A robust multi-conductor transmission line model to simulate EM transients in underground cables

H.M.J. De Silva, L.M. Wedepohl and A.M. Gole

***Abstract*--This paper introduces a new mesh-domain model to simulate time-domain transients in underground cables accurately considering frequency dependent effects. The model is formulated in mesh domain using mesh voltages and currents. The advantages of the mesh domain is that mesh currents and voltages in co-axial cables are naturally decoupled at high frequencies, so that the mesh domain functions (characteristic admittance, propagation functions etc.) behave smoothly compared with phase domain functions. The transformation matrices between mesh and phase domain are real and constant. The paper validates the proposed model by comparing the time domain simulations with solution obtained using inverse Laplace transform method for simple linear terminations.**

***Keywords*: transmission line models, underground cables, direct phase domain, mesh domain.**

I. INTRODUCTION

TIME domain transmission line and cable models for electromagnetic transient simulation are required to be accurate over a very wide frequency range from few Hertz to several tens of kilohertz [1]. The approach for time domain modelling begins with the frequency domain representation. Then the frequency domain parameters (entries of the characteristic admittance and propagation matrices) are approximated using rational functions (curve-fitting). The conventional phase domain formulation, which uses conductor voltages and currents, can be numerically challenging due to the superposition of independent traveling waves arising from the coupling of modes. This results in oscillatory behaviour in the frequency response of elements of propagation matrix, which makes delay extraction and low order rational function approximation difficult [1], [8].

This problem can be significantly reduced by reformulating the phase domain transmission line expressions into mesh domain expressions involving mesh voltages and currents. The mesh domain analysis for cables is not a new idea. For example, the EMTDC theory book formulates mesh domain equations as an intermediate step to obtain parameters, but immediately converts them to phase domain [5]. The resulting system is then simulated using a modal domain or phase domain approach.

However this paper shows that there are advantages in trying to solve the problem directly in the mesh domain. The mesh equations are naturally decoupled at high frequencies in a manner similar to modes. Also the characteristics of propagation and characteristic admittance functions are well behaved and easy to fit using rational functions. The resulting transformation matrix between phase and mesh domains is frequency independent.

Once mesh domain formulation is made, known methods can be applied. This model exploits a feature in the Vector Fitting algorithm [3], in which two or more functions can be curve-fitted using common set of poles. The entries of each column of the propagation matrix are approximated using rational functions with a common set of poles; hence the numerical efficiency of the model is improved. The modal order reduction technique [4] further improves the solution by removing poles and residues contributing insignificantly to the accuracy of the approximated rational function.

Unlike the phase domain, the oscillatory behavior of mesh propagation function is less. However sometimes there is still a problem and this problem can be overcome by employing multiple delays (modal delays) in the functional form of the fitted propagation function [2], [7]. The multiple delay approach notably improves the accuracy of the fitted propagation function, compared with the single common delay assumption (particularly in case of highly frequency dependent cable systems).

Finally time domain simulations involving example multi-conductor underground cables are presented in order to verify the validity of the proposed model. The time domain results of the proposed model are compared with the solution obtained via Numerical Inverse Laplace transform technique.

II. PHASE DOMAIN MODELLING

This section briefly describes the basics of traditional phase domain modelling and the same equations are applicable to mesh domain modelling.

In the discussion to follow, the term 'line' refers to both the overhead line and the underground cable systems, as the treatment developed is common to both. For an n-phase transmission line having length *l*, the frequency domain solution of the traveling wave equation can be expressed by the well know matrix-vector equations at each end-of the line

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Paper submitted to the International Conference on Power Systems Transients (IPST2009) in Kyoto, Japan June 3-6, 2009

given by [1],

$$I_k = Y_c V_k - A(Y_c V_m + I_m) \tag{1}$$

$$I_m = Y_c V_m - A(Y_c V_k + I_k) \tag{2}$$

In the above equations, V and I are n dimensional voltage and current vectors and subscripts ‘k’ and ‘m’ denote sending-end and receiving-end of the line. Y and Z are (n×n) shunt admittance and series impedance matrices per unit length respectively.

The (n×n) characteristic admittance matrix Yc and the (n×n) propagation matrix A are calculated as below using matrix functions [1]:

$$Yc = \sqrt{(YZ)^{-1}}\,Y \tag{3}$$

$$A = e^{-\sqrt{YZ}l} \tag{4}$$

In order to implement the model in the time domain, the elements of Yc and A are approximated with rational functions of suitable orders M and N [1] in the form shown below in (5) and (6). Such forms can easily be converted into differential equations which can be numerically integrated.

$$A_{i,j}(s) = \sum_{p=1}^{N} \frac{c_p e^{-s\tau}}{s - a_p} \tag{5}$$

$$Yc_{i,j}(s) = \sum_{q=1}^{M} \frac{c_q}{s - a_q} + d \tag{6}$$

The unknown coefficients, c_p and a_p (p = 1: N) in equations (5) and a_q, c_q (q = 1: M) and d in (6) are calculated using an efficient robust technique called Vector Fitting [3]. Note that the time delay (τ) in equation (5) is estimated before the fitting procedure. Sometimes for accurate curve fitting additional terms are added to (5) with different time delays (see equation (17)). For most practical transient simulation studies, it is sufficient to consider frequencies from zero Hz to 1 MHz for the fitting procedure.

III. Mesh Domain Formulation

The phase domain approach for the previous section uses phase voltages and currents, i.e. the voltages in equations (1) & (2) are defined as voltages of each conductor with respect to ground and similarly currents are defined as currents through each conductor with return path through earth as shown in figure 1.

In this mesh domain approach, the currents and voltages of a cable are defined in the mesh domain as shown in figure 2. Instead of selecting inner conductor to ground, sheath to ground, armour to ground voltages, the new set of voltages are chosen as voltage between inner conductor and sheath, sheath and armour, and armour and ground. Similarly the currents are selected from conductor to conductor, instead of conductor to ground. Later explained in this paper, there are advantages in formulating transmission line equations in mesh domain using mesh voltages and currents than in conventional phase domain.

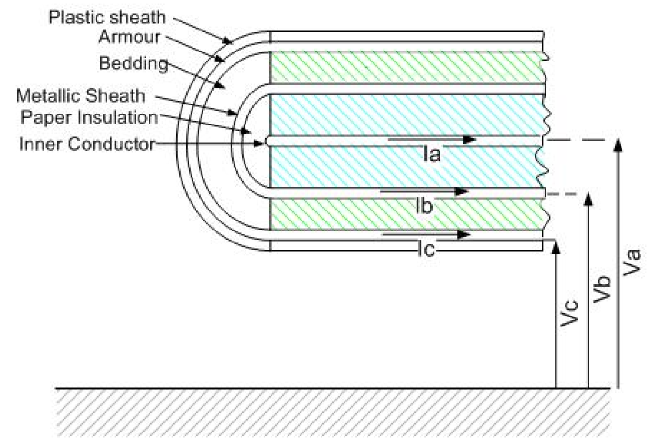

Figure 1: Phase voltages and currents in a coaxial cable system

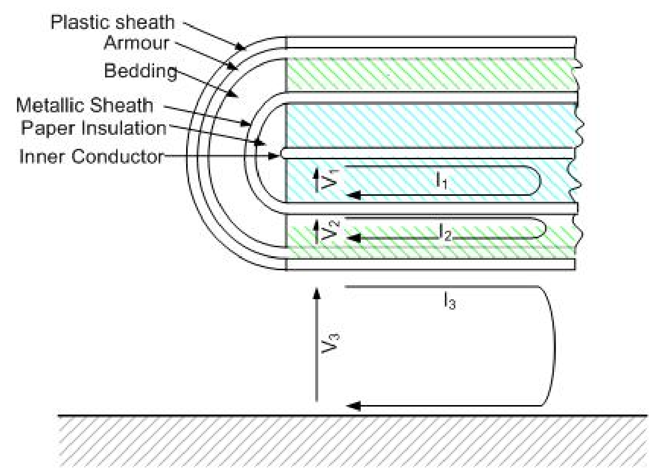

Figure 2: Mesh voltages and currents in a coaxial cable system

IV. Series Impedance and Shunt Admittance Matrices

This section briefly describes the derivation of impedance and admittance matrices in mesh domain. The self and mutual impedances and admittances between the cable conductors can be represented using the per unit length equivalent circuit shown in figure 3. Here, Z_{jj} is the self impedance of j^{th} loop. And Z_{ij} is the mutual impedance between i^{th} and j^{th} loops.

Z_{11} = impedances of internal conductor + insulator between first sheath and conductor + inner first sheath.

Z_{mi} = mutual impedance of i^{th} sheath

Z_{ii} = sum of impedances of outer $(i\text{-}1)^{th}$ sheath + $(i\text{-}1)^{th}$

insulator + inner i[th] sheath.

Z_{nn} = sum of impedances of outer (n-1)[th] sheath + the (n-1)[th] insulator + self earth return .

Y_{ii} = shunt admittance between two adjacent conductor layers.

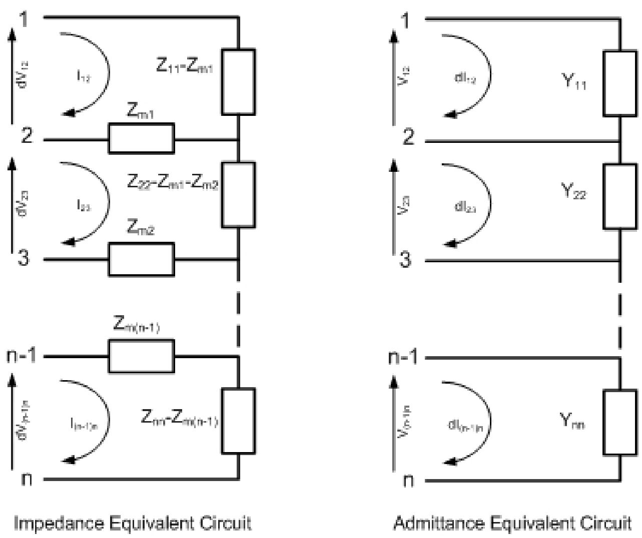

Figure 3: Equivalent circuits for unit length admittance and impedance

This difficulty can be significantly reduced by choosing different formulation for Z and Y matrices. The new current meshes or meshes are selected as 1-2, 2-3, 3-4…, and (n-1)-n. The Z and Y matrices formulated in this new mesh domain are,

$$Z = \begin{bmatrix} Z_{11} & -Z_{m1} & . & 0 \\ -Z_{m1} & Z_{22} & . & 0 \\ . & . & . & -Z_{m(m-1)} \\ 0 & 0 & -Z_{m(m-1)} & Z_{mm} \end{bmatrix} \tag{7}$$

$$Y = \begin{bmatrix} Y_{11} & 0 & . & 0 \\ 0 & Y_{22} & . & 0 \\ . & . & . & 0 \\ 0 & 0 & 0 & Y_{nn} \end{bmatrix} \tag{8}$$

The mutual impedances tend to zero as frequency increases. Thus the matrix formulated in the new mesh domain becomes increasingly diagonal at high frequencies. Since Y is already a diagonal matrix, the currents (as well as voltages) defined in the new mesh domain become decoupled at high frequencies. In other words the system supports almost independent traveling waves. This is a good approximation to a pure modal domain system at very high frequencies.

Both magnitude and phase of the frequency response of elements of propagation matrix (A) behave smoothly compared with that of direct phase domain formulation (phase to ground). This is a significant advantage for rational function approximation of the propagation function. A low order fit can be always found and this leads to an efficient model. This is particularly true for the elements of characteristic admittance matrix as well.

A. Phase domain Z and Y matrices

The series impedance and shunt admittance matrices are traditionally formulated in the phase domain choosing the conductor currents 1-g, 2-g, 3-g...n-g as the variables. Phase currents and phase voltages (conductor to ground) have significantly mutual coupling within the frequency range from 0 Hz to 1 MHz. This results in oscillatory behavior in propagation and characteristic admittance functions, when plotted as a function of frequency. A higher order fit is usually required for the rational function approximation due to the oscillatory nature of elements of propagation matrix both in magnitude and phase.

<u>Transfer impedance and transfer admittance in mesh domain</u>

The electromagnetic field penetration through cable shields can be explained using the transfer impedance and admittance. In traditional phase domain approach, the transfer impedance consists of outer sheath internal impedance, impedance due to time varying flux in the outer insulation, self impedance of the earth return path and sheath mutual impedance [9]. As the frequency increases, all the sub-impedance terms increase except sheath mutual impedance. Hence, the transfer impedance increases with frequency, resulting in a significant coupling in phase voltages and currents. The phase domain transfer admittance is the admittance of the insulator between two adjacent conductors (e.g. between the inner conductor and the sheath) [9]. In the mesh domain approach, the transfer impedance only contains sheath mutual impedance $\left(Z_{mi}\right)$ (see equation (7)), which tends to zero at higher frequencies. The transfer admittance in mesh domain is zero (see equation (8)).

V. TRANSFORMATION MATRICES

The goal of mesh domain model is to develop a time domain equivalent circuit compatible with emtp-type software. Since final quantities are the phase quantities, at the highest level the time domain model must be expressed in phase domain. It is necessary to transform voltages and currents in phase domain to new mesh domain and vice versa. The voltage transformation matrix can be derived as follows.

$$\begin{aligned} V_{1g} &= V_{12} + V_{23} + V_{34} + ...V_{(n-1)n} \\ V_{2g} &= V_{23} + V_{34} + ...V_{(n-1)n} \\ &.......................... \\ V_{ng} &= V_{(n-1)n} \end{aligned} \tag{9}$$

Note that suffix ‘g’, ‘phase’, ’mesh’ denote ground, traditional phase domain and the new proposed mesh domain respectively.

$$V_{phase} = K_V V_{mesh} \tag{10}$$

The voltage transformation matrix K_V is defined as,

$$K_V = \begin{bmatrix} 1 & 1 & 1 & . & 1 \\ 0 & 1 & 1 & . & 1 \\ 0 & 0 & 1 & . & 1 \\ . & . & . & . & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

Similarly the current transformation matrix can be derived by considering phase to mesh transformation for currents. The current transformation matrix K_I is defined as,

$$K_I = \begin{bmatrix} 1 & 0 & 0 & . & 0 \\ -1 & 1 & 0 & . & 0 \\ 0 & -1 & 1 & . & 0 \\ . & . & . & . & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \tag{12}$$

Another advantage of the mesh method is that current and voltage transformation matrices are real and constant avoiding the difficulties with frequency dependent transformation matrices. It can be easily shown that,

$$K_V = \left(K_I^{\,T}\right)^{-1} \tag{13}$$

VI. Rational function approximation of Yc and A

Once mesh domain Z and Y matrices are computed (as discussed in section IV), mesh A and mesh Yc are then calculated using equations (3) and (4). Time domain models require rational function approximation (curve-fitting) of entries of the characteristic admittance (Yc) and propagation (A) matrices.

A. Rational function approximation of Yc

Although the discussion below is strategic to mesh domain formulation, these methods have been applied to phase domain approaches by previous researchers [2], [4].

For the curve-fitting of mesh Yc, the technique used to fit phase Yc in the Universal Line Model [2] is employed. The elements of mesh Yc matrix are approximated using rational functions (in the form (6)) with a common set of poles. The common poles are identified by curve-fitting the trace of the matrix [2].

B. Time delay estimation for A

The functional form of the fitted propagation function (as shown in equation (7)) has a single delay for each entry. However the use of multiple delays (modal delays) for the fitted propagation function gives better accuracy [2]. The modal delays are the delays attributed to the modes of propagation matrix (A). These modes are computed in the standard manner using a frequency dependent transformation matrix as for the case of the phase domain Universal Line Model [2]. The modal delays are estimated using Bode’s gain-phase formula from modes of mesh A matrix [2].

The rational form of the propagation function with multiple delays (τ_i) is,

$$A_{i,j}(s) \approx e^{-s\tau_1}\sum_{n_1=1}^{N_1}\frac{c'_{n_1}}{s-a_{n_1}} + e^{-s\tau_2}\sum_{n_2=1}^{N_2}\frac{c'_{n_2}}{s-a_{n_2}} + ... \tag{14}$$

Here, τ_i is the delay corresponding to the i[th] mode. The unknown residues and poles c'_{n_i}'s and a_{n_i}'s’ are obtained using a least squares method.

C. Identification of poles and residues in equation (14)

This model exploits a feature in the Vector Fitting algorithm, in which two or more functions can be simultaneously curve-fitted using a common set of poles [3]. Each column of the mesh propagation matrix is approximated with a common set of poles using the modified Vector fitting algorithm (i.e. the unknown residues and poles in (14) are calculated by curve-fitting each column of the propagation matrix and the entries in a column share same set of poles); hence the numerical efficiency of the model is improved. The modal order reduction technique further improves the solution by removing poles and residues contributing insignificantly to the accuracy of the approximated rational function [4].

VII. Time domain equivalent circuit

The final objective of mesh domain model is to develop a time domain equivalent circuit, which can be implemented in emtp-type software. The mesh domain transmission line equations are,

$$I_{m,mesh} = Y_{c,mesh}V_{m,mesh} - A_{mesh}(Y_{c,mesh}V_{k,mesh} + I_{k,mesh}) \tag{15}$$

$$I_{k,mesh} = Y_{c,mesh}V_{k,mesh} - A_{mesh}(Y_{c,mesh}V_{m,mesh} + I_{m,mesh}) \tag{16}$$

Although core of the method is the mesh domain solution, the ultimate real quantities are the phase quantities, at the highest level the time domain model must be expressed in phase domain. If equation (18) is multiplied by current transformation matrix K_I,

$$K_I I_{m,mesh} = K_I Y_{c,mesh} V_{m,mesh} - K_I F_k \tag{17}$$

where,

$$F_k = A_{mesh}(Y_{c,mesh} V_{k,mesh} + I_{k,mesh}) \tag{18}$$

Since $I_{m,phase} = K_I I_{m,mesh}$ Then equation (17) becomes,

$$I_{m,phase} = K_I Y_{c,mesh} V_{m,mesh} - K_I F_K \tag{19}$$

The time domain form of (19) is,

$$i_{m,phase}(t) = K_I y_{c,mesh}(t) \times v_{m,mesh}(t) - K_I f_k(t) \tag{20}$$

where, the term "$\times$" denotes convolution and,

$$f(t) = a_{mesh}(t) \times \left(y_{c,mesh}(t) \times v_{k,mesh}(t) + i_{k,mesh}(t) \right) \tag{21}$$

The lower case variables are the corresponding time domain form of the upper case variables. The above convolutions can be efficiently evaluated by recursive convolution [1] and the equation (20) now becomes,

$$i_{m,phase}(t) = \overline{y}_{eq\_m} v_{m,mesh}(t) + i_{hist\_m}(t) \tag{22}$$

The term $i_{hist\_m}(t)$ is calculated using past values of voltages and currents. Since $y_{eq\_m} = \overline{y}_{eq\_m} \cdot K_I^T$, the equation (22) becomes,

$$i_{m,phase}(t) = y_{eq} v_{m,phase}(t) + i_{hist\_m}(t) \tag{23}$$

Similarly for the other end of the transmission line (see equation (16)),

$$i_{k,phase}(t) = y_{eq} v_{k,phase}(t) + i_{hist\_k}(t) \tag{24}$$

Equations (23) and (24) are the mathematical representation of a time domain equivalent circuit, which can be realized in popular electromagnetic transient programs as two current sources in parallel with two conductances as shown in figure 4 for single conductor case.

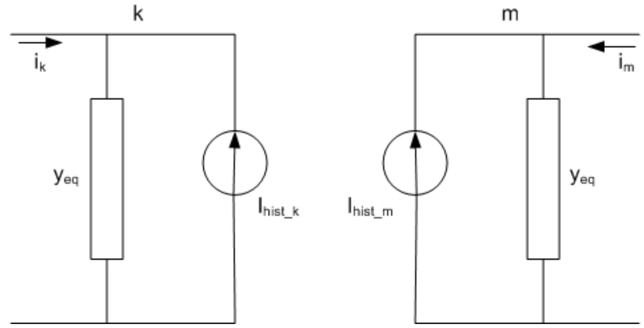

Figure 4: Time domain equivalent circuit

VIII. Application Example

In order to explain the advantages of the proposed mesh domain method, an example underground cable system is considered. Figure 5 shows a three-cable system (each has an inner conductor and sheath) with data shown in Table I [22]. The typical plots of entries of the propagation matrix formulated in traditional phase domain are shown in Figure 6, clearly indicating the oscillatory behavior of phase domain elements when plotted as a function of frequency.

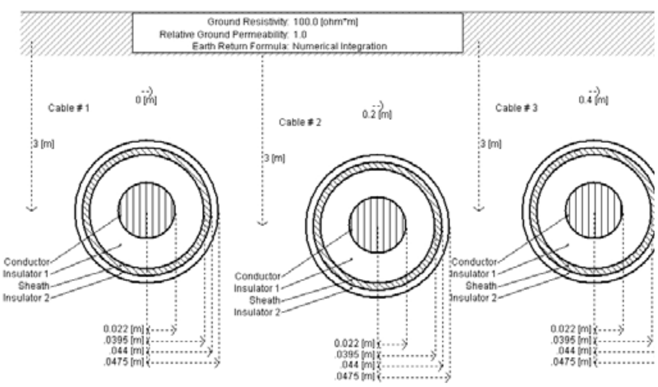

Figure 5: Three cable system

TABLE I
CABLE DATA

RESISTIVITY OF THE INNER CONDUCTOR (Ωm)	1.68E-8
RESISTIVITY OF THE OUTER CONDUCTOR (Ωm)	2.2E-7
RELATIVE PERMITTIVITY OF INNER AND OUTER INSULATORS	4.1/3.2
EARTH RESISTIVITY (Ωm)	100
LENGTH OF THE CABLE SYSTEM (KM)	30

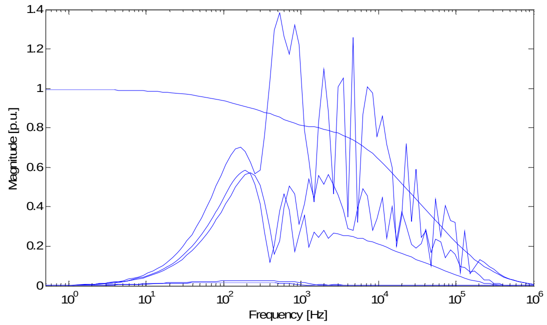

Figure 6: First column of the phase propagation function

If the propagation matrix is formulated in mesh domain, the frequency domain response of elements of mesh propagation matrix (see Figure 8) show smooth behaviour in contrast to response of the phase domain functions. The fitted first column of mesh domain propagation matrix is shown in Figure 8 as a function of frequency. Also shown in the graphs are the theoretical (actual) curves (the solid lines in the same figure) indicating that the fitting is accurate. In comparison with phase domain elements, mesh domain elements are smooth and hence easy to curve-fit using low order rational functions. Table II compares the order of rational function for mesh domain with phase domain formulation. For the propagation function, order of the transfer function is reduced noticeably by 30%, if mesh formulation is used and hence leads to a numerically efficient model.

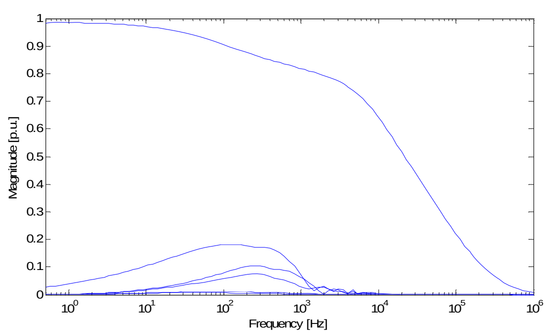

Figure 7: First column of the mesh propagation function

TABLE II
ORDER COMPARISON (PER ELEMENT)

ORDER OF PHASE YC	10
ORDER OF MESH YC	10
ORDER OF PHASE A	43
ORDER OF MESH A	33

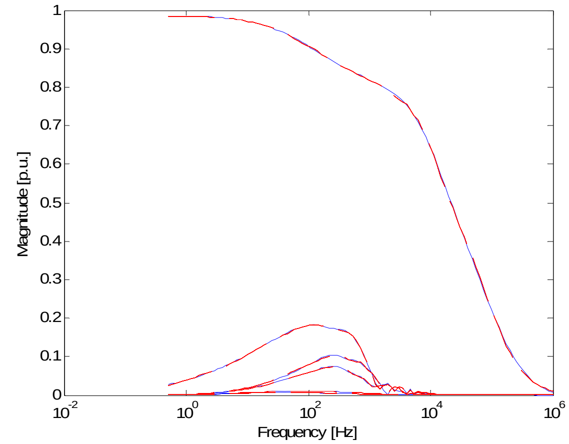

Figure 8: Actual and fitted mesh propagation functions

Time domain simulations are conducted to validate the proposed mesh domain model. A short circuit test is conducted for the same cable system. The inner conductor of cable 1 is energized with 1V step, while all other conductors are connected to the ground. Figure 9 shows the sending-end current waveform (solid conductor) through inner conductor of cable 1 and the theoretical solution (dotted line) obtained via numerical inverse Laplace transform method. The current waveform from mesh domain model is in a close agreement with theoretical solution, indicating that simulation results from mesh domain model are accurate.

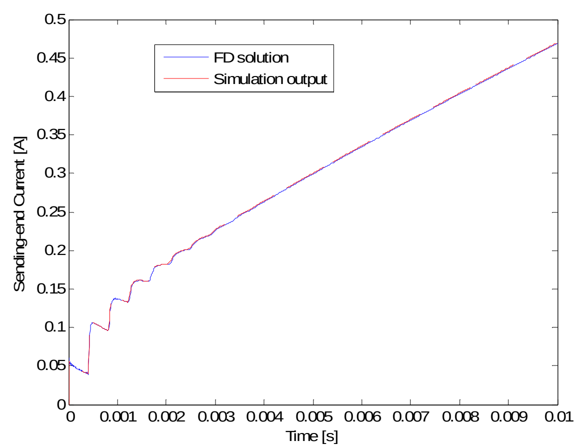

Figure 9: Sending-end current

Another simulation is conducted for the same cable system. A 1V step is applied to the inner conductor of cable 1 and all other terminations are kept open. Figure 10 shows the induced voltage (solid line) at the sending-end of the inner conductor of cable 2. Also shown in the graph (dotted line) is the theoretical solution. This confirms that the induced voltage is in a close agreement with theoretical results.

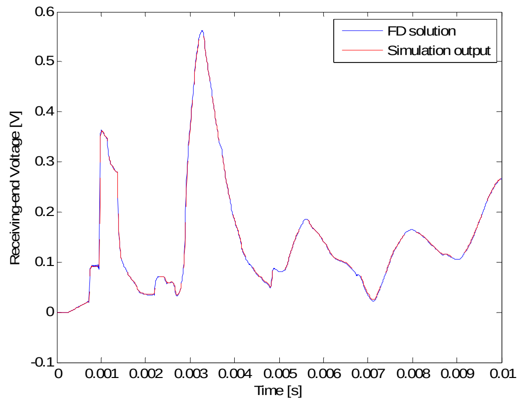

Figure 10: Induced voltage in the second conductor

IX. CONCLUSION

This paper proposed a new mesh-domain method to simulate time-domain transients in underground cables accurately considering frequency dependent effects. One advantage of this approach is that the resulting transformation matrix between phase and mesh domains is frequency independent. The mesh currents and voltages in co-axial cables are naturally decoupled at high frequencies hence the propagation function shows relatively smooth behaviour. The rational function approximation is relatively easier than in basic phase domain methods.

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