# Representation of a Group of Three-phase Induction Motors Using Per Unit Aggregation Model

A.Kunakorn and T.Banyatnopparat

***Abstract*--This paper presents a per unit aggregation model for representing a group of three-phase induction motor loads connected in the same bus. All parameters of the model are derived based on no load tests and block rotor tests with the development for a generalized aggregation of induction motors with similar and different synchronous speeds. Dynamic equations on the d-q axis model with arbitrary reference frames are used for predicting transient responses of the three-phase induction motors. The validity of the model is verified by simulating the starting current due to a group of induction motors using MATLAB/Simulink compared with measurements. It is found that the proposed aggregation model gives acceptably accurate results and will be useful for load modeling techniques in power system studies**.

***Keywords:* Aggregation model, Induction motors, MATLAB**

## I. INTRODUCTION

IN a power system, there are various types of load. An accurate and reasonable representation of the load is very important for studying and analyzing the transient responses and dynamic performances of the power system such as faults and stabilities. Induction motors are employed in many applications for industries, and in some particular cases, there are a number of induction motors connected in the same bus. To study the dynamic and transient performance of such a system, an accurate load model for the group of induction motors is required. The computer models representing the group of induction motor have been developed [1,2,3,4]. Various tests and simulations have been performed in both transient and steady state operations to implement such models [1,2,3,4]. This type of such models is normally known as an aggregation model of induction motors. The main purpose for developing of the aggregation model is to substitute a group of induction motors as a single unit load. Although, the previous aggregation models have been proved and implemented with many case studies, and satisfactory results have been obtained, none of these models have mentioned on the aggregation representation for a group of induction motors with different synchronous speeds for being used in the transient analysis on the time domain.

This paper presents a generalized aggregation model for representing a group of three-phase induction motors using per unit method. This approach can overcome the difficulty in modeling the group of the induction motors with different synchronous speeds connected in the same bus in an industrial power system. The aggregation model proposed in this paper is developed on the basis of parameters and general equations obtained from the previous works [2,3,4]. All parameters, then, are converted to per unit forms. The per unit model proposed in this paper is successful in representing the group of induction motors which have different synchronous speeds. The d-q model of a three-phase induction motor is modified so that the transient responses for a group of three-phase induction motors can be predicted. The aggregation model is constructed and implemented on MATLAB/Simulink and PSCAD/EMTDC. The aggregation model is verified with various case studies as well as experimental results for dynamic performances of the induction motors such as starting current.

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## II. PER UNIT AGGREGATION MODEL

There have been many aggregation models proposed by researchers. Among these models, parameters in the model developed by Kataoka et al are determined from a per phase equivalent circuit, and using d-q analysis to form equations for predicting dynamic performances due to a group of two induction motors connected as a load [2]. On the basis of the same procedure for calculating the parameters, an equivalent circuit for the aggregation model for the case of N-induction motor has been illustrated as Fig 1.

All parameters in the equivalent circuit, then, are derived, based on no load tests and lock rotor tests. When performing the no load test, the slip of all induction motors in a group is approaching zero. As a result, the no load per unit impedance of each motor is:

$$\overline{Z}_{n,i} = \overline{R}_{s,i} + j(\overline{X}_{ls,i} + \overline{X}_{m,i}) \tag{1}$$

i = 1,2,…N

All motors are connected in parallel at the same bus. As a result, an equivalent per unit impedance of the group of induction motors during no load is:

$$\frac{1}{\overline{Z}_{n,ag}} = \frac{1}{\overline{Z}_{n1}} + \frac{1}{\overline{Z}_{n2}} + \frac{1}{\overline{Z}_{n3}} + ... + \frac{1}{\overline{Z}_{nN}} \tag{2}$$

$$\overline{Z}_{n,ag} = \frac{\prod_{i=1}^{N} \overline{Z}_{ni}}{\sum_{i=1}^{N}\left[\prod_{k=1,k\neq i}^{N} \overline{Z}_{nk}\right]} \tag{3}$$

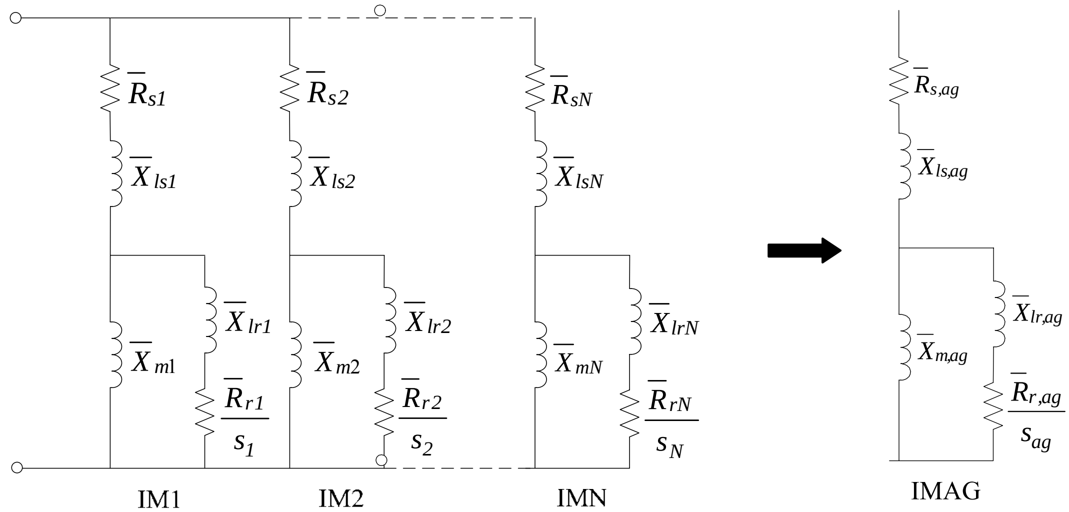



Fig 1. An equivalent circuit of the per unit aggregation model

Considering the aggregation model shown in Fig 1, the no load per unit impedance of the model is:

$$\overline{Z}_{n,ag} = \overline{R}_{s,ag} + j(\overline{X}_{ls,ag} + \overline{X}_{m,ag}) \tag{4}$$

where,
N = number of induction motors in a group
$\overline{R}_{s,i}$ = per unit stator resistance of $i^{th}$ motor
$\overline{X}_{ls,i}$ = per unit stator leakage reactance of $i^{th}$ motor
$\overline{X}_{m,i}$ = per unit magnetizing reactance of $i^{th}$ motor
$\overline{R}_{s,ag}$ = per unit stator resistance of aggregation motor
$\overline{X}_{ls,ag}$ = per unit stator leakage reactance of aggregation motor
$\overline{X}_{m,ag}$ = per unit magnetizing reactance of aggregation motor

When performing the lock rotor test, the slip of all induction motors in a group is approaching 1. As a result, the lock rotor per unit impedance of each motor is:

$$\overline{Z}_{b,i} = (\overline{R}_{s,i} + \overline{R}_{r,i}) + j(\overline{X}_{ls,i} + \overline{X}_{lr,i}) \tag{5}$$

An equivalent per unit impedance of the group of induction motors during the lock rotor test is:

$$\frac{1}{\overline{Z}_{b,ag}} = \frac{1}{\overline{Z}_{b1}} + \frac{1}{\overline{Z}_{b2}} + \frac{1}{\overline{Z}_{b3}} + \ldots + \frac{1}{\overline{Z}_{bN}} \tag{6}$$

$$\overline{Z}_{b,ag} = \frac{\prod_{i=1}^{N} \overline{Z}_{bi}}{\sum_{i=1}^{N} \left[ \prod_{k=1,k\neq i}^{N} \overline{Z}_{bk} \right]} \tag{7}$$

Considering the aggregation model shown in Fig 1, the lock rotor per unit impedance of the model is:

$$\overline{Z}_{b,ag} = (\overline{R}_{s,ag} + \overline{R}_{r,ag}) + j(\overline{X}_{ls,ag} + \overline{X}_{lr,ag}) \tag{8}$$

where,
N = number of induction motors in a group
$\overline{R}_{s,i}$ = per unit stator resistance of $i^{th}$ motor
$\overline{R}_{r,i}$ = per unit referred rotor resistance of $i^{th}$ motor
$\overline{X}_{ls,i}$ = per unit stator leakage reactance of $i^{th}$ motor
$\overline{X}_{lr,i}$ = per unit referred rotor leakage reactance of $i^{th}$ motor
$\overline{R}_{s,ag}$ = per unit stator resistance of aggregation motor
$\overline{R}_{r,ag}$ = per unit referred rotor resistance of aggregation motor
$\overline{X}_{ls,ag}$ = per unit stator leakage reactance of aggregation motor
$\overline{X}_{lr,ag}$ = per unit referred rotor leakage reactance of aggregation motor

$(\overline{R}_{s,ag} + \overline{R}_{r,ag})$ = real part of $\overline{Z}_{b,ag}$
$(\overline{X}_{ls,ag} + \overline{X}_{lr,ag})$ = imaginary part of $\overline{Z}_{b,ag}$

From (4) and (8), the values of $\overline{R}_{s,ag}$ and $\overline{R}_{r,ag}$ can be determined. To obtain the values of $\overline{X}_{ls,ag}$ and $\overline{X}_{lr,ag}$, the ratio of both parameters can be taken from the IEEE standard 112-1996 which specify the empirical distribution of leakage inductances in induction motors depending on the motor class [5]. This is correct approximation when all motors in the group are in the same motor class. However, if there are various classes of the motor, it is proper to divide the motors into subgroups, and each subgroup consists of the same class of the motors. The value of $\overline{X}_{ls,ag}$ and the imaginary part of (4), then, are employed for calculating the value of $\overline{X}_{m,ag}$.

In addition, slip $s_{ag}$ can be determined using equations proposed by Kataoka et al[4] as follows:

$$\alpha S_{ag}^2 + \beta S_{ag} + \gamma = 0 \tag{9}$$

where,

$$\alpha = 2\overline{R}_{r,ag}\overline{X}^{2}_{m,ag}\overline{V}^{2}_{1} + \left\{\overline{R}^{2}_{s,ag}\overline{X}^{2}_{r,ag} + (\overline{X}_{s,ag}\overline{X}_{r,ag} - \overline{X}^{2}_{m,ag})^{2}\right\}\overline{P}_{ag}$$

$$\beta = -\overline{R}_{r,ag}\overline{X}^{2}_{m,ag}(2\overline{V}^{2}_{1} - 2\overline{R}_{s,ag}\overline{P}_{ag})$$

$$\gamma = \overline{R}^{2}_{r,ag}(\overline{R}^{2}_{s,ag} + \overline{X}^{2}_{s,ag})\overline{P}_{ag}$$

$$S_{ag} = \frac{-\beta \pm \sqrt{\beta^{2} - 4\alpha\gamma}}{2\alpha} \tag{10}$$

There are two roots of (10), the slip of the aggregation motor is the root with a smaller value[2].

Choosing base values as follows:

$V_{base}$ = peak value of the phase voltage

$I_{base}$ = peak value of the phase current

$$P_{base} = \frac{3}{2}V_{base}I_{base}$$

The moment of inertia of the aggregation motor can be obtained [1,2,3,4]:

$$\overline{P}_{ag} = \sum_{i=1}^{N}\overline{P}_{i} \tag{11}$$

$$J_{ag} = \frac{\sum_{i=1}^{N} J_{i}N^{2}_{s,i}}{N^{2}_{s,ag}} \tag{12}$$

where,

$\overline{P}_{ag}$ = per unit rated power of aggregation motor

$\overline{P}_{i}$ = rated power of each motor in a group

$J_{ag}$ = moment of inertia of aggregation motor

$J_i$ = moment of inertia of each motor in a group

$N_{s,i}$ = synchronous speed of each motor in a group

$N_{s,ag}$ = synchronous speed of aggregation motor

Normally, the synchronous speed of an induction motor is calculated using the number of poles of the motor. If induction motors in the group which are required to be aggregated have different poles numbers, this means that there are various synchronous speeds among the motors. As a result, the motors cannot be aggregated. In order to overcome such a problem, a pseudo number of poles of the aggregation motor has been invented so that a synchronous speed of the aggregation motor can be determined [6]. The pseudo number of poles of the aggregation motor is:

$$pole_{ag} = \frac{120\, fP_{ag}}{\sum_{i=1}^{N} N_{s,i}P_{i}} \tag{13}$$

where,

$f$ = supply frequency

$N_{s,i}$ = synchronous speed of each motor in a group

## III. MATLAB/SIMULINK MODELS AND CASE STUDIES

Mathematical models of the aggregation induction motors are developed on the basis of MATLAB/Simulink so that the transient analysis on time domain can be performed. The block diagram is constructed with dynamic equations of the induction machine on the arbitrary reference frame modified from standard equations of an induction machine detailed by Chee-Mun Ong[7]. The block diagram developed for an aggregation model is shown in Fig 2. To verify the aggregation model, transient starting current waveforms of a group of induction motors were measured. Direct online starting tests were performed on various sizes of induction motors. All motors under tests were class A motors. The details of the induction motors under the tests are as shown in Table I.

TABLE I
NAME PLATES OF INDUCTION MOTORS USED IN EXPERIMENTS

| No. | | | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| Output Power | P | | 1 hp | 1 hp | 3 hp | 5 hp |
| Voltage (line-line) | V | [V] | 380/220 | 380/220 | 380/220 | 380/220 |
| Frequency | f | [Hz] | 50 | 50 | 50 | 50 |
| Number of Poles | | | 6 | 4 | 4 | 4 |
| Base Quantities: | | | | | | |
| Voltage | $V_b$ | [V] | 220 | 220 | 220 | 220 |
| Current | $I_b$ | [A] | 7.9 | 7.9 | 7.9 | 7.9 |
| Impedance | $Z_b$ | [Ω] | 27.85 | 27.85 | 27.85 | 27.85 |
| Parameters | | | | | | |
| Resistances | $R_s$ | [pu.] | 0.2496 | 0.3770 | 0.1357 | 0.0738 |
| | $R_r$ | [pu.] | 0.2420 | 0.3638 | 0.1181 | 0.0705 |
| Reactances | $X_{ls}$ | [pu.] | 0.3384 | 0.3521 | 0.1385 | 0.0750 |
| | $X_{lr}$ | [pu.] | 0.3384 | 0.3521 | 0.1385 | 0.0750 |
| | $X_m$ | [pu.] | 5.1268 | 7.7785 | 2.9121 | 1.9839 |
| Moment of Inertia | $J$ | [kg.m$^2$] | 0.0049 | 0.0028 | 0.0056 | 0.0101 |

Due to the limitation of instruments, all measurements were recorded with no-load condition only. However, the steady state current when the motors were loaded was observed, and it was found that the aggregation model was able to calculate correctly the current when comparing with the measured values.

1st case study

(motors with similar synchronous speed in the group)

Using induction motors No.2, No.3 and No.4

2nd case study

(motors with different synchronous speeds in the group)

Using induction motors No.1, No.3 and No.4

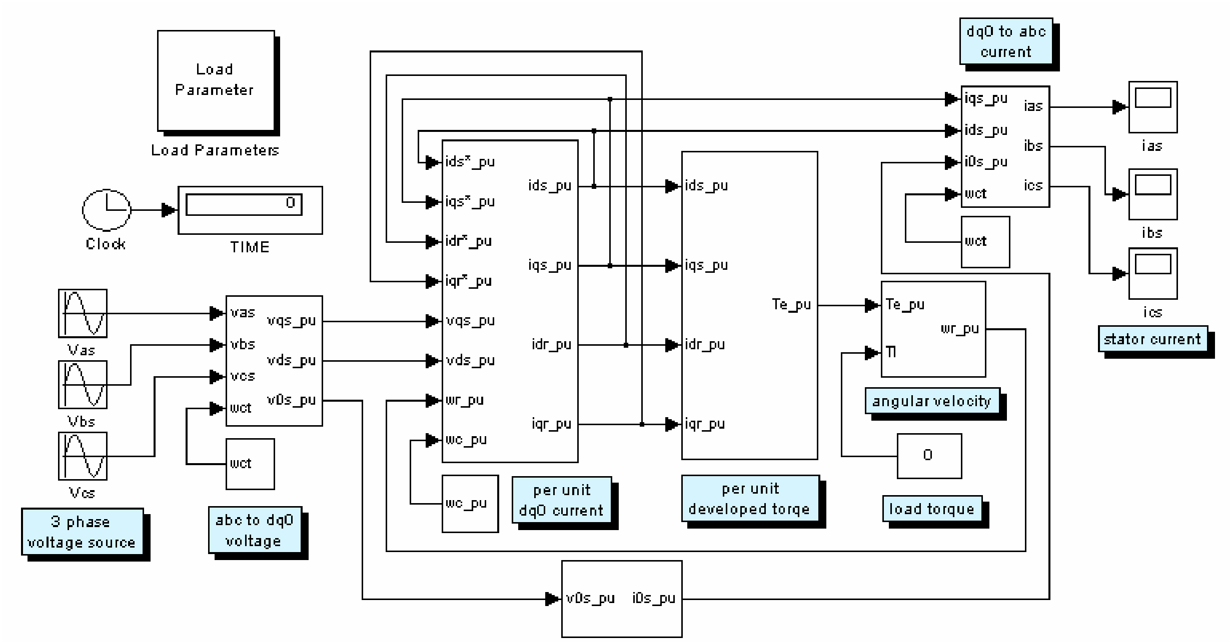



Fig 2. The block diagram on MATLAB/Simulink of the per unit aggregation model

## IV. RESULTS

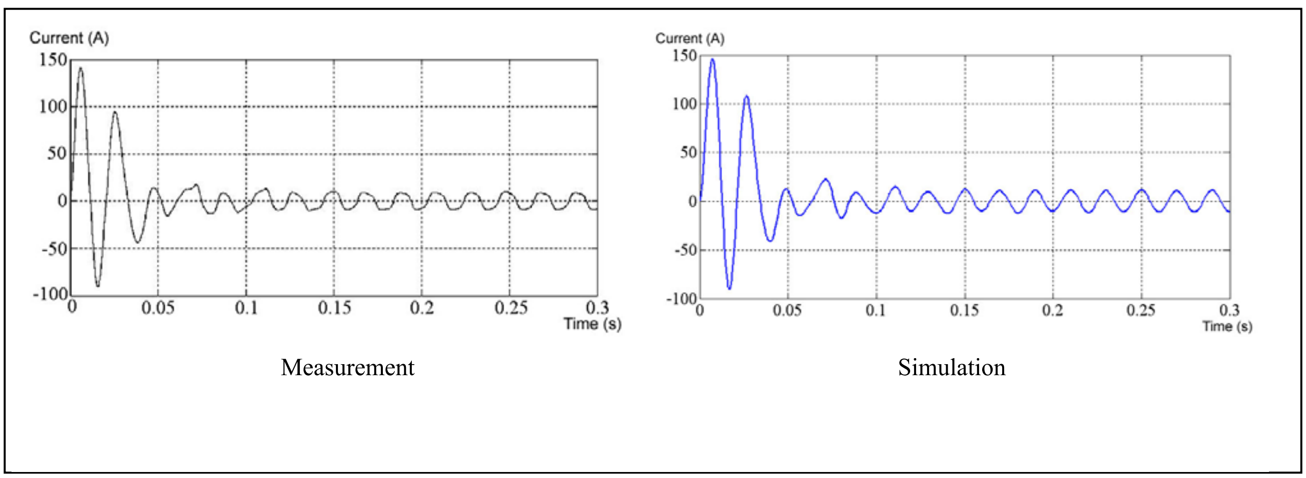



Fig 3. 1st case study, Comparison of starting current waveforms of the group of induction motors (Phase A)

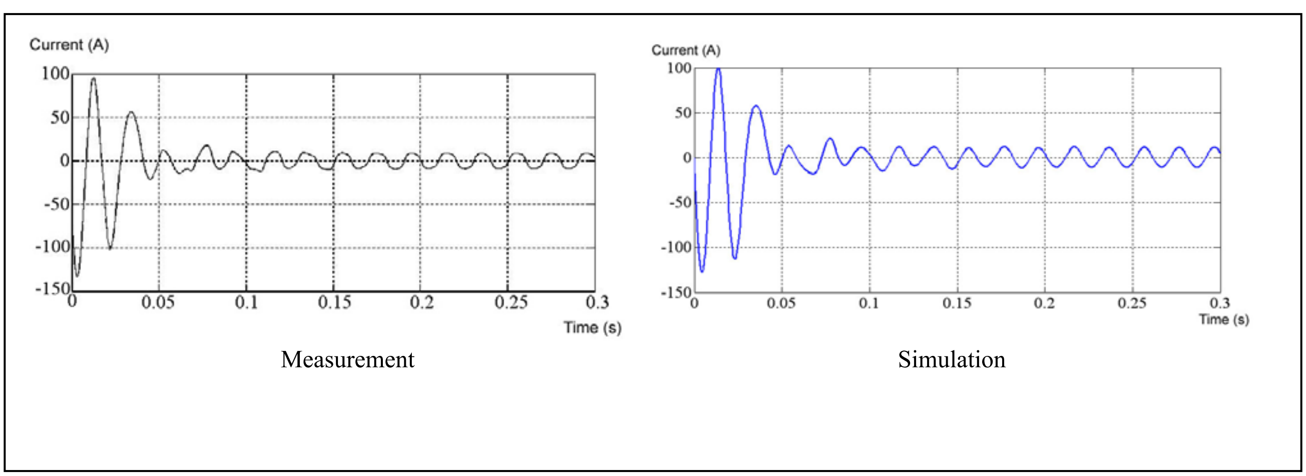



Fig 4. 1st case study, Comparison of starting current waveforms of the group of induction motors (Phase B)

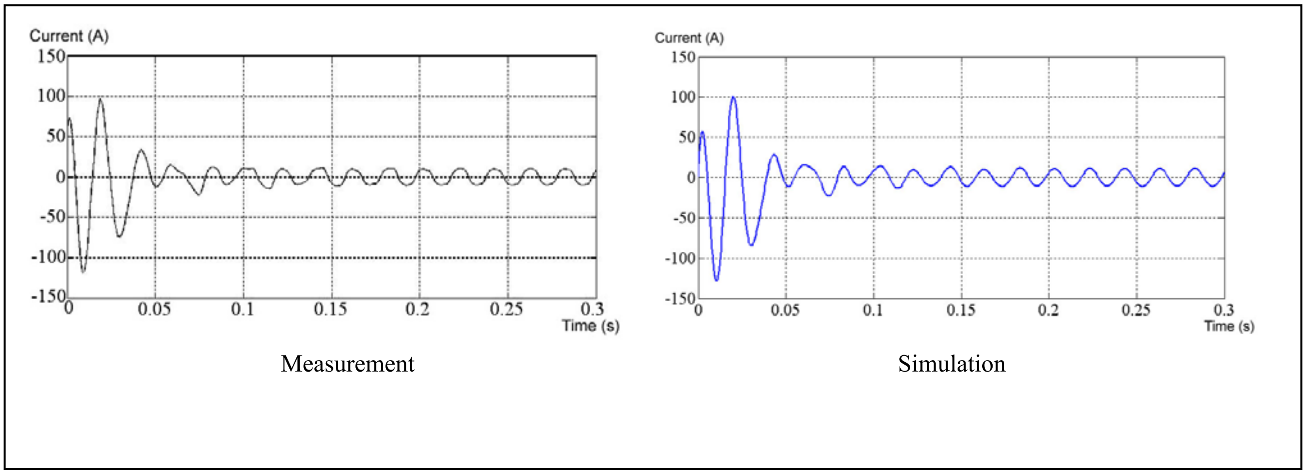



Fig 5. 1st case study, Comparison of starting current waveforms of the group of induction motors (Phase C)

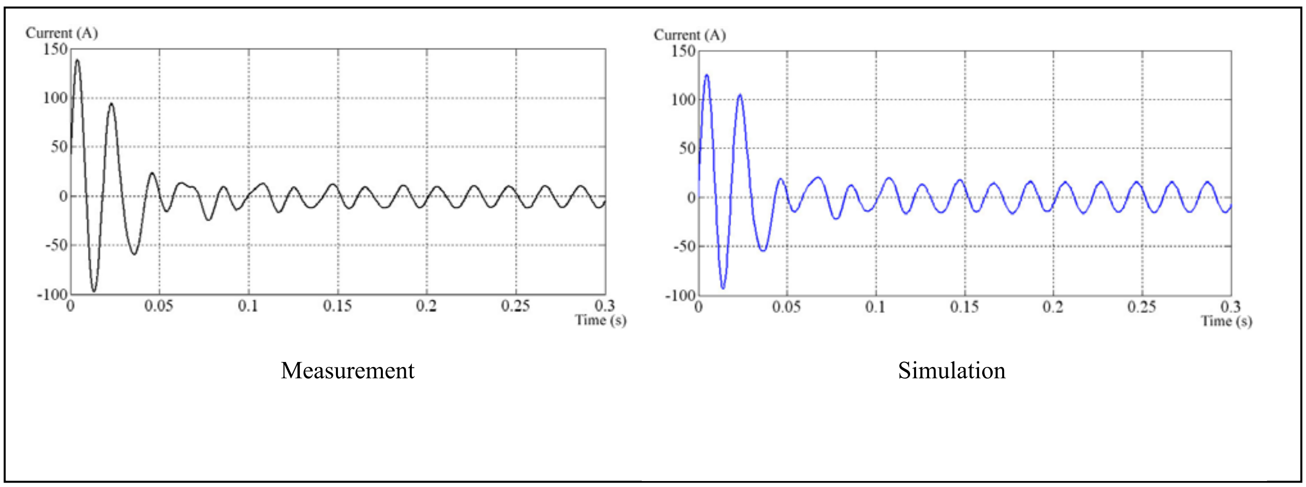



Fig 6. 2nd case study, Comparison of starting current waveforms of the group of induction motors (Phase A)

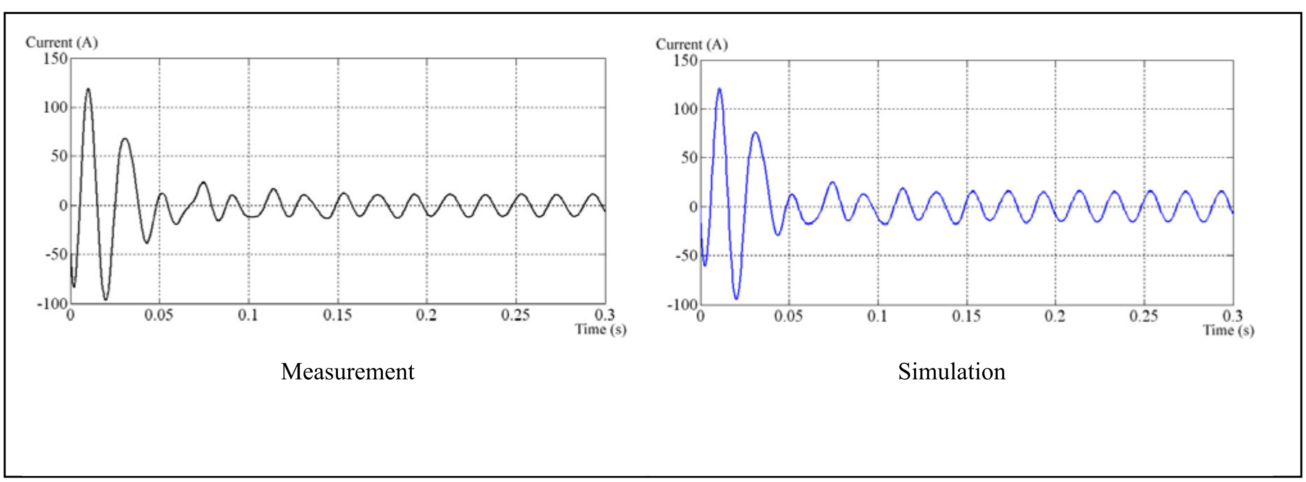
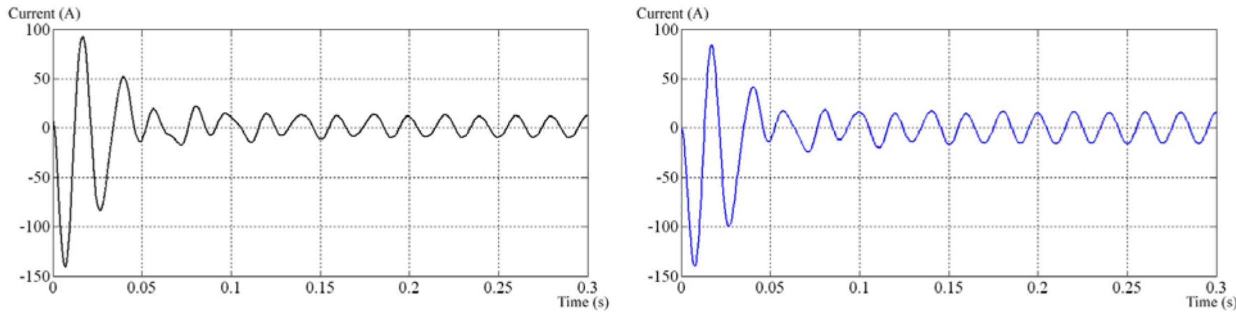



Fig 7. 2nd case study, Comparison of starting current waveforms of the group of induction motors (Phase B)




Measurement Simulation

Fig 8. 2nd case study, Comparison of starting current waveforms of the group of induction motors (Phase C)

TABLE II

NAME PLATES OF INDUCTION MOTORS USED IN FURTHER CASE STUDIES IN ORDER TO MAKE COMPARISON WITH PSCAD/EMTDC [1]

| No. | | | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|
| Output Power | P | | 3 hp | 15 hp | 30 hp | 50 hp | 100 hp |
| Voltage (line-line) | V | [V] | 460 | 460 | 460 | 460 | 460 |
| Frequency | f | [Hz] | 60 | 60 | 60 | 60 | 60 |
| Number of Poles | | | 4 | 4 | 4 | 4 | 4 |
| **Base Quantities :** | | | | | | | |
| Voltage | $V_b$ | [V] | 265.58 | 265.58 | 265.58 | 265.58 | 265.58 |
| Current | $I_b$ | [A] | 185.39 | 185.39 | 185.39 | 185.39 | 185.39 |
| Impedance | $Z_b$ | [Ω] | 1.433 | 1.433 | 1.433 | 1.433 | 1.433 |
| **Parameters :** | | | | | | | |
| Resistances | $R_s$ | [pu.] | 3.3925 | 1.0331 | 0.5096 | 0.2932 | 0.1745 |
| | $R_r$ | [pu.] | 1.2844 | 0.2164 | 0.1117 | 0.0977 | 0.0558 |
| Reactances | $X_{ls}$ | [pu.] | 1.8638 | 0.1256 | 0.1117 | 0.1047 | 0.0698 |
| | $X_{lr}$ | [pu.] | 1.8638 | 0.1256 | 0.1117 | 0.1047 | 0.0698 |
| | $X_m$ | [pu.] | 59.1111 | 17.3745 | 10.4429 | 6.6106 | 2.7713 |
| Moment of Inertia | $J$ | [kg.m$^2$] | 0.09 | 0.50 | 1.00 | 1.66 | 2.70 |

## V. CASE STUDIES WITH PSCAD/EMTDC

An additional case study was performed in order to make comparison between simulation results from the model developed on MATLAB/Simulink and those obtained from standard models on PSCAD/EMTDC. In this case study, the system used by Pillay et al [1] was employed. The system consists of five induction motors connected at the same bus as shown in Fig 9, while Table II shows the details of all the motors. After starting the group of the induction motors, the system reached the steady state operation, the circuit breaker, then, was open for 8 cycles, and closed. Electromagnetic transients of the system were simulated on both MATLAB/Simulink and PSCAD/EMTDC.

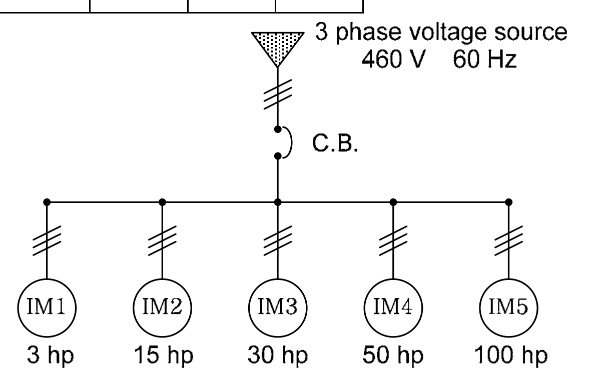



Fig 9. The system used in verifying the MATLB/Simulink model with a standard model on PSCAD/EMTDC [1]

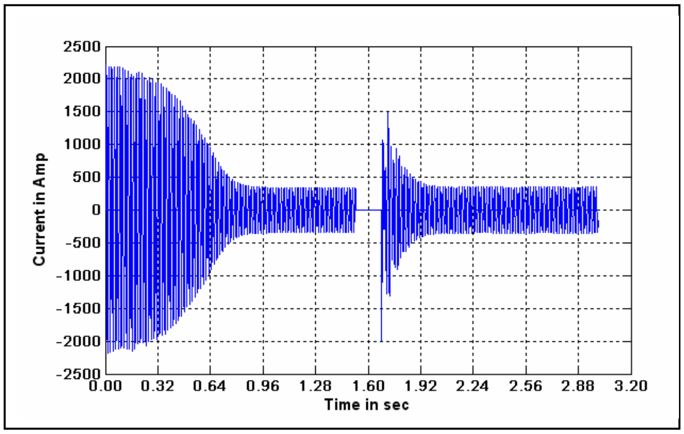



Fig 10. MATLAB/Simulink result for the transient current

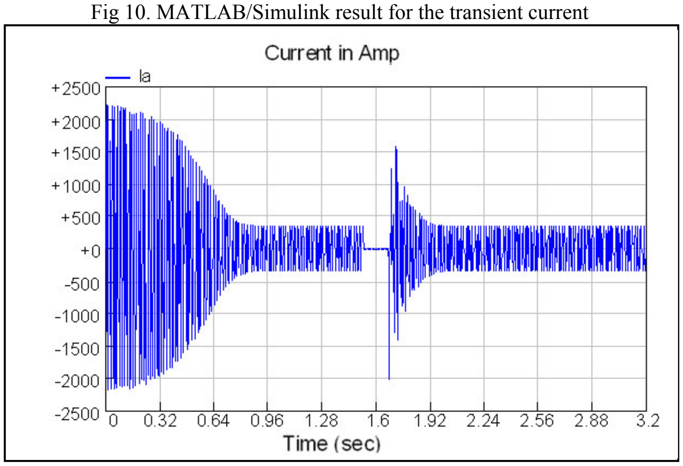



Fig 11. PSCAD/EMTDC result for the transient current

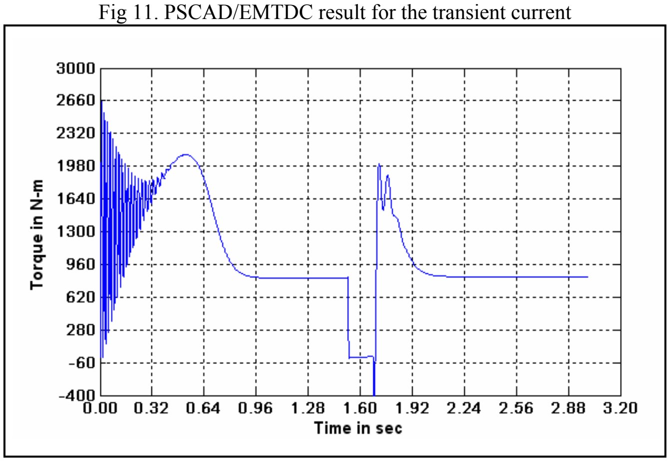



Fig 12. MATLAB/Simulink result for the transient electromagnetic torque

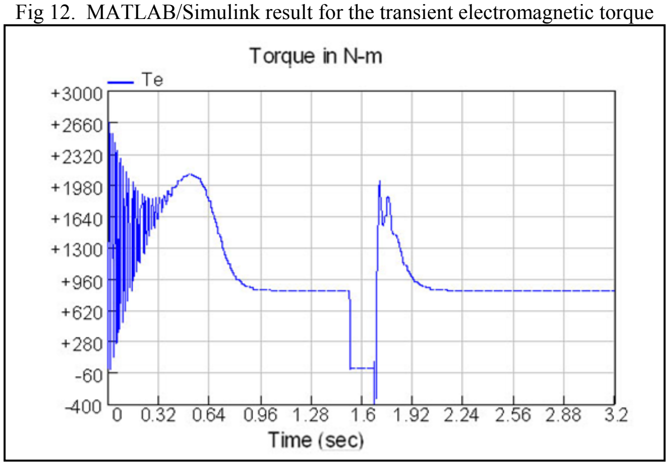



Fig 13. PSCAD/EMTDC result for the transient electromagnetic torque

Transient responses of the Phase A current and the electromagnetic torque of the group of the induction motors are obtained as shown from Fig 10 to Fig 13. It can be seen that there are small discrepancies in time domain responses simulated from the model developed on MATLAB/Simulink and those from PSCAD/EMTDC. However, these simulations give a good agreement compared with the results reported by Pillay et al [1].

## VI. Conclusions

The per unit aggregation model to represent a group of induction motors has been presented. Parameters in such a model have been determined. The pseudo number of poles of the aggregation model has been employed with an advantage in overcoming the difficulty in modeling the group of the induction motors with different synchronous speeds connected in the same bus in an industrial power system. Per unit impedances are used in the model so that the model is more properly useful in power system studies. The dynamic equations of the aggregation model have been constructed as a block diagram using MATLAB/Simulink in order to perform transient analysis on the time domain. The model has been implemented in predicting starting current for two groups of induction motors with various sizes and different synchronous speeds. It has been found that the model gives satisfactory simulation results when comparing with measurements. The error obtained from the model is about less that 10%. In addition, the proposed model has been verified by making the comparison between the results simulated on MATLAB/Simulink with those from the standard model available on PSCAD/EMTDC, and the correlation has been obtained. The limit of the model is that all induction motors in the aggregation must be in the same motor class.

## VII. References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