Surge Propagation on Two Conductors System Consisting of an Overhead Wire and a Grounding Conductor

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***Abstract*—This paper proposes an approximate formula to estimate overvoltages on multiconductor consisting of a horizontal grounding conductor and an overhead wire. The approximate formula is derived in time domain on the basis of a lattice diagram method, and is very simple. This paper discusses propagation characteristics of voltages on the multiconductor using the proposed formula.**

***Keywords*: multiconductor, grounding conductor, lattice diagram method, surge paropgation.**

I. Introduction

HORIZONTAL grounding conductors are used to ground power apparatuses in electric power and substations. Surge impedances and propagation constants of the grounding conductors are expressed by a function of frequency even if series resistance is neglected. Surge characteristics of the grounding conductor are usually estimated using numerical simulation. The authors derived approximate formulas to describe the surge characteristics on a horizontal grounding conductor [1, 2]. The approximate formulas are very simple, and enable engineers to do rough estimation of the surge characteristics.

Lightning overvoltage is one of significant factors to cause such troubles as damage and malfunction in control circuits in the power and substations [3]. Secondary voltage on the control circuits through measurement apparatuses is an important factor. Another overvoltage to cause the troubles is induced voltage on a control cable generated by currents in the horizontal grounding conductors in the station. Rise time of lightning overvoltages observed in a station is relatively short [4], and the currents in the grounding conductor also show steep front. The induced voltages on the control circuit for the steep-front currents are high enough to cause the troubles. Thus, the induced voltage on the control cable due to the currents in the grounding conductor is serious for the low-voltage control circuits, and it is very important to estimate the induced voltages. However, it is difficult to calculate the induced voltages, because there is no formula in time domain to describe the surge characteristics of a multiconductor system consisting of a grounding conductor and an overhead wire.

Numerical simulations based on an electromagnetic field theory are recently used to estimate the induced voltages on a control cable [5]. Engineers require making great efforts to obtain the induced voltages for many parameters. Therefore, a convenient method to calculate the induced voltages is required. This paper proposes an approximate formula of the induced voltage at both ends of a multiconductor system consisting of a horizontal grounding conductor and an overhead wire as illustrated in Fig. 1 in time domain based on the lattice diagram method. This paper treats only a circuit of which the sending-end is terminated through the surge impedance matrix of the multiconductor for simplicity. Then, propagation characteristics of the voltages on the multiconductor system are investigated using the proposed formula.

II. Formulas of Impedance and Admittance

Series impedance and shunt admittance matrices of multiconductor illustrated in Fig. 2 are described in this chapter. The multiconductor is approximated by loss-less lines.

A. Overhead Conductor

Impedance and admittance per length of an overhead loss-less conductor are calculated using the following formulas.

$$Z_o = j\omega\frac{\mu_0}{2\pi}\ln\frac{2h_o}{r_o} = j\omega L_o \tag{1}$$

$$Y_o = j\omega\frac{2\pi\varepsilon_0}{\ln\dfrac{2h_o}{r_o}} = j\omega C_o \tag{2}$$

where h_o is the conductor height, r_o is the conductor radius, and ω is the angular frequency

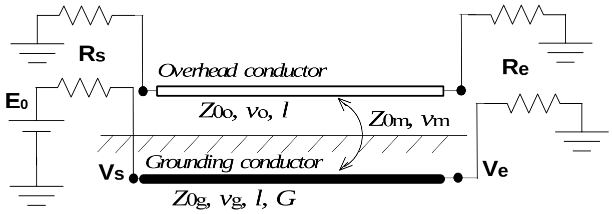

Fig. 1. Multiconductor consisting of a grounding conductor and an overhead wire

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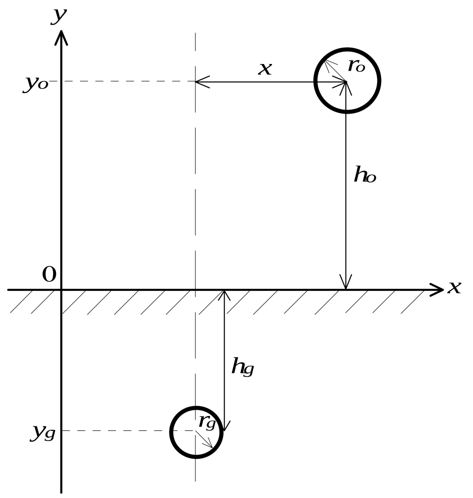

Fig. 2. Configuration of the multiconductor

The velocity along the loss-less overhead wire is equal to the velocity of light in free space v_o.

B. Grounding Conductor

Sunde derived the following formulas of the constants per length of a horizontal grounding conductor [6].

$$\text{Inductance} \quad : \; L_g = \frac{\mu_0}{2\pi} W \tag{3}$$

$$\text{Capacitance} : \; C_g = 2\pi\varepsilon_r\varepsilon_0 W^{-1} \tag{4}$$

$$\text{Conductance:} \; G = \frac{\pi}{\rho} W^{-1} \tag{5}$$

where $W = \ln\left(2l/\sqrt{2r_g d_g}\right) - 1$, ρ is the soil resitivity, ε_r is the soil relative permittivity, d_g is the conductor burial depth, r_g is the conductor radius, and l is the conductor length.

Strictly speaking, the Sunde's formula (3) is not correct. The formula is often used in many papers to study propagation characteristics on a buried conductor. Therefore, this paper uses the formula in a preliminary study of the propagation characteristics.

The surge impedance Z_{0g} and the velocity v_g of the loss-less line of an equivalent grounding conductor are given by

$$Z_{0g} = \sqrt{\frac{L_g}{C_g}} = \frac{60W}{\sqrt{\varepsilon_r}} \tag{6}$$

$$v_g = \frac{1}{\sqrt{L_g C_g}} = \frac{v_0}{\sqrt{\varepsilon_r}} . \tag{7}$$

C. Mutual Impedance between an Overhead Wire and a Grounding Conductor

The mutual impedance per length between the overhead wire and an isolated conductor in the ground is given by Pollaczek [7]:

$$Z_m = j\omega \frac{\mu_0}{2\pi} \int_{-\infty}^{\infty} \frac{\exp\left[-h_o|s| - h_g\sqrt{s^2+k^2}\right]}{|s| + \sqrt{s^2+k^2}} \exp(jxs)ds \tag{8}$$

where $k^2 = j\omega\mu_0/\rho$, x is the horizontal distance between the conductors.

The above formula needs an infinite integration, and it is difficult to calculate the mutual impedance. A simplified calculation method of the mutual impedance is proposed in [8]. A practical formula of the impedance is given by [9]:

$$Z_m = j\omega L_m = j\omega \frac{\mu_0}{2\pi} \ln \frac{S_m}{s_m} \tag{9}$$

where $S_m = \sqrt{(h_o - h_g + 2h_e)^2 + x^2}$, $s_m = \sqrt{(h_o + h_g)^2 + x^2}$, $h_e = \sqrt{\dfrac{\rho}{j\omega\mu_o}}$.

Fig. 3 shows a comparison of the mutual inductance by the practical formula with the exact solution given by (8) [8]. It is clear from Fig. 3 that the practical formula gives sufficient accuracy.

The mutual shunt admittance between the overhead wire and the grounding conductor can be regarded to be zero assuming the ground is conductor. Thus, this paper treats the mutual shunt admittance to be Y_m=0 [8].

D. Surge Impedance Matrix of Multiconductor

Neglecting the shunt conductance, the surge impedance matrix $\mathbf{Z}_0$ of the multiconductor is given by [9]

$$\mathbf{Z_0} = \mathbf{P}^{1/2}\mathbf{Y}^{-1} \tag{10}$$

where $\mathbf{Z} = j\omega\mathbf{L} = j\omega\begin{bmatrix} L_g & L_m \\ L_m & L_o \end{bmatrix}$, $\mathbf{Y} = j\omega\mathbf{C} = j\omega\begin{bmatrix} C_g & 0 \\ 0 & C_o \end{bmatrix}$,

and **P**=**ZY**.

Using the eigenvalue theory, the surge impedance matrix is given by

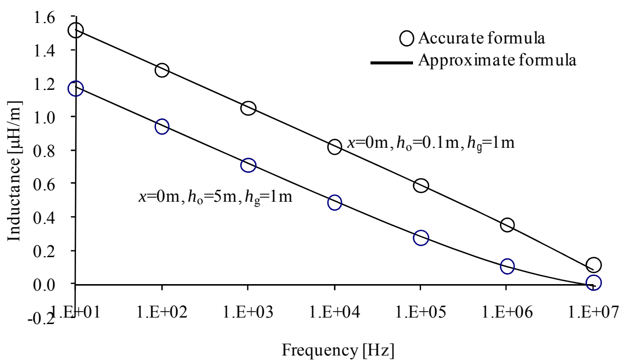

Fig. 3. Comparison of mutual inductance in the multiconductor.

$$\mathbf{Z}_0 = \frac{1}{\sqrt{Q_1}+\sqrt{Q_2}}\begin{bmatrix} L_g + \dfrac{\sqrt{Q_1Q_2}}{C_g} & L_m \\ L_m & L_o + \dfrac{\sqrt{Q_1Q_2}}{C_o} \end{bmatrix} \qquad (11)$$

where $Q_1, Q_2 = \frac{1}{2{v_0}^2}\left[\varepsilon_r + 1 \pm \sqrt{(\varepsilon_r - 1)^2 + 4\varepsilon_r \frac{{L_m}^2}{L_g L_o}}\right]$

III. Equivalent Circuit of Multiconductor Consisting of an Overhead Wire and a Horizontal Grounding Conductor

Fig. 4 illustrates a part of an equivalent circuit of multi-conductor consisting of a long grounding conductor and an overhead wire. The equivalent circuit is represented by shunt conductances due to finite soil conductivity and loss-less distributed-parameter lines. Coefficient matrices **A**, **B** and **A'** in Fig. 4 are given by

$$\begin{aligned} \mathbf{A} &= 2\mathbf{R}(\mathbf{Z_0} + 2\mathbf{R})^{-1} \\ \mathbf{B} &= \mathbf{A} - \mathbf{U} = -\mathbf{Z_0}(\mathbf{Z_0} + 2\mathbf{R})^{-1} \\ \mathbf{A'} &= \mathbf{A} \end{aligned} \qquad (12)$$

where $\mathbf{R}=(\mathbf{G}\Delta x)^{-1}$, **R** is the grounding resistance matrix, **G** is the shunt conductance matrix, **A** is the refraction coefficient matrix from k to k+1, **B** is the reflection coefficient matrix from k th segment to k+1 one, **A'** is the refraction coefficient matrix from k+1 th segment to k th one, **U** is the unit matrix, $\mathbf{Z}_0$ is the surge impedance matrix of the loss-less lines, $\boldsymbol{v}$ is the surge velocity vector on the loss-less lines, and Δx is the elementary length of the lines.

R, **G** and $\mathbf{Z_0}$ are given by

$$\mathbf{R} = \begin{bmatrix} R_g & 0 \\ 0 & \infty \end{bmatrix},\ \mathbf{G} = \begin{bmatrix} G & 0 \\ 0 & 0 \end{bmatrix},\ \mathbf{Z_0} = \begin{bmatrix} Z_{0g} & Z_{0m} \\ Z_{0m} & Z_{0o} \end{bmatrix}$$

where G is the shunt conductance per length of the grounding conductor, suffices g, o and m mean the grounding conductor, the overhead wire and the mutual component between the conductors.

The coefficient matrices **A** and **B** can be rewritten as follows:

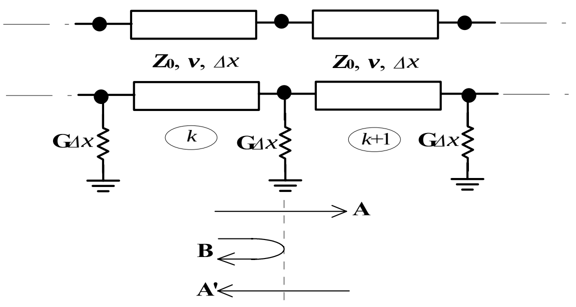

Fig. 4. An equivalent circuit of multiconductor consisting of a grounding conductor and an overhead wire

$$\mathbf{A} = (\mathbf{U} + \boldsymbol{\theta}\Delta x)^{-1} = \begin{bmatrix} \dfrac{1}{1+\theta_g\Delta x} & 0 \\ \dfrac{-\theta_m\Delta x}{1+\theta_g\Delta x} & 1 \end{bmatrix} \equiv \begin{bmatrix} a_g & 0 \\ a_m & 1 \end{bmatrix} \qquad (13)$$

$$\mathbf{B} = -\boldsymbol{\theta}\Delta x(\mathbf{U} + \boldsymbol{\theta}\Delta x)^{-1} = \begin{bmatrix} \dfrac{-\theta_g\Delta x}{1+\theta_g\Delta x} & 0 \\ \dfrac{-\theta_m\Delta x}{1+\theta_g\Delta x} & 0 \end{bmatrix} \equiv \begin{bmatrix} b_g & 0 \\ b_m & 0 \end{bmatrix}. \qquad (14)$$

$$\boldsymbol{\theta} = \frac{1}{2}\mathbf{Z_0}\mathbf{G} = \begin{bmatrix} \dfrac{1}{2}Z_{0g}G & 0 \\ \dfrac{1}{2}Z_{0m}G & 0 \end{bmatrix} \equiv \begin{bmatrix} \theta_g & 0 \\ \theta_m & 0 \end{bmatrix} \qquad (15)$$

From (13), (14) and $a_m=b_m$,

$$\mathbf{AB} = \mathbf{BA} = \begin{bmatrix} a_g b_g & 0 \\ a_g b_m & 0 \end{bmatrix} \qquad (16)$$

The matrices **A** and **B** are commutative. Therefore, the same manner for the derivation of approximate formulas of a grounding conductor alone [1, 2] is applicable to the multi-conductor.

IV. Derivation of Approximate Formula of Voltages on Multiconductor Consisting of an Overhead Wire and a Horizontal Grounding Conductor

A. Assumptions for Derivation of Formula

The following assumptions are adopted to derive an approximate expression.

(a) Series resistance is neglected.

(b) Constants of the conductors are independent of frequency, and the soil resistivity is homogenously distributed.

(c) Voltages reflected more than once on the grounding conductor are neglected.

B. Lattice Diagram Method

The lattice diagram method [10] is applicable to a non-uniform line such as a tapered line [11] and nonparallel multiconductor [12]. The surge impedance of the nonuniform line varies along the line, and voltages reflected on the line due to the discontinuity of the surge impedance are generated. An approximate formula of the reflection voltages on the non-uniform line can be obtained as $\Delta x \to 0$.

Considering the reflection voltages within length x, where x is the distance from the sending end, the number of segments n is given by:

$$n = \frac{x}{\Delta x} \qquad (17).$$

No attenuation on the overhead wire is observed if the series resistance and the shunt conductance are neglected. On the other hand, the surge impedance of a horizontal grounding conductor is independent of location, and the shunt conductance is distributed along the grounding conductor. As a result, voltages reflected and refracted on the grounding conductor appear at the both ends of the multiconductor.

Fig. 5 illustrates a lattice diagram of a single long grounding conductor for simplicity. Reflection voltage at the sending end does not occur because this paper considers a condition of $\mathbf{R}_s$=$\mathbf{Z}_0$. First-order *B* components are drawn on the figure considering the assumption (c). The diagram for an overhead wire shows no reflection, but induced voltage caused by the current in the grounding conductor must be considered.

The lattice diagram for the multiconductor is represented by matrices, and can be given by the same expression as the single grounding conductor.

C. Voltage Propagation on Multiconductor

$\mathbf{A}^k$ is expressed as follows:

$$\mathbf{A}^k = \begin{bmatrix} a_g{}^k & 0 \\ a_m \sum_{j=1}^{k} a_g{}^{j-1} & 1 \end{bmatrix} \quad (17)$$

$$\begin{bmatrix} V_g \\ V_o \end{bmatrix} = \mathbf{A}^k \mathbf{E} = \begin{bmatrix} a_g{}^k E_g \\ a_m \sum_{j=1}^{k} a_g{}^{j-1} E_g + E_o \end{bmatrix} \quad (18)$$

where $\mathbf{E}$=$(E_g, E_o)^t$, and $\mathbf{E}$ is the original sending-end voltage vector with step waveform.

From (18), the voltage propagation characteristic on the grounding conductor is the same as that for the grounding conductor alone considering the constant $a_g{}^k$ coincides with that for the grounding conductor [1]. The voltage on the overhead wire is given by the sum of non-attenuated component and induced voltages from the grounding conductor.

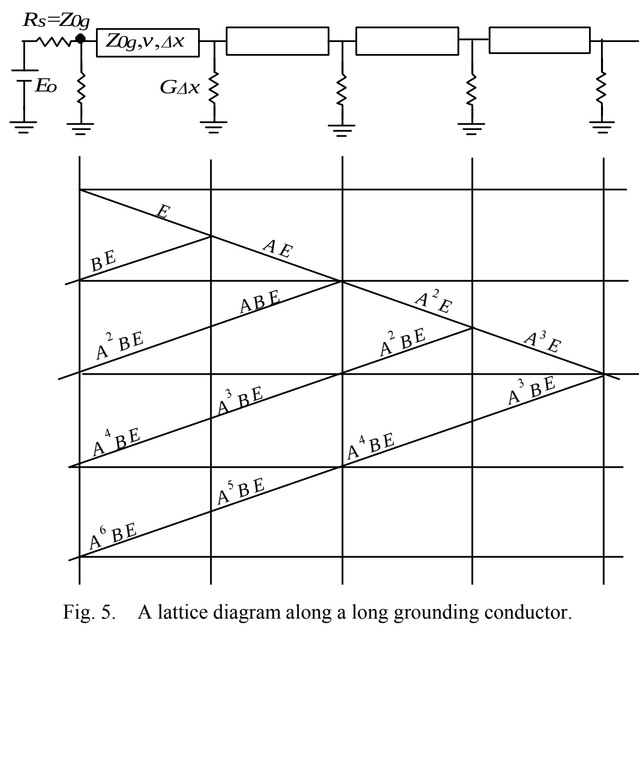

Fig. 5. A lattice diagram along a long grounding conductor.

The velocity of the voltage on the grounding conductor is v_g, which is less than v_0. Considering the traveling time on the multiconductor, the voltages can be written by the following expression.

$$\begin{bmatrix} V_g \\ V_o \end{bmatrix} = \begin{bmatrix} a_g{}^k E_g u(t - k\Delta\tau_g) \\ a_m \sum_{j=1}^{k} a_g{}^{j-1} E_g u(t - k\Delta\tau_g) + E_o u(t - k\Delta\tau_o) \end{bmatrix} \quad (19)$$

where $u(t)$ is the unit step function, $\tau_g = \Delta x / v_g$, and $\tau_o = \Delta x / v_o$

D. First-Order B Reflection Voltage on Multiconductor at Sending End

Voltage $\mathbf{V}_b$', which is reflected once on the grounding conductor, is given by:

$$\mathbf{V_b}' = \sum_{i=1}^{n} \mathbf{A}'^{i-1} \mathbf{B} \mathbf{A}^{i-1} \mathbf{E} = \sum_{i=1}^{n} \mathbf{B} \mathbf{A}^{2(i-1)} \mathbf{E} \quad (20)$$

Considering $\mathbf{B}\mathbf{A}^{2n} = \begin{bmatrix} b_g a_g{}^{2n} & 0 \\ b_m a_g{}^{2n} & 0 \end{bmatrix}$,

$$\mathbf{V_b}' = \begin{bmatrix} V_{bg}' \\ V_{bo}' \end{bmatrix} = \sum_{i=1}^{n} \begin{bmatrix} b_g a_g{}^{2n} E_g \\ b_m a_g{}^{2n} E_g \end{bmatrix} \quad (21)$$

is obtained.

The reflection voltage $\mathbf{V_b}$' as a limit is derived using equations in the appendix is given by

$$\lim_{\Delta x \to 0} V_{bg}' = -\frac{1}{2} E_g \left(1 - e^{-2\theta_g x_g}\right) \quad (22)$$

$$\lim_{\Delta x \to 0} V_{bo}' = -\zeta \frac{1}{2} E_g \left(1 - e^{-2\theta_g x_g}\right) \quad (23)$$

where $\zeta = Z_{0m}/Z_{0g}$, x_g is the location of the head of the traveling voltage on the grounding conductor, and

$$x_g = \frac{1}{2} v_g t \quad (23).$$

The sending-end voltage $\mathbf{V}_s$ until reflection voltages from the receiving end does not reach is calculated by the following equation.

$$\mathbf{V}_s = \mathbf{E} + \mathbf{P}_s \mathbf{V_b}' \quad (24)$$

where $\mathbf{P}_s$ is the refraction coefficient matrix at the sending end, and is a unit matrix in this paper.

E. Receiving-end Voltage

Fig. 6 illustrates reflection voltages. The voltage $\mathbf{V}_f$ with no reflection on the grounding conductor, and the voltage $\mathbf{V}_f$', which is reflected on the grounding conductor after $\mathbf{V}_f$ is reflected at the receiving end, are observed at the receiving end.

$\mathbf{V}_f$ is given by

$$\mathbf{V}_f = \mathbf{A}^N \mathbf{E} \quad (25)$$

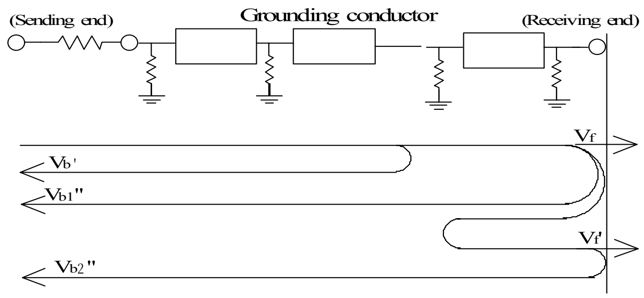

Fig. 6. An illustration of traveling wave propagation on a multiconductor.

where $N=l/\Delta x$.

From (19), $\mathbf{V}_f$ as a limit is expressed by

$$V_{fg} = E_g e^{-\theta_g l} u(t - l/v_g) \quad (26)$$

$$V_{fo} = -\zeta E_g \left(1 - e^{-\theta_g l}\right) u(t - l/v_g) + E_o u(t - l/v_o) \quad (27)$$

When $\mathbf{Q}_r$, which is the reflection coefficient matrix at the receiving end, and $\mathbf{A}$ are commutative, the receiving-end voltage $\mathbf{V}_f$' is given by

$$\mathbf{V}_f' = \sum_{i=1}^{n} \mathbf{A}^{i-1}\mathbf{B}\mathbf{A}^{i-1}\mathbf{Q}_r\mathbf{A}^N\mathbf{E} = \mathbf{Q}_r\mathbf{A}^N \sum_{i=1}^{n} \mathbf{B}\mathbf{A}^{2(i-1)}\mathbf{E}$$
$$= \mathbf{Q}_r \begin{bmatrix} -\frac{1}{2} E_g e^{-\theta_g l}\left(1 - e^{-2\theta_g x_g'}\right) u(t - l/v_g) \\ -\zeta \frac{1}{2} E_g e^{-\theta_g l}\left(1 - e^{-2\theta_g x_g'}\right) u(t - l/v_g) \end{bmatrix} \quad (28)$$

where $x_g' = \frac{1}{2}(v_g t - l)$.

Total receiving-end voltage is obtained by

$$\mathbf{V}_e = \mathbf{P}_r(\mathbf{V}_f + \mathbf{V}_f') \quad (29)$$

where $\mathbf{P}_r$ is the refraction coefficient matrix at the receiving end.

F. Sending-end voltage for $t>2l/v_g$

Reflection voltages from the receiving end are observed at the sending end for $t>2l/v_g$. The reflection voltages are given by

$$\mathbf{V}_{b1}'' = \mathbf{Q}_r\mathbf{A}^{2N}\mathbf{E}$$
$$= \mathbf{Q}_r \begin{bmatrix} E_g e^{-2\theta_g l} u(t - 2l/v_g) \\ -\zeta E_g \left(1 - e^{-2\theta_g l}\right) u(t - 2l/v_g) + E_o u(t - 2l/v_o) \end{bmatrix}$$
(30)

$$\mathbf{V}_{b2}'' = \sum_{i=1}^{n} \mathbf{A}^N\mathbf{A}^{i-1}\mathbf{B}\mathbf{A}^{i-1}\mathbf{Q}_r\mathbf{A}^N\mathbf{E} = \mathbf{Q}_r\mathbf{A}^{2N}\sum_{i=1}^{n}\mathbf{B}\mathbf{A}^{2(i-1)}\mathbf{E}$$
$$= \mathbf{Q}_r \begin{bmatrix} -\frac{1}{2} E_g e^{-2\theta_g l}\left(1 - e^{-2\theta_g x_g''}\right) u(t - 2l/v_g) \\ -\zeta \frac{1}{2} E_g e^{-2\theta_g l}\left(1 - e^{-2\theta_g x_g''}\right) u(t - 2l/v_g) \end{bmatrix}$$
(31)

where $x_g'' = \frac{1}{2}(v_g t - 2l)$.

The sending-end voltage for $t>2l/v_g$ is given by the voltages reflected at the receiving end plus the $\mathbf{V}_b$' at $t=2l/v_g$ [11]. The sending-end voltage is given by

$$\mathbf{V}_s = [\mathbf{V}_b']_{t=2l/v_g} + \mathbf{P}_s(\mathbf{V}''_{b1} + \mathbf{V}''_{b2}) . \quad (32)$$

The formulas to describe voltage on the multiconductor in time domain are very simple, and it is very convenient to carry out rough estimation of surge characteristics of multi-conductor.

V. Discussion

A. Wave Propagation Characteristics of Multiconductor Consisting of a Grounding Conductor and an Overhead Wire

(1) Voltage on grounding conductor

Wave propagation characteristics on the grounding conductor are same as those on the grounding conductor alone [1, 2].

(2) Voltage on overhead wire

V_{bg} is backward traveling voltage which is reflected on the grounding conductor. $I_{bg}=V_{bg}/Z_{0g}$ corresponds to the current in the grounding conductor. Distributed-parameter circuit theory [9] suggests that induced voltage is given by inducing current in a conductor multiplied by mutual surge impedance. The current I_{bg} induces voltage $Z_{0m}I_{bg}$. This voltage coincides with V_{bo}. Thus, (24) means the induced voltage on the overhead wire is caused by the reflection voltages on the grounding conductor.

From (27), receiving-end voltage with no reflection on the grounding conductor is given by the sum of voltage which is energized at the sending end plus the induced voltage from the grounding conductor.

The proposed method is based on some approximations. Reference [1] discusses the accuracy of the approximate formula for a single grounding conductor, and shows the proposed formulas have sufficient accuracy for $2\theta l<1$.

B. Voltage Waveforms on Multiconductor

An opened circuit at the receiving end is discussed, and yields $\mathbf{Q}r=\mathbf{U}$, and $\mathbf{P}r=2\mathbf{U}$. A simulation circuit includes conductors of r_g= 5 mm, r_o=1 mm, h_g= 1 m, h_o=0.1 m. The conductor length is a parameter: l=20 m.. The soil resistivity is ρ=500 Ωm, and the relative soil permittivity is 9. The mutual impedance is estimated for f=1 MHz ($\approx 1/(4l/v_g)$). The overhead wire is located above the grounding conductor. Only the grounding conductor is energized. The original sending-end voltage $\mathbf{E}$ of the circuit in Fig. 5 is given by

$$\mathbf{E} = \mathbf{Z_0}(\mathbf{Z_0} + \mathbf{R_s})^{-1}\mathbf{E_0} = \begin{bmatrix} \frac{1}{2}E_0 \\ 0 \end{bmatrix} \quad (33)$$

where $\mathbf{E_0}=(E_0, 0)^t$ is the source voltage vector.

Fig. 7 shows calculated results of waveforms of terminal voltages on the overhead wire (OW) and the grounding conductor (GC) using the approximate formula. E_0=2 V. Fig. 7 includes calculated results of the sending-end and receiving-end voltages by the exact solution with solid line. The exact solution is calculated by the following equations [9] using a numerical Laplace transform [13].

$$\mathbf{V}_s = L^{-1}\left\{\left[\mathbf{R}_s\mathbf{Y}_0(\mathbf{U}-\mathbf{F})(\mathbf{U}+\mathbf{F})^{-1}+\mathbf{U}\right]\mathbf{E}\right\} \tag{34}$$

$$\mathbf{V}_r = L^{-1}\left\{2\exp(-\mathbf{\Gamma}l)(\mathbf{U}+\mathbf{F})^{-1}\mathbf{V}_s\right\} \tag{35}$$

where $\mathbf{F}$=exp(-2Γl), $\mathbf{\Gamma}$ is the propagation characteristic constant matrix, $\mathbf{Y}_0$ is the characteristic admittance matrix, *s* is the Laplace operator, and L^{-1} is the Laplace inverse transform

From Fig. 7, the approximate formula shows relatively good accuracy in comparison with the exact solution. The error is caused by the lack of high-order B components, and neglecting the difference of surge velocities on the grounding conductor and the overhead wire.

The voltage on the overhead wire shows negative polarity. On the other hand, the voltage on the grounding conductor shows positive one. The induced voltage on the overhead wire is caused by voltages reflected on the grounding conductor.

VI. Conclusions

This paper has described an approximate formula of voltage on multiconductor consisting of an overhead wire and a grounding conductor. The approximate formula is derived in time domain using the lattice diagram method, and is very simple. Therefore, the proposed formula is convenient for rough estimation of the overvoltages induced on a control cable in low-voltage control circuit in substations caused by currents in a grounding conductor, and makes clear the surge characteristics of the multiconductor.

Mutual admittance is neglected in this paper. Ground potential rise is observed in experiments when currents are applied to a grounding electrode. From this fact, the mutual admittance should be considered. The author will discuss the admittance in another paper.

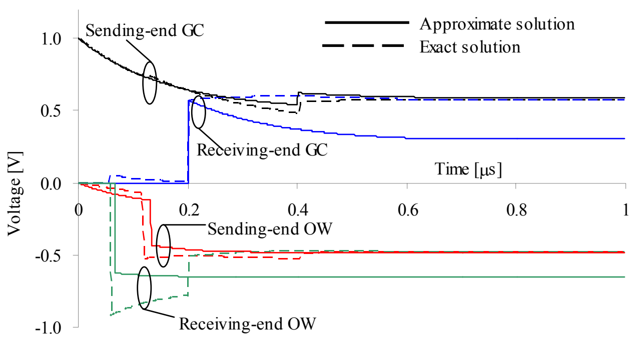

Fig. 7. Calculated waveforms of terminal voltages of the multiconductor consisting of a grounding conductor and an overhead wire

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Appendix

Derivation of Equations as a Limit

$$\lim_{\Delta x\to 0}\ln a^{2n} = \lim_{\Delta x\to 0}\frac{-2x}{\Delta x}\ln(1+\theta\Delta x)$$

$$= \lim_{\Delta x\to 0}(-2x)\frac{\theta}{1+\theta\Delta x}$$

$$= -2\theta x$$

$$\therefore \lim_{\Delta x\to 0} a^{2n} = e^{-2\theta x}$$

$$\lim_{\Delta x\to 0}\frac{b}{1-a^2} = \lim_{\Delta x\to 0}\frac{-1}{1+a}$$

$$= -\frac{1}{2} \qquad \left(\because \lim_{\Delta x\to 0} a = 1\right)$$