorithm is verified under various conditions including magnetic inrush and over-excitation from the EMTP generated data. The algorithm can estimate the winding currents very accurately under magnetic inrush and over-excitation. This paper will conclude by implementing the algorithm into the digital signal processor based prototype test bed.

### III. REAL-TIME ESTIMATION OF A DELTA WINDING CURRENT IN A WYE-WYE-DELTA TRANSFORMER

A three-winding Y-Y- $\Delta$  transformer is studied in this paper. The connections of a three-winding Y-Y- $\Delta$  transformer are shown in Fig. 1. The voltages of the primary, secondary and tertiary windings can be represented by:

$$v_{1A} = R_{1A}i_{1A} + L_{11A} \frac{di_{1A}}{dt} + e_{1A} \quad (1)$$

$$v_{1B} = R_{1B}i_{1B} + L_{11B} \frac{di_{1B}}{dt} + e_{1B} \quad (2)$$

$$v_{1C} = R_{1C}i_{1C} + L_{11C} \frac{di_{1C}}{dt} + e_{1C} \quad (3)$$

$$v_{2A} = -R_{2A}i_{2A} - L_{12A} \frac{di_{2A}}{dt} + e_{2A} \quad (4)$$

$$v_{2B} = -R_{2B}i_{2B} - L_{12B} \frac{di_{2B}}{dt} + e_{2B} \quad (5)$$

$$v_{2C} = -R_{2C}i_{2C} - L_{12C} \frac{di_{2C}}{dt} + e_{2C} \quad (6)$$

$$v_{3AB} = R_{3AB}i_{3AB} + L_{13AB} \frac{di_{3AB}}{dt} + e_{3AB} \quad (7)$$

$$v_{3BC} = R_{3BC}i_{3BC} + L_{13BC} \frac{di_{3BC}}{dt} + e_{3BC} \quad (8)$$

$$v_{3CA} = R_{3CA}i_{3CA} + L_{13CA} \frac{di_{3CA}}{dt} + e_{3CA} \quad (9)$$

The delta winding currents  $i_{3AB}$ ,  $i_{3BC}$  and  $i_{3CA}$  can be decomposed into the two components, i.e. non-circulating and circulating components.

$$i_{3AB} = i_{p3A} + i_{\Delta}, \quad i_{3BC} = i_{p3B} + i_{\Delta}, \quad i_{3CA} = i_{p3C} + i_{\Delta} \quad (10)$$

In [3], the non-circulating components were obtained as:

$$i_{p3A} = \frac{i_{3B} - i_{3A}}{3}, \quad i_{p3B} = \frac{i_{3C} - i_{3B}}{3}, \quad i_{p3C} = \frac{i_{3A} - i_{3C}}{3} \quad (11)$$

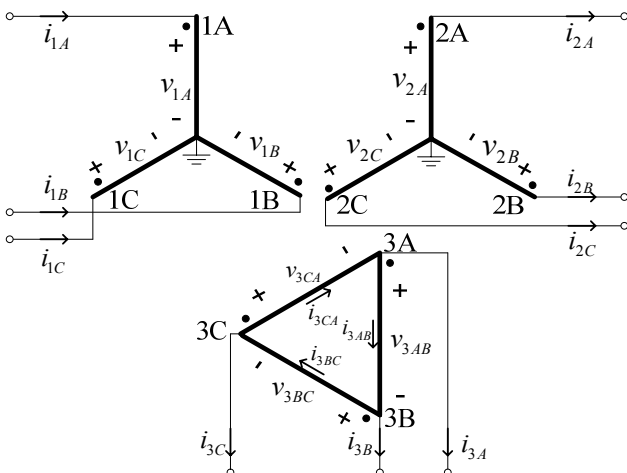


Fig. 1 Three-winding three-phase wye-wye-delta transformer

However,  $i_A$  can not be obtained directly from the line currents. This paper proposes a procedure of estimating  $i_A$ . In this paper, it is assumed for convenience that  $R_{1A} \approx R_{1B} \approx R_{1C} = R_1$ ,  $L_{11A} \approx L_{11B} \approx L_{11C} = L_{11}$ ,  $R_{2A} \approx R_{2B} \approx R_{2C} = R_2$ ,  $L_{12A} \approx L_{12B} \approx L_{12C} = L_{12}$ ,  $R_{3AB} \approx R_{3BC} \approx R_{3CA} = R_3$  and  $L_{13AB} \approx L_{13BC} \approx L_{13CA} = L_{13}$ . Rearranging (1)–(3) yields:

$$e_{1A} = v_{1A} - R_1 i_{1A} - L_{11} \frac{di_{1A}}{dt} \quad (12)$$

$$e_{1B} = v_{1B} - R_1 i_{1B} - L_{11} \frac{di_{1B}}{dt} \quad (13)$$

$$e_{1C} = v_{1C} - R_1 i_{1C} - L_{11} \frac{di_{1C}}{dt} \quad (14)$$

$e_{1A}$ ,  $e_{1B}$ , and  $e_{1C}$  can be obtained using (12)–(14) and  $e_{3A}$ ,  $e_{3B}$ , and  $e_{3C}$  can be obtained from (12)–(14) and (15).

$$\frac{e_{3AB}}{e_{1A}} = \frac{N_3}{N_1}, \quad \frac{e_{3BC}}{e_{1B}} = \frac{N_3}{N_1}, \quad \frac{e_{3CA}}{e_{1C}} = \frac{N_3}{N_1} \quad (15)$$

Summing (7)–(9) and rearranging them yields:

$$e_{3AB} + e_{3BC} + e_{3CA} = -L_{13} \frac{d(3i_{\Delta})}{dt} - R_3(3i_{\Delta}) \quad (16)$$

As the left hand side of (16) can be obtained from  $e_{1A}$ ,  $e_{1B}$ ,  $e_{1C}$  and (15),  $i_A$  can be estimated by solving (16). Finally,  $i_{3AB}$ ,  $i_{3BC}$  and  $i_{3CA}$  can be estimated from (10).

### IV. CASE STUDIES

Fig. 2 shows a single line diagram of the system studied in this paper. The three-winding three-phase Y-Y- $\Delta$  transformer (345kV/154kV/23kV, 500 MVA) is modeled using EMTP and the sampling rate is 32 samples/cycle (s/c).

The hysteresis characteristics of the core is modeled using a type-96 element; the saturation point of (100 A, 822 Vs) is selected to HYSDAT, a subroutine of EMTP. Butterworth 2nd order filters with a stop-band cut-off frequency of 960 Hz (sampling frequency/2) are used as anti-aliasing low pass filters.

Equations (12)–(14) contain differentiation terms, which should be approximated numerically to calculate the delta winding currents. Among the approximation methods using the two successive samples, the trapezoidal rule causes a smaller error than the backward Euler method. However it can cause numerical oscillation. Thus, in this paper, the differentiation terms are approximated by the trapezoidal rule in the form of (17), a method for damping the numerical oscillations with a parallel damping resistance,  $R_p$ , as shown in Fig. 3 [5].

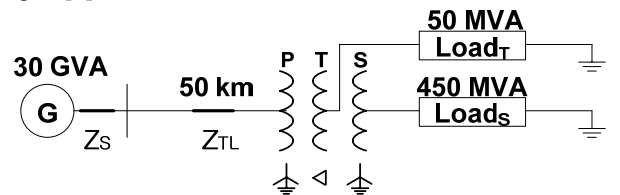


Fig. 2 Single line diagram of the simulated system

$$v_L(t) = \frac{1}{\frac{\Delta t}{2L} + \frac{1}{R_p}} \{i(t) - i(t - \Delta t)\} - \frac{R_p - \frac{2L}{\Delta t}}{R_p + \frac{2L}{\Delta t}} v(t - \Delta t) \quad (17)$$

$R_p$  is calculated using (18), as suggested in [5].

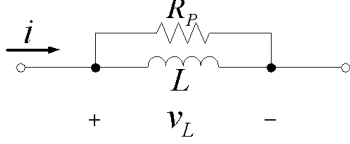


Fig. 3 Inductance model for the parallel damping

The inductance model for the parallel damping is selected by:

$$R_p = \frac{20}{3} \frac{2L}{\Delta t} \quad (18)$$

The performance of the proposed method was compared with the winding currents of EMTP under operating conditions such as magnetizing inrush and over-excitation.

This paper employs the errors described by (19). The errors evaluate the difference in per cent between the estimated value  $i_{3AB}$  and the correct value  $i_{3ABcorrect}$ .

$$error_{3AB} = \frac{i_{3AB} - i_{3ABcorrect}}{I_{3AB}} \times 100 \quad (19)$$

where,  $I_{3AB}$  is the maximum value of  $i_{3AB}$ .

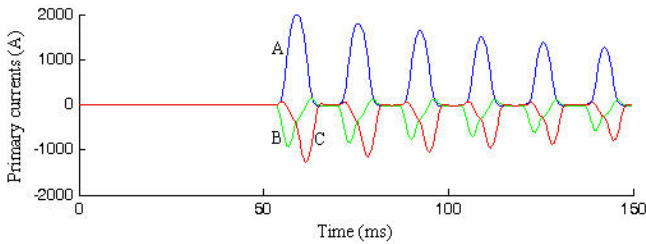
#### A. Magnetizing inrush

The magnitude of the inrush current depends on the energization angle, the remanent flux in the core, and the load current. Thus, inrush data is generated by varying the above three parameters.

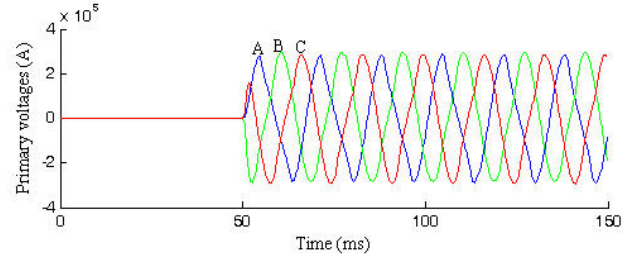
##### 1) Case 1: energization angle of 0 deg, no remanent flux, no load

Figs. 4–6 show the results for Case 1. An energization angle of 0 deg and no remanent flux results in very large inrush currents in the primary windings. The transformer is energized at 50.2ms. The three-phase voltages are severely distorted as shown in Fig. 4.

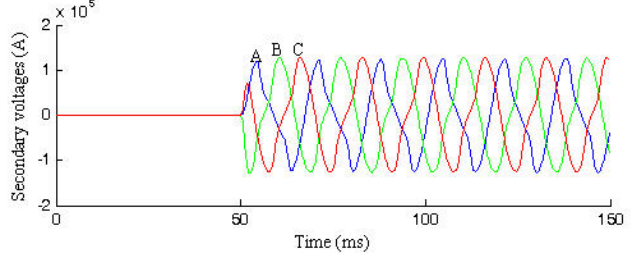
Fig. 5 shows estimated the winding currents in Case 1. A circulating component is produced after energization. The magnitude of the first pulse is 3,200A. Non-circulating components are not produced after energization, because all the tertiary line currents are zero. In this case, the winding currents are the same as the circulating currents because the non-circulating currents are zero.



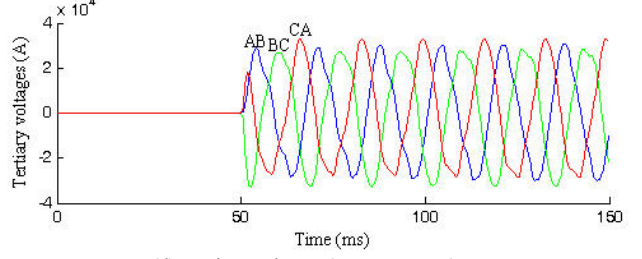
(a) Primary currents ( $i_{1A}$ ,  $i_{1B}$ ,  $i_{1C}$ )



(b) Primary voltages ( $v_{1A}$ ,  $v_{1B}$ ,  $v_{1C}$ )

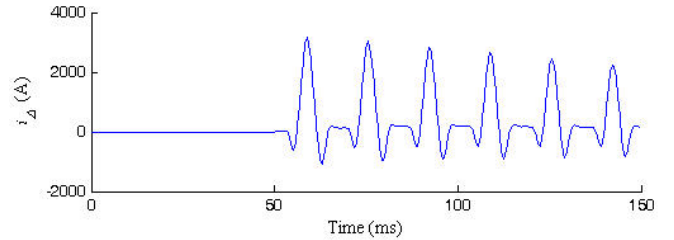


(c) Secondary voltages ( $v_{2A}$ ,  $v_{2B}$ ,  $v_{2C}$ )

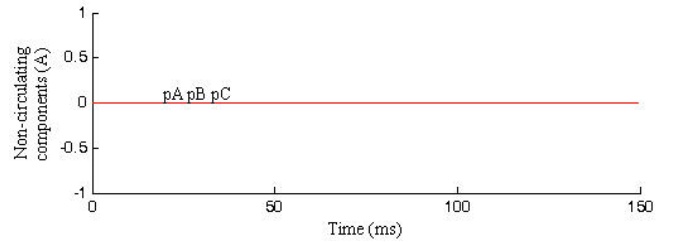


(d) Tertiary voltages ( $v_{3AB}$ ,  $v_{3BC}$ ,  $v_{3CA}$ )

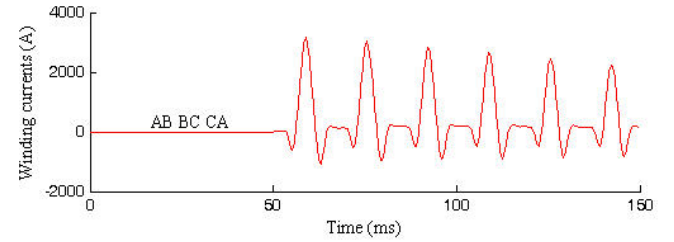
Fig. 4 Three-phase currents and voltages in Case 1



(a) Circulating component ( $i_d$ )



(b) Non-circulating components ( $i_{pA}$ ,  $i_{pB}$ ,  $i_{pC}$ )



(c) Winding currents ( $i_{AB}$ ,  $i_{BC}$ ,  $i_{CA}$ )

Fig. 5 Estimated winding currents in Case 1

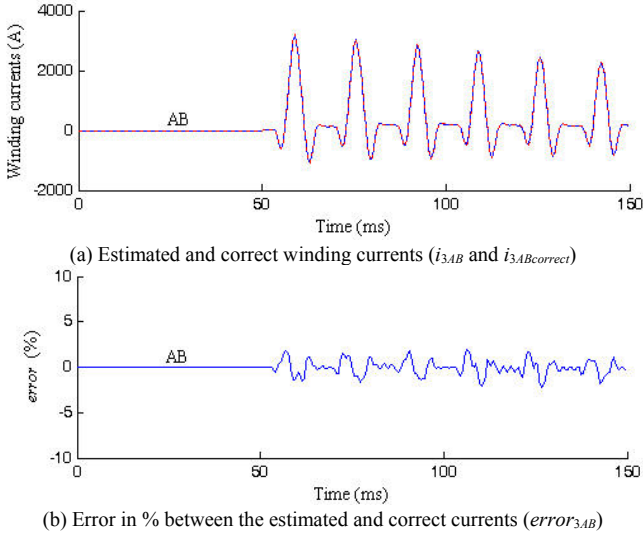


Fig. 6 Error of the estimated winding currents in Case 1

Fig. 6 shows the error of the estimated winding currents in Case 1. Fig. 6a is estimated and correct winding currents. The solid and dotted lines are the results for the estimated and correct currents, respectively. The results indicate that the estimated and correct currents are nearly the same even though inrush currents are very large and the voltages are severely distorted. Fig. 6b is the error in per cent between the estimated and correct currents. The error remains below 5%. The results indicate that the algorithm accurately estimates the delta winding currents during magnetic inrush.

### 2) Case 2: energization angle of 0 deg, 80% remanent flux, no load

Figs. 7 and 8 show the results for Case 2, which is the same as Case 1 except for the remanent flux. The transformer is energized at 50.2ms.

Fig. 7 shows the estimated winding currents in Case 2. Circulating component is produced after energization. The magnitude of the first pulse is 6,600A.

Fig. 8 shows the error of the estimated winding currents in Case 2. Fig. 8a is the estimated and correct winding currents. The solid and dotted lines are the results for the estimated and correct currents, respectively. The results indicate that the estimated and correct currents are nearly the same even though inrush currents are very large and the voltages are severely distorted due to a high remanent flux. Fig. 8b shows the error in per cent between the estimated and correct winding currents. The error remains below 5%. The results indicate that the algorithm accurately estimates the delta winding currents during magnetic inrush with a high remanent flux.

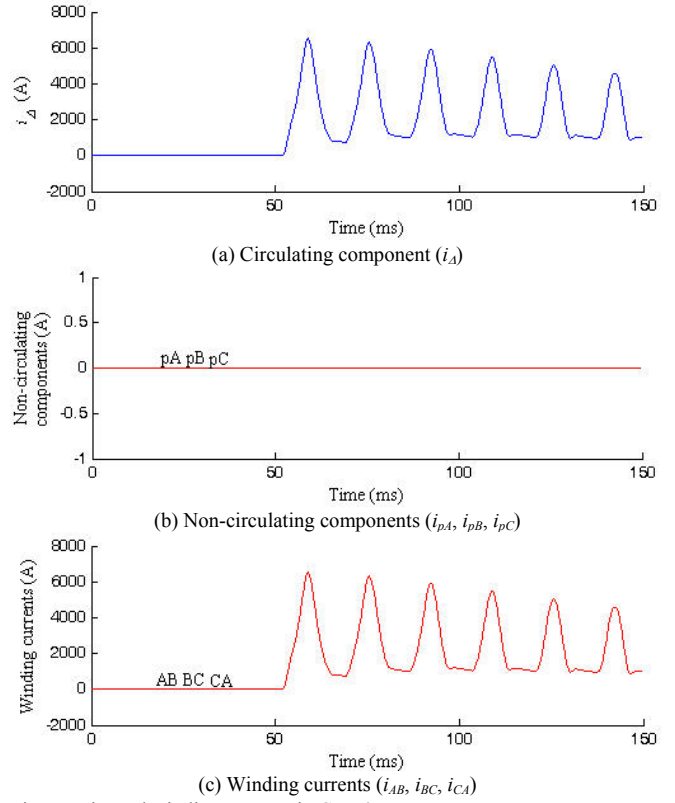


Fig. 7 Estimated winding currents in Case 2

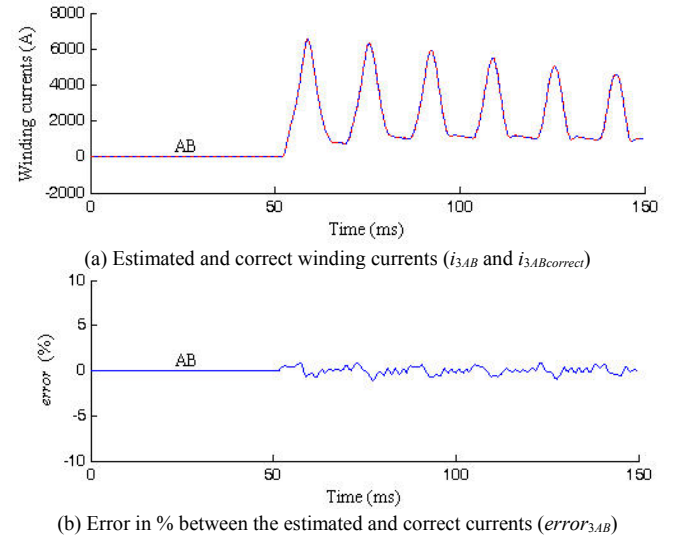


Fig. 8 Error of the estimated winding currents in Case 2

## B. Over-excitation

### 1) Case 3: overvoltage of 150% applied, full load

The algorithm was tested in the case of over-excitation. As an extreme case, 150% of the primary rated voltage at 50.2 ms, i.e., 517.5 kV is applied.

Fig. 9 shows the estimated winding currents in Case 3. A large circulating component is produced during over-excitation. The magnitude of the first pulse is 3,500 A.

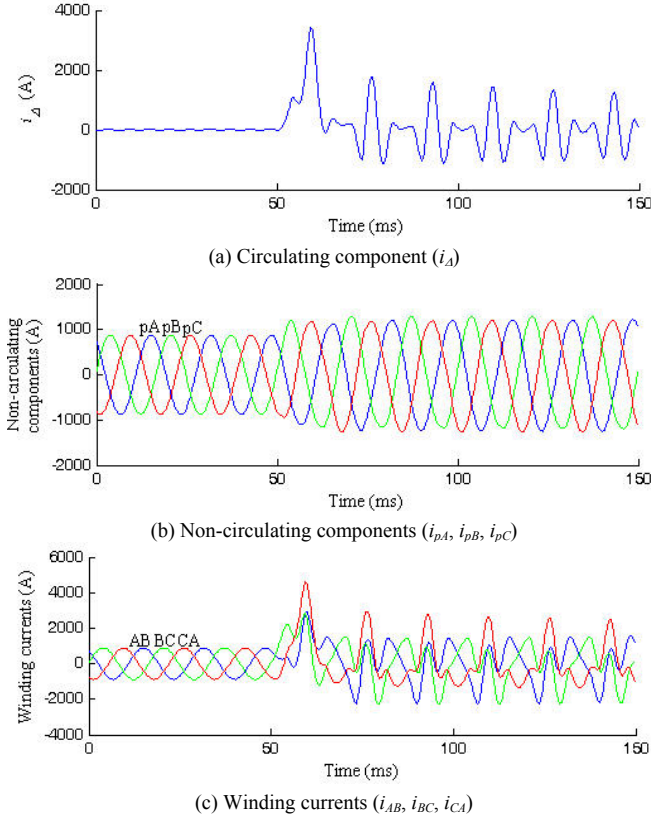


Fig. 9 Estimated winding currents in Case 3

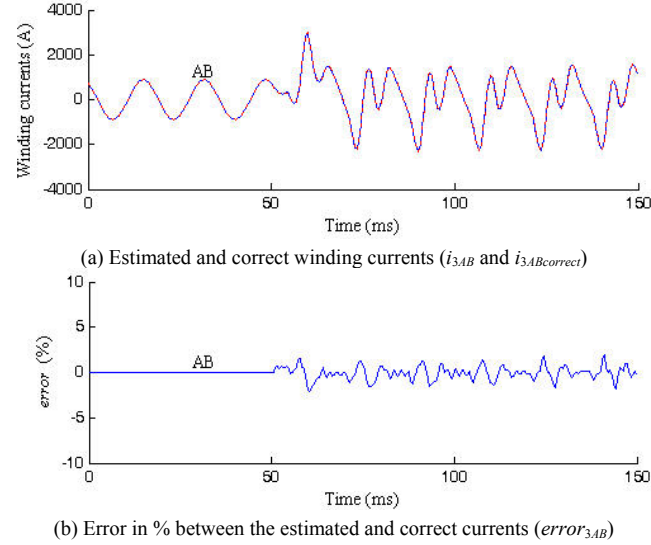


Fig. 10 Error of the estimated winding currents in Case 3

Fig. 10 shows the error of the estimated winding currents in Case 3. Fig. 10a shows the estimated and correct winding currents. The solid and dotted lines are the results for the estimated and correct currents, respectively. The results indicate that the estimated and correct currents are nearly the same during over-excitation. Fig. 10b shows the error in per cent between the estimated and correct currents. The error remains below 5%. The results indicate that the algorithm accurately estimates the delta winding currents even though currents and voltages are severely distorted during over-excitation.

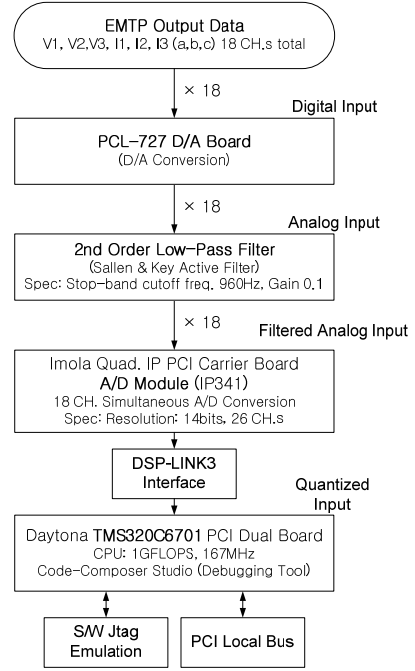


Fig. 11 Hardware implementation of the test system

## V. HARDWARE IMPLEMENTATION TEST

This section shows the results obtained when the proposed method was implemented on a hardware platform based on a TMS320C6701 digital signal processor, see Fig. 11. The nine voltages and nine currents generated by EMTP are converted into analog signals using a PCL-727 D/A board. The signals are then passed through Butterworth 2nd order filters ( $f_c = 960$  Hz) to the 14-bit A/D converters operating at a sampling rate of 32 s/c.

Figs. 12 and 13 show the results obtained when the hardware implementation was evaluated using the voltage and current signals described in Case 2. Non-circulating components are not zero because of noises and A/D conversion errors of PCL-727 D/A board. In the case of real time test, the error of the estimated winding current is somewhat higher than the simulation result (Fig. 8). However, the error remains below 10.5%, see Fig. 13b.

## VI. CONCLUSIONS

This paper proposes an algorithm for estimating a delta winding currents for a Y-Y- $\Delta$  transformer in real-time. The primary induced voltages of the three phases are calculated from the primary currents and voltages. The induced voltages are referred into the tertiary side considering the turns ratio of the transformer. Applying the Kirchhoff's voltage law on the delta winding yields a first-order differential equation for the delta winding currents. The non-circulating components of the winding currents are obtained from the line currents of the delta winding. Thus, a differential equation for the circulating component is derived. Solving the equation gives the circulating component, which is then added to the non-circulating components to estimate the delta winding currents.

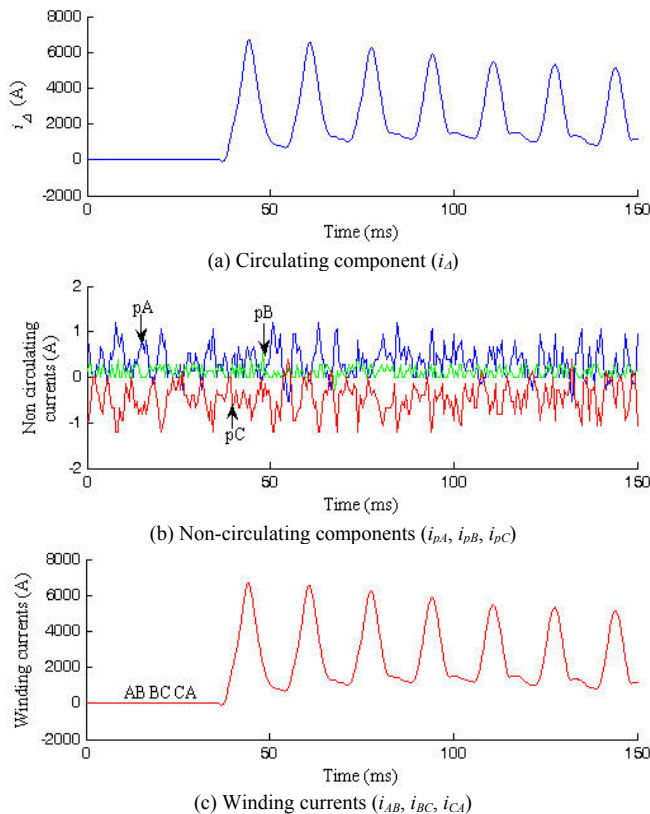


Fig. 12 Estimated winding currents in Case 2

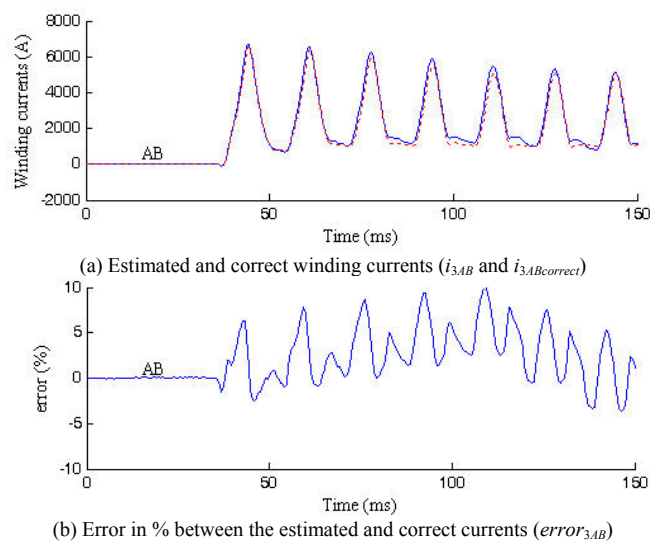


Fig. 13 Hardware implementation results in Case 2

The performance of the proposed algorithm was investigated under various operating conditions including magnetic inrush and over-excitation. Simulated results indicate that the difference between the estimated and correct delta winding currents remains very small during magnetic inrush and over-excitation. The results indicate that the algorithm accurately estimates the delta winding currents even though currents and voltages are severely distorted. The real time test result shows that the algorithm performs properly when it is implemented into a TMS320C6701 DSP. The

algorithm can improve sensitivity of the current differential relay for transformers.

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## VIII. BIOGRAPHIES

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