

Impacts of Various Representations of Core Saturation Curve on Ferroresonance Behavior of Transformers

Afshin Rezaei-Zare and Reza Iravani

Abstract—This paper investigates and compares the impacts of two widely used types of the transformer core representation, i.e. piecewise linear magnetization characteristic and two-term polynomial-based saturation curve, on the ferroresonance behaviors of a power transformer. The polynomial-based saturation curve is implemented in the EMTP-RV environment, using Dynamic Link Library (DLL) programming feature. Such an implementation participates in the solution of the equation set of the system and results in true nonlinear solutions of the ferroresonance phenomenon.

The simulation results indicate that the ferroresonance behaviors of the transformer under study, based on the piecewise linear and the polynomial saturation characteristics, are significantly different. The two-term polynomial has limited flexibility to represent the saturation characteristic of the transformer around the knee point and in both linear and saturation regions. Although ferroresonance behavior of a transformer highly depends on the characteristic of the core above the rated excitation level, an inaccurate representation of the characteristic in the linear part can result in erroneous ferroresonance conditions.

Keywords: Power transformer, ferroresonance, saturation curve, EMTP.

I. INTRODUCTION

FERRORESONANCE is a highly nonlinear phenomenon which is sensitive to the parameters and initial conditions of the transformer and the power system. The main element of a ferroresonance circuit is the nonlinear inductance of the transformer core, and the accuracy of a ferroresonance study depends on the accuracy of the transformer core representation.

Majority of ferroresonance studies, including analytical methods, e.g. harmonic balanced method, and time domain methods are based on single-valued magnetization characteristics of the transformer core [1]-[10]. The core saturation curve is usually represented based on either a piecewise linear characteristic or a two-term polynomial function. The piece-wise linear characteristic is constructed based on

no-load test data of the transformer which are recorded at a few excitation levels. The available electromagnetic transient programs, e.g. the EMTP and the EMTP-RV [11],[12], usually provide a subroutine to convert the rms values of the transformer no-load test data to the corresponding peak values which are required for the construction of the magnetization characteristics.

The two-term polynomial-based saturation curve is deduced based on a function with a linear term and a term with the order of n . The reported ferroresonance studies recommend lower order of the polynomial, e.g. 5th or 7th order, for the representation of voltage transformer (VT) core [1] and higher orders, e.g. 11th and 13th order, for the saturation curve of power transformers [3]. As an advantage, such a function represents a simple form of the asymptotic saturation curve. Another advantage is that in analytical ferroresonance studies, e.g. harmonic balanced method [2]-[4], there is only one nonlinear term which appears in the differential equations of the system under study. Therefore, majority of the investigations of ferroresonance bifurcation and ferroresonance stability domains are based on the polynomial function [2]-[4], [6]-[10]. In addition, in the DCG/EPRI version of the EMTP, the Type-92 true nonlinear reactor is developed based on such a function [13].

Accordingly, this paper investigates and compares the impacts of the two representations of the saturation curves on the ferroresonance behavior of a power transformer. The study is carried out in the EMTP-RV program environment. The piece-wise linear model is available in the program. However, the polynomial model is not available and therefore is implemented as a user-defined model. Unlike the other existing electromagnetic transient programs, based on the DLL programming feature of the EMTP-RV, The core model participates in the iteration loop of the equation set of the whole system and represents true nonlinear solutions of the ferroresonance phenomenon.

The simulation results indicate that the ferroresonance behaviors of the transformer under study, based on the piecewise linear and the polynomial saturation characteristics, are significantly different. This study highlights the significance of accurate representations of both linear and saturation the saturation curves for the ferroresonance analysis and the calculation of the stability domains of the ferroresonance modes in power transformers and systems.

Afshin Rezaei-Zare and Reza Iravani are with the Center of Applied Power Engineering, Department of Electrical and Computer Engineering, University of Toronto, Toronto, ON, Canada, M5S 3G4. (e-mails: a.rezaei@utoronto.ca, iravani@ecf.utoronto.ca, Phone: +1(416)978-3126).

II. TIME DOMAIN IMPLEMENTATION OF THE POLYNOMIAL TRANSFORMER CORE MODEL

Based on the polynomial-based representation of the transformer core, the magnetization characteristic is given by the polynomial function (1),

$$i_m = a\lambda + b\lambda^n, \quad (1)$$

where i_m and λ are the magnetizing current and the core linkage flux, respectively. In addition, constant coefficients a and b respectively impact the linear and saturated regions of the core magnetization characteristic. The curvature of the characteristic in saturation region is mainly deduced based on constant exponent n .

The polynomial magnetization characteristic is not available in the existing electromagnetic transient programs. In this study, the polynomial model is created in the EMTP-RV environment as a user-defined model based on the DLL programming feature of the EMTP-RV. The model is represented based on a constant core loss resistance and a nonlinear inductance with the characteristic (1). In general, the characteristic of a nonlinear inductance is a relationship between the flux and the magnetizing current of the inductance given by,

$$\lambda = f(i_m). \quad (2)$$

Based on the trapezoidal integration method, the representation of the inductance in time-domain is represented by

$$\begin{aligned} i_m(t) &= \frac{\Delta t}{2L} V_L(t) + \frac{\Delta t}{2L} V_L(t - \Delta t) + i_m(t - \Delta t) \\ &= \frac{\Delta t}{2L} V_L(t) + i_h, \end{aligned} \quad (3)$$

where V_L is the inductor voltage and i_h is a history term which is deduced based on the inductance current and voltage values at previous time step of the simulation. For the nonlinear inductance, L is the inductance at time t and defined as the slope of the magnetization characteristic (2).

Fig. 1 shows the flowchart of the model implementation in a time-domain program, e.g. the EMTP-RV. The model is initialized by setting initial conditions and remnant flux, and (2) is constructed based on the known remnant flux. An initial guess i_m should be made and equation (2) is solved based on Newton-Raphson iterations. When the solution converges, parameters L and i_h are calculated and inserted in the equation set of the system under study. The system equations are solved and if the convergence is met, the program proceeds to the next time step. Otherwise, (2) is reconstructed based on the new calculated flux value. To represent the flowchart of Fig. 1, a FORTRAN program was developed and converted to a DLL file. The DLL file is used in the simulations described in the next section.

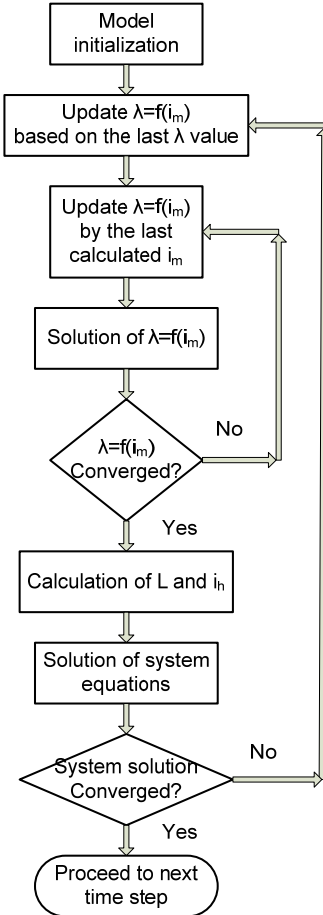


Fig. 1. Implementation flowchart of a nonlinear core model

III. STUDIED SYSTEM AND SIMULATION RESULTS

In this study, the ferroresonance behavior of a 50MVA, 230kV/66kV power transformer is investigated, based on the piecewise linear and the polynomial magnetization characteristics. The transformer model consists of three single-phase transformers. The saturation characteristic of the studied transformer is selected based on the per unit value of the core saturation flux and the per unit value of the core no-load magnetizing current. The saturation flux of large power transformers are usually in the range of $1.1pu-1.3pu$. In addition, in modern power transformers, the no-load current is in the range of $0.5\%-2\%$ [14]. Accordingly, the saturation flux density $1.2pu$ and the no-load current 1% are assumed for the transformer under study. Based on $a=0.001$, $b=0.01$, and $n=25$, Fig. 2 depicts the polynomial magnetization characteristic of the transformer. The piecewise linear characteristic with the point data of Table I is constructed based on the polynomial saturation curve of Fig. 2. Fig. 3 illustrates the polynomial characteristic and the corresponding piecewise linear saturation curve. The short circuit impedance and the no-load power loss of the transformer are 10% and $50kW$, respectively. The ferroresonance behavior of the

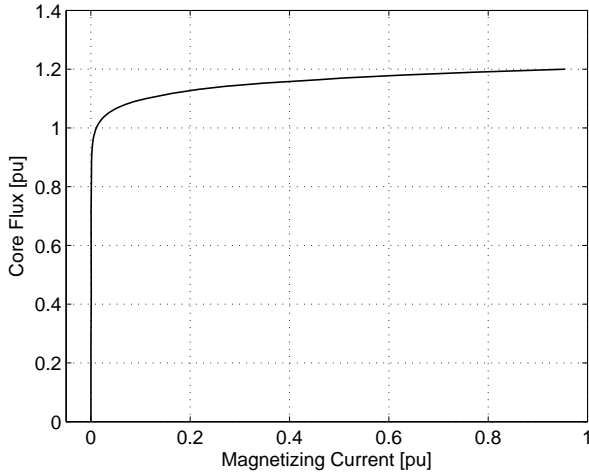


Fig. 2. Magnetization characteristic of the transformer under study

transformer is investigated in the following two case studies.

A. Case Study 1: transformer energized through grading capacitance of circuit breakers

Fig. 4 depicts the single-line diagram of the three-phase system under study. The power system is represented by a $230kV$ - $60Hz$ voltage source and impedance $Z_s=0.2+j5\Omega$. The power transformer is connected to the system through a circuit breaker with grading capacitance C_g . At high voltage levels, circuit breakers are usually equipped with more than one interrupter per phase. To equally divide the transient voltage among the interrupters, the grading capacitors are used in parallel with the interrupters. The equivalent grading capacitance per each pole of a circuit breaker is usually less than $2nF$. However, in a high voltage substation with multiple incoming and outgoing feeders and depending on the busbar configuration, the equivalent grading capacitance can reach a few nFs. In this study, the equivalent grading capacitance is $C_g=8nF$. In addition, the stray capacitance C_s of the conductor and busbar, between the circuit breaker and the transformer, is assumed to be $1nF$.

When the system of Fig. 4 operates in a steady-state no-load condition, a temporary three-phase short circuit fault with the amplitude of $1.8kA$ -rms occurs on the secondary side and close to the transformer at $t=0.06s$, Fig. 5. The fault is cleared by opening the circuit breaker CB after five cycles at $t=0.146s$, Fig. 5. Figs. 6 and 7 depict the transformer Phase A voltage when the magnetization characteristic of the transformer is represented based on the piecewise linear and the polynomial characteristics, respectively. Subsequent to the opening of the circuit breaker, the transformer is fed through the equivalent grading capacitance C_g . Based on the piecewise linear magnetization characteristic, Fig. 6 depicts a normal operating condition for the transformer. However, based on the polynomial saturation curve, subsequent to the opening of the circuit breaker, the transient of the transformer voltage is followed by a fundamental mode ferroresonance oscillation

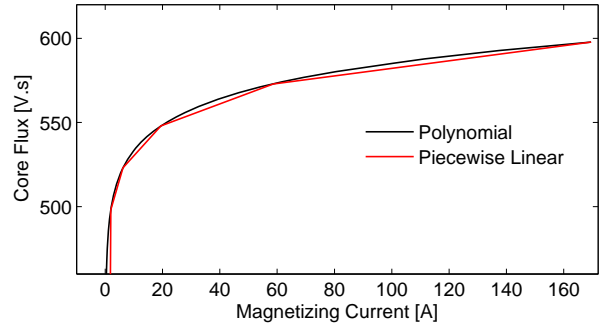


Fig. 3. Polynomial and piecewise linear magnetization characteristics

TABLE I
PIECEWISE LINEAR CHARACTERISTIC DATA

Magnetizing current [A]	Core flux [V.s]
0	0
1.95	498.14
6.2	523.047
19.427	547.95
58.635	572.86
169.54	597.77

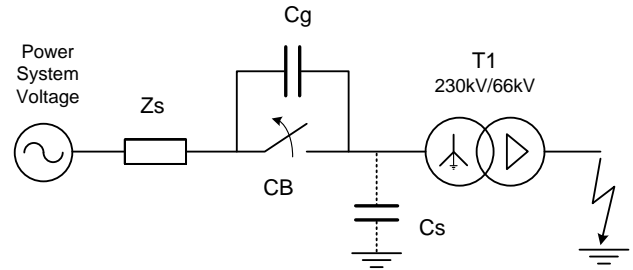


Fig. 4. System of Case Study 1, the transformer is energized through the grading capacitance of the circuit breaker

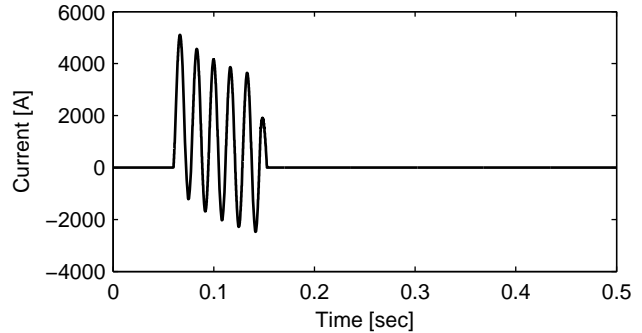


Fig. 5. Short circuit fault current on the secondary side of transformer T1 of Fig. 4.

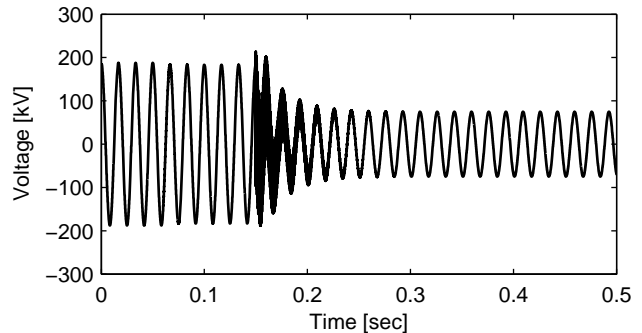


Fig. 6. Case Study 1, Transformer terminal voltage based on the piecewise linear magnetization characteristic

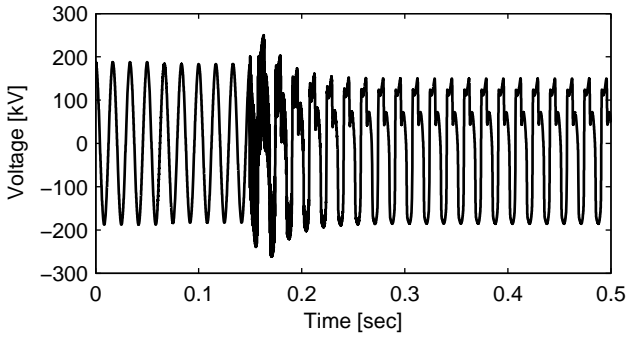


Fig. 7. Case Study 1, Transformer terminal voltage based on the polynomial saturation curve

shown in Fig. 7. The amplitude of the ferroresonance voltage is slightly higher than $1pu$. Although both the polynomial and the piecewise linear characteristics conceptually represent the same core nonlinearity, the corresponding results for Case Study 1 are significantly different.

B. Case Study 2: transformer connected to a double circuit transmission line

Fig. 8 depicts the system of this case study. The power system voltage and impedance, and the power transformer under study are the same as those of Case Study 1. In this case, the transformer is connected to the end of a three-phase circuit of a double-circuit transmission line. When the line which corresponds to the transformer is disconnected from the source side, the transformer is still fed through the capacitive coupling C_c between the two circuits of the transmission system, Fig. 8. It is well documented that such a configuration is favorable to ferroresonance [15],[16].

When the system of Fig. 8 with the no-load transformer T1 are in the normal steady-state operating conditions, circuit breaker CB is opened at $t=0.146s$. In this case, the value of C_c is $1\mu F$ which is a typical value for a double-circuit high voltage transmission line with an average length. Figs. 9 and 10 respectively illustrate the transformer terminal voltage based on the piecewise linear and the polynomial magnetization characteristics. Unlike Case Study 1, the piecewise linear characteristic results in dangerous ferroresonance overvoltages with the peak values of $860kV(4.58pu)$ while the polynomial saturation curve does not show any significant overvoltage.

IV. DISCUSSION AND CONCLUSIONS

Based on two widely used magnetization characteristics, i.e. the piecewise linear and the polynomial magnetization characteristics, the ferroresonance behaviors of a 230kV/66kV power transformer are studied in this paper. The piecewise linear characteristic is available in most electromagnetic transient programs, e.g. the EMTP. However, the polynomial magnetization characteristic is not usually available in the

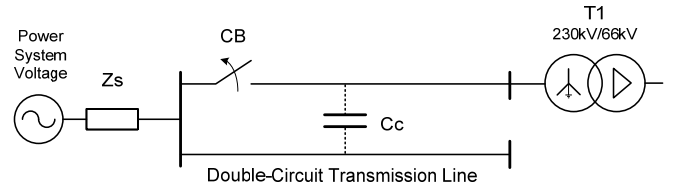


Fig. 8. System of Case Study 2, the transformer is connected at the end of a disconnected line of a double circuit transmission line

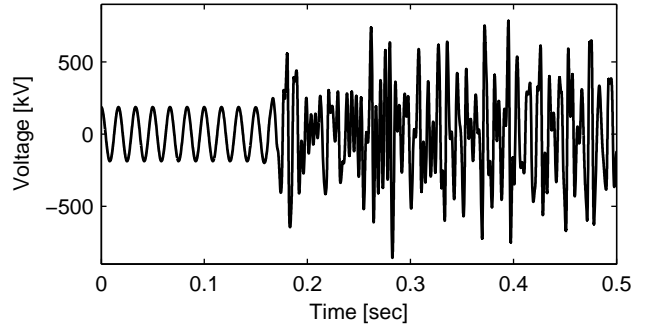


Fig. 9. Case Study 2, Transformer terminal voltage based on the piecewise linear magnetization characteristic

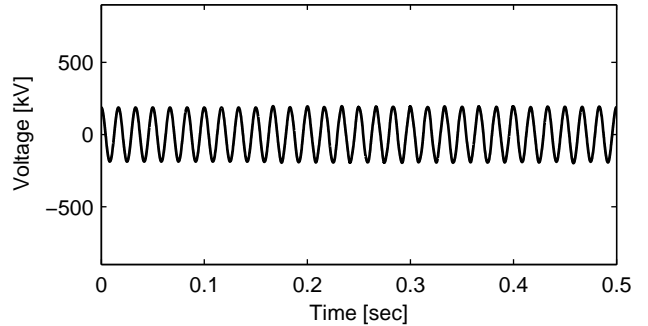


Fig. 10. Case Study 2, Transformer terminal voltage based on the polynomial saturation curve

programs and is developed in this study based on the DLL user defined modeling feature of the EMTP-RV program.

Under ferroresonance conditions, the core flux exceeds the rated core flux. Therefore, for the ferroresonance analysis, the magnetization characteristic shape around the knee point and in the saturation region is of great importance and discussed in the technical literature. However, this study indicates that if the linear part of the saturation curve, which is less discussed in the ferroresonance studies, is not accurately represented, it can result in significant errors.

The simulation results indicate that the ferroresonance behaviors of the transformer under study, based on the piecewise linear and the polynomial saturation characteristics, are significantly different. Although both the characteristics can fairly represent the same core nonlinearity at and above the rated excitation level, Fig. 3, they do not provide the same characteristic in the linear part. The piecewise linear characteristic represents an inductance of $255.45H$, below and up to the rated excitation level. However, the inductance of the linear part deduced from the polynomial curve has a significantly larger value of $2806H$. The polynomial function has only three parameters and therefore, limited flexibility to

accurately represent the core nonlinearity in the linear part, around the knee point, and in the saturation region. An inaccurate representation of each part of the magnetization characteristic can result in erroneous ferroresonance conditions. Consequently, such a function provides limited accuracy for ferroresonance analysis and investigation of ferroresonance stability domains.

V. REFERENCES

- [1] Z. Emin, B.A.T. Al Zahawi, D.W. Auckland, Y.K. Tong, "Ferroresonance in Electromagnetic Voltage Transformers: A Study Based on Nonlinear Dynamics", *IEE Proc.-Gener. Trasm. Distrib.*, Vol. 144, No. 4, July 1997, pp. 383-387.
- [2] N. Janssens, "Direct Calculation of the Stability Domains of Three-Phase Ferroresonance in Isolated Neutral Networks with Grounded-Neutral Voltage Transformers", *IEEE Trans. on Power Delivery*, Vol. 11, No. 3, July 1996, pp. 1546-1553.
- [3] A.E.A. Araujo, A.C. Soudack, J.R. Marti, "Ferroresonance in Power Systems: Fundamental Solutions", *IEE Proc.-C*, Vol. 138, No. 4, July 1991, pp. 321-329.
- [4] D. Jacobson, P.W. Lehn, R.W. Menzies, "Stability Domain Calculations of Period-1 Ferroresonance in a Nonlinear Resonant Circuit", *IEEE Trans. on Power Delivery*, Vol. 17, No. 3, July 2002, pp. 865-871.
- [5] S. Mozaffari, S. Henschel, A.C. Soudack, "Chaotic Ferroresonance in Power Transformers", *IEE Proc.-Gener. Trasm. Distrib.*, Vol. 142, No. 3, May 1995, pp. 247-250.
- [6] K. Al-Anbari, R. Ramanujam, T. Keerthiga, K. Kuppusamy, "Analysis of Nonlinear Phenomena in MOV Connected Transformer", *IEE Proc.-Gener. Trans. Distrib.*, Vol. 148, No. 6, Nov. 2001, pp 562 – 566.
- [7] Z. Emin, Y.K. Tong, "Ferroresonance Experience in UK: Simulations and Measurements", International Conference on Power System Transients (IPST), Brazil, June 24-28, 2001.
- [8] T. Van Craenenbroeck, W. Michiels, D. Van Dommelen, K. Lust, "Bifurcation Analysis of Three-Phase Ferroresonant Oscillations in Ungrounded Power Systems", *IEEE Trans. on Power Delivery*, Vol. 14, No. 2, April 1999, pp. 531-536.
- [9] F. Wornle, D.K. Harrison, Chengke Zhou, "Analysis of a Ferroresonant Circuit Using Bifurcation Theory and Continuation Techniques", *IEEE Trans. on Power Delivery*, Vol. 20, No. 1, January 2005, pp. 191 – 196.
- [10] A. Ben-Tal, V. Kirk, G. Wake, "Banded Chaos in Power Systems", *IEEE Trans. on Power Delivery*, Vol. 16, No. 1, January 2001, pp. 105 – 110.
- [11] H.W. Dommel, "Electromagnetic Transients Program Reference Manual", (EMTP Theory Book). Portland, Aug. 1986.
- [12] J. Mahseredjian, EMTP-RV Program, EMTWork Version 2.1, IREQ/Hydro Quebec, 2007.
- [13] E.J. Tarasiewicz, A.S. Morched, A. Narang, E.P. Dick, "Frequency dependent eddy current models for nonlinear iron cores", *IEEE Trans. on Power App. and Systems*, Vol. 8, No. 2, May 1993, pp. 588 – 597.
- [14] K. Karsai, D. Kerenyi, and L. Kiss, "Large power transformers," New York, NY, U.S.A. Elsevier, 1987.
- [15] E.J. Dolan, D.A. Gillies, E.W. Kimbark, "Ferroresonance in a Transformer Switched with an EHV Line", *IEEE Trans. on Power App. and Systems*, Vol. PAS-91, No. 3, May 1972, pp. 1273 – 1280.
- [16] M. R. Iravani, A. K. S. Chaudhary, W. J. Giesbrecht, et. al. "Modeling and Analysis Guidelines for Slow Transients—Part III: The Study of Ferroresonance", Slow Transients Task Force of the IEEE Working Group on Modeling and Analysis of Systems Transients Using Digital Programs, *IEEE Trans. Power Del.*, Vol. 15, No. 1, January 2000, pp. 255-265.