Parameter Estimation of a Nonlinear EDLC Model for EMTP Simulation

Naoto Nagaoka, Shota Fujiyama, Hiroshi Nonoyama, Akihiro Ametani

***Abstract*--A parameter estimation method for a nonlinear model of an Electric Double Layer Capacitor (EDLC) is modified and a model for the Electromagnetic Transients Program (EMTP) is proposed in this paper. The model consists of a nonlinear capacitor and two linear resistors. The nonlinear capacitor is expressed by a voltage source controlled by the Transient Analysis of Control Systems (TACS) of the EMTP. Simulation results by the EMTP using the proposed model agree well with measured results.**

***Keywords*: Electric Double Layer Capacitor (EDLC), numerical model, nonlinear capacitor, EMTP**

I. Introduction

Although EDLC is widely used in an electronics field, its high current charging/discharging characteristic is suitable for power equipments. For example, the EDLC is applicable to electric vehicles and power stabilizers for wind-generation systems. A numerical simulation is indispensable for designing the power equipments. An accurate numerical model of the EDLC is required for reliable system design.

Numerical models of EDLCs are broken down into either linear or nonlinear models [1]-[3]. The basic model of an EDLC is a linear capacitor. The model is, however, unable to express the loss of the EDLC. The model can be improved by appending a series resistor and a parallel resistor to compensate the weakness. The model has been further modified to express the frequency characteristic of the impedance of the EDLC. These models have been widely used for numerical simulations of the EDLC. A nonlinear model is recently proposed [3] for an accurate numerical simulation. A modeling method of the nonlinear model based on an experimental result is proposed and an application example of the proposed model is presented in this paper.

II. Numerical Model of EDLC

Fig. 1 shows a schematic diagram of a current and voltage characteristic when an EDLC is charged by a Constant-Current/Constant-Voltage (CCCV) charging/discharging method. EDLC has to be charged by a constant current from a view point of efficiency. However, the working voltage of EDLC has to be strictly kept below its rated voltage for preventing its break down. The CCCV charging/discharging is a common method of the usage of an EDLC.

Fig. 1 illustrates typical voltage and current waveforms of a CCCV charging and discharging. A sudden voltage change ΔV is observed at the beginning of either charging or discharging. The voltage change indicates that the EDLC has a series resistor.

The terminal voltage is increased by a constant-current charging as time passes. If the capacitance of the EDLC is linear, the voltage linearly increases as illustrated by a broken line. In a practical circuit, the slope of the voltage rise slightly decreases, i.e. the voltage waveform is convex upward, even if the EDLC is charged by a constant current. This characteristic could be expressed by a frequency dependent impedance of the EDLC.

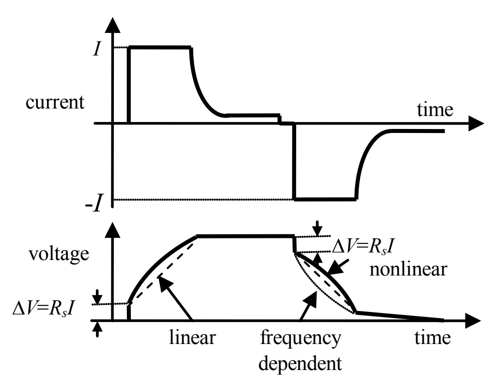

Fig. 1 A schematic diagram of voltage and current waveform.

Fig. 2 illustrates a conventional EDLC model, which is able to express the charging characteristic. The resistor R_s expresses the sudden voltage change ΔV and the electrical charge is mainly stored into the capacitor C_p. The voltage increase trend is expressed by the resistor R_i and capacitor C_i in Fig. 2. The resistor R_p expresses a leakage current.

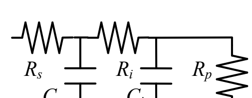

Fig. 2 A conventional equivalent circuit of EDLC.

The conventional model shown in Fig. 2 is a kind of a low-pass filter. If the terminal voltage waveform is

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determined by the frequency characteristic, the falling edge becomes a convex downward waveform. The waveform observed across a practical EDLC is, however, convex upward as shown in Fig. 1. The conventional model shown in Fig. 2 cannot express the voltage waveform. It can be assumed that the characteristic of the voltage waveform is due to a nonlinear characteristic of the EDLC. In this paper, a nonlinear model illustrated in Fig. 3 is proposed. The circuit parameter of the equivalent circuit is determined by a measured result as described in the next chapter.

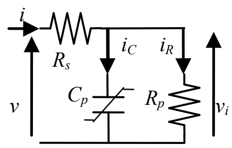
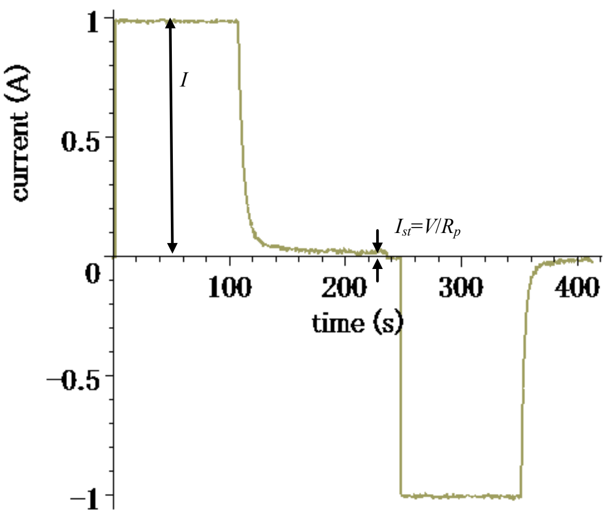
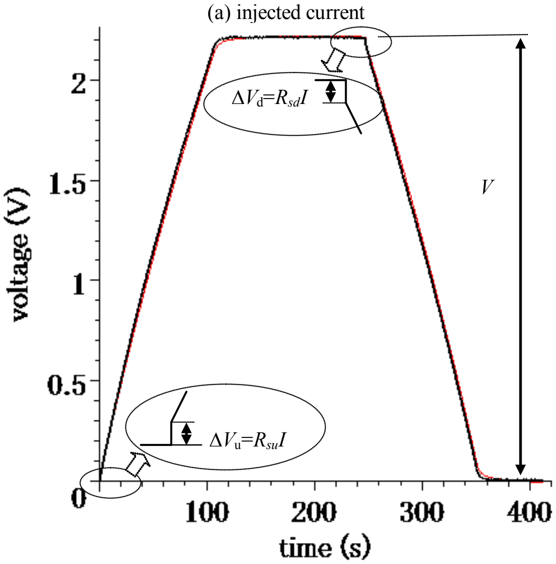


Fig. 3 Nonlinear equivalent circuit of EDLC.



(a) injected current



(b) terminal voltage

Fig. 4 Measured and calculated waveforms. (a) for a measured current waveform whose amplitude is *I*, (b) for measured and calculated voltage waveform across the EDLC. (bold line: measured result, thin line: calculated result when the current (a) is injected into the nonlinear equivalent circuit (Fig. 6)).

III. Circuit Parameters of Nonlinear EDLC Model

A. Voltage and Current Characteristics

A measurement of a charging/discharging characteristic is carried out using a bipolar current source with a voltage limiter which is indispensable to keep the terminal voltage of the EDLC below its maximum voltage. Fig. 4 (a) illustrates a current waveform injected into an EDLC whose capacitance is 50 F (EECHW0D506, manufactured by Panasonic Corp.).

A constant current of 1 A (=*I*) is injected at first, and a constant voltage is applied after the terminal voltage reaches to 2.2 V (=*V*) as shown in Fig. 4 (b). The current rapidly decreases from the constant charging current of 1 A. The current, however, remains after 100 s from the operation mode enters into the constant voltage mode. The electric charges in the EDLC are discharged by a constant current of 1 A (=-*I*) after the constant voltage charging.

B. Series Resistance

The voltage jumps up by ΔV_u just after a step-like current is injected into the EDLC as shown in Fig. 4 (b). The ratio of the voltage rise and the current expresses the series resistance of the EDLC (R_{su}). The sudden voltage change of ΔV_d at the time of the discharging by a step-like current also expresses the series resistance R_{sd}. The series resistances can be evaluated by the following equations.

$$R_{su} = \frac{\Delta V_u}{I} = 27.3[\mathrm{m\Omega}], \quad R_{sd} = \frac{\Delta V_d}{I} = 29.7[\mathrm{m\Omega}] \qquad (1)$$

The small difference between the resistances obtained at the charging and discharging indicates that the series resistance is independent from the current direction. Their averaged value is adopted as a series resistance of the EDLC in this paper.

$$R_s = \frac{R_{su} + R_{sd}}{2} = \frac{27.3 + 29.7}{2} = 28.5[\mathrm{m\Omega}] \qquad (2)$$

C. Nonlinear Capacitance

The simplified equivalent circuit shown in Fig. 3 is adopted in this paper. In this figure, the capacitor C_p is assumed to be a nonlinear capacitor whose capacitance is determined by the voltage across the capacitor, i.e. the internal voltage v_i.

The internal voltage $v_i(t)$ is calculated from the measured terminal voltage $v(t)$, injected current $i(t)$ and the internal resistance R_s.

$$v_i(t) = v(t) - R_s i(t) \qquad (3)$$

In general, a nonlinear parameter estimation method is required to determine the resistance R_p and the nonlinear characteristic of the capacitor C_p. It is however, difficult to keep its numerical stability. In this paper, the loss resistance R_p is assumed to be linear. The linear resistance R_p can be determined from the measured results by three methods: (1) steady state current when a constant voltage is applied, (2) circuit loss, (3) self discharging characteristic. In this paper, the first and the second methods are employed because the last method requires a long duration measurement.

From the averaged steady state current $\overline{I_{st}}$ and voltage $\overline{V_{st}}$, the resistance R_p can be obtained as shown in Fig. 4 (b).

$$R_p = \frac{\overline{V_{st}}}{\overline{I_{st}}} = \frac{2.21}{0.0198} = 112[\Omega] \tag{4}$$

The total injected charge $q(T_{\max})$ is obtained as follows:

$$\begin{aligned} q(T_{\max}) &= q_C(T_{\max}) + q_R(T_{\max}) = \int_0^{T_{\max}} i(t)\,dt \\ q_C(T_{\max}) &= \int_0^{T_{\max}} i_C(t)\,dt \\ q_R(T_{\max}) &= \int_0^{T_{\max}} i_R(t)\,dt \end{aligned} \tag{5}$$

where $T_{\max}$ denotes the maximum observation time, and the EDLC is assumed to be perfectly discharged at that time. The currents flowing into capacitor C_p and resistor R_p are expressed by i_C and i_R, respectively.

If the capacitor is completely discharged, the charge q_C becomes:

$$q_C(T_{\max}) = 0 \tag{6}$$

The resistance R_p is obtained from the injected charge $q(T_{\max})$.

$$q(T_{\max}) = q_R(T_{\max}) = \int_0^{T_{\max}} i_R(t)\,dt = \int_0^{T_{\max}} \frac{v_i(t)}{R_p}\,dt \tag{7}$$

$$R_p = \frac{\int_0^{T_{\max}} v_i(t)\,dt}{q(T_{\max})} = \frac{\int_0^{T_{\max}} v(t) - R_s i(t)\,dt}{\int_0^{T_{\max}} i(t)\,dt} = 124[\Omega] \tag{8}$$

The difference between the resistances obtained by the two methods is only 5%. In this paper the average of the both results is used as the leakage resistance R_p (= (112+124)/2 = 118 Ω).

The offset (DC level) included in the measured voltage or current causes significant error on the integration calculations in the above equations. To obtain the offsets of the voltage and current, the data before the current injection are averaged.

A charge vs. voltage (Q-V) curve is an essential characteristic for a numerical simulation of a capacitor. Fig. 5 illustrates the Q-V curves. The injected charge $q(t)$ is obtained by a numerical integration of the injected current. Because the series resistance R_s is already known, the Q-V curve for internal voltage (q_C-v_i) can be illustrated. The capacitor current $i_C(t)$ is obtained from the measured current $i(t)$, internal voltage $v_i(t)$ and the resistance R_p.

$$i_C(t) = i(t) - i_R(t) = i(t) - \frac{v_i(t)}{R_p} = i(t) - \frac{v(t) - R_s i(t)}{R_p} \tag{9}$$

The charges stored in the capacitor q_C are obtained as an integration of the capacitor current $i_C(t)$. The q_C-v_i characteristic is illustrated in Fig. 5 by a bold line. The characteristic shows a minor hysteresis. It indicates further modification is required for a more accurate simulation. In this paper, this minor hysteresis is ignored. The q_C-v_i characteristic can be approximated by a quadratic function.

$$q_C = a_1 v_i + a_2 v_i^2 \tag{10}$$

The coefficients for the EDLC are obtained by a least square method.

$$a_1 = 39.9[\text{F}], \quad a_2 = 4.31[\text{F/V}] \tag{11}$$

The nonlinear characteristic of the capacitance is expressed by:

$$\frac{dq}{dv_i} = C(v_i) = a_1 + 2a_2 v_i \tag{12}$$

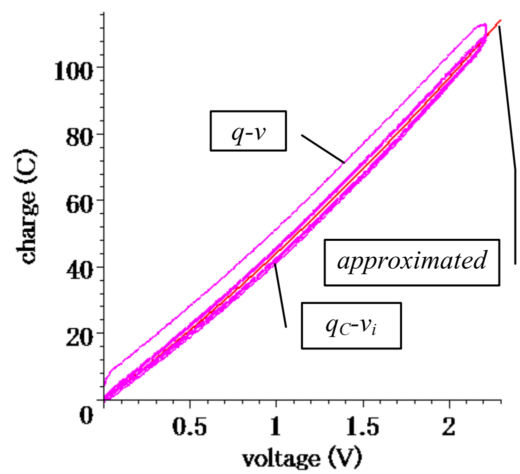


Fig. 5 Q-V curve for EDLC

D. Model for EMTP

Fig. 6 illustrates a nonlinear equivalent circuit for the EMTP. Because the EMTP has no nonlinear capacitor model, the nonlinearity of the capacitance is expressed by an equivalent voltage source E_p controlled by the TACS according to eq. (13).

$$\begin{aligned} E_p(t) &= \frac{-a_1 + \sqrt{a_1^2 + 4a_2 q_C(t)}}{2a_2} = \frac{-a_1 + \sqrt{a_1^2 + 4a_2 \int_0^t i_C(t)\,dt}}{2a_2} \\ &\approx \frac{-a_1 + \sqrt{a_1^2 + 4a_2 \int_0^{t-\Delta t} i_C(t)\,dt}}{2a_2} \end{aligned} \tag{13}$$

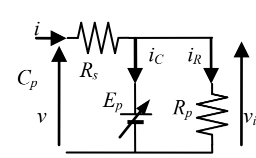


Fig. 6 Nonlinear equivalent circuit for EMTP simulation.

Fig. 4 (b) illustrates a calculated voltage using the nonlinear model by a thin line. The calculated terminal voltage shows a good agreement with the measured result.

IV. Simulation Results

A. Power Source with DC-DC Converter and EDLCs

A voltage stabilizer is required for a usage of an EDLC as a power source of electrical equipments, because the voltage across EDLC is almost proportional to the charges stored in the EDLC. This characteristic is the major difference between a battery and an EDLC. A switching converter is a practical solution for compensating the voltage fluctuation at the present time. Especially for a power source of a motor drive system, a bidirectional DC-DC converter is suitable from a viewpoint of efficient energy usage of regenerative power.

A power source is developed as an alternative battery whose rated voltage is 24 V (=V). It consists of 10 series connected EDLC cells and a bidirectional DC-DC converter. The rated voltage of the EDLC cell is 2.3 V and its capacity of 50 F. Fig. 7 illustrates the circuit diagram of the power source. Its output voltage V is stabilized by a controller using a microcomputer (PIC® microcontroller manufactured by Microchip Technology Inc.)

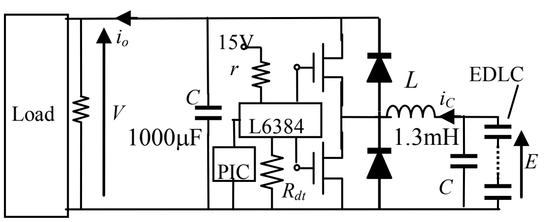


Fig. 7 A circuit diagram of a bidirectional DC-DC converter.

B. Model for Series Connected EDLCs

The equivalent circuit of the EDLC unit for n-series connected cells is obtained from Fig. 6. The series resistance R_{ns} and the parallel resistance R_{np} becomes:

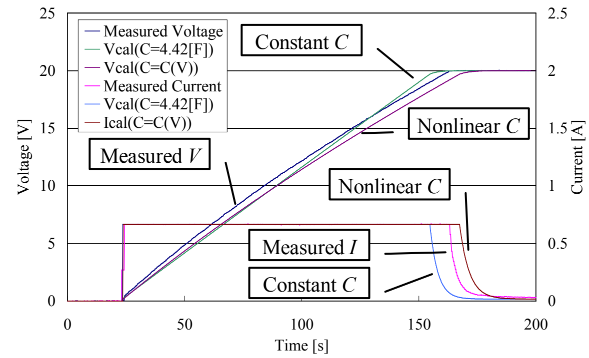
$$R_{ns} = nR_s, \quad R_{np} = nR_p \tag{14}$$

The nonlinear characteristic is obtained from the characteristic of the single cell (eq. (10)).

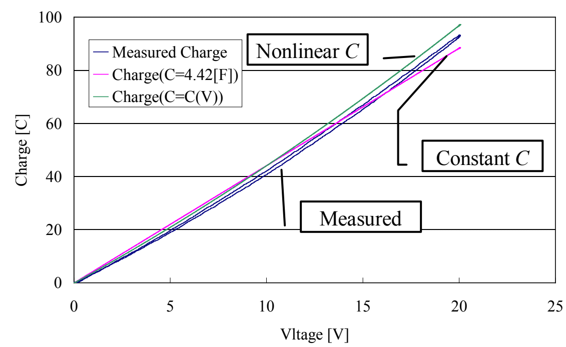
$$q_{nC} = a_1 \frac{v_{ni}}{n} + a_2 \left(\frac{v_{ni}}{n} \right)^2 = \frac{a_1}{n} v_{ni} + \frac{a_2}{n^2} v_{ni}^2 = a_{n1} v_{ni} + a_{n2} v_{ni}^2 \tag{15}$$

$$\begin{aligned} a_{n1} &= \frac{a_1}{n} = \frac{39.9}{10} = 3.99[\mathrm{F}] \\ a_{n2} &= \frac{a_2}{n^2} = \frac{4.31}{10^2} = 0.0431[\mathrm{F/V}] \end{aligned} \tag{16}$$

The numerical model for the series connected EDLCs is identical to the circuit illustrated in Fig. 6. Fig. 8 illustrates the measured and calculated voltage and Q-V characteristic of the EDLC unit.



(a) voltage characteristic



(b) Q-V characteristic

Fig. 8 Characteristics of 10-series connected cells when a constant current of 0.67A is injected.

The difference between the measured and calculated results comes from piece-to-piece variations of the EDLC cells. Fig. 9 illustrates the characteristics of another cell. The model obtained in the previous chapter shows a minor error and it causes the difference shown in Fig. 8. If the parameters of the nonlinear model for the series connected cells are obtained from the total voltage and current characteristic, the error will be minimized.

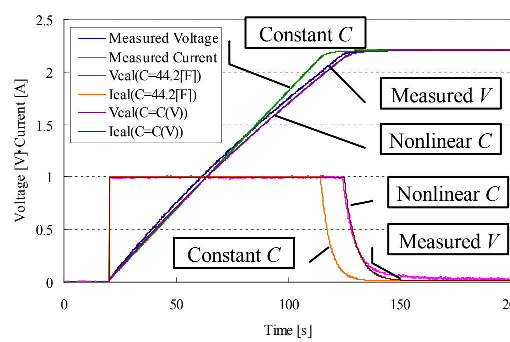
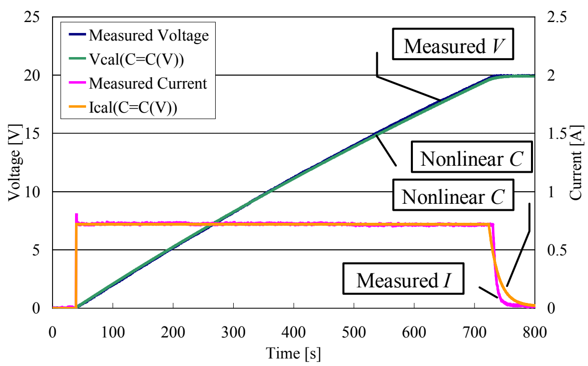


Fig. 9 Characteristics of another cell when a constant current of 1 A is injected.

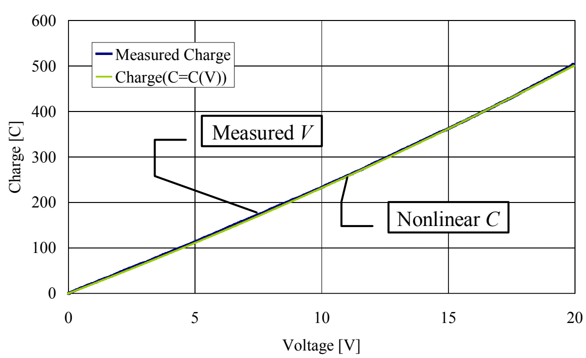
The capacitance at the internal voltage v_i=10 V which is the average voltage of the EDLC unit becomes 4.42 F(=3.99+0.431). The calculated characteristics for the linear (constant parameter) model are also shown in the figure. The characteristics of the nonlinear numerical model gives a better result compared with that by the linear model.

C. Accuracy for high-power density EDLC

Recently, some high power-density EDLCs for power equipments are developed. Their working voltage is higher than that of the conventional EDLC cell. Fig. 10 (a) illustrates the voltage and current characteristic and Fig. 10 (b) shows its *Q*-*V* characteristic. The results show that the proposed model and modeling method is applicable to the high-voltage EDLCs.



(a) voltage characteristic



(b) *Q*-*V* characteristic

Fig. 10 Characteristics of an EDLC whose rated voltage is 64 V and capacitance is 34.5 F.

D. Simulation Results of Electrical Wheel Chair

The EDLC unit consisting of 10-series connected cells is applied to a power source of an electrical wheel chair in this paper. The capacity of the power unit is smaller than that of the practical unit for a confirmation of the proposed numerical model.

Fig. 11 shows the voltage and current characteristic of an electrical wheel chair powered by the EDLC unit with a bidirectional DC-DC converter. The high frequency oscillation on the current waveform is caused by the switching of the DC-DC converter. The calculated voltage agrees well with the measured result. If a constant capacitance of 4.42 F is assumed, the error of the capacitor voltage at the maximum observation time becomes 15 %. The result indicates that the nonlinearity should be taken into account to the numerical simulation of the practical design of the electrical equipment. The result obtained by the nonlinear model without the parallel resistor R_p gives almost identical to that by the model with the resistor R_p. The results show the accuracy of the nonlinear model heavily depends on the nonlinear parameter of the capacitance, and the leakage resistor has minor effect on the simulated result.

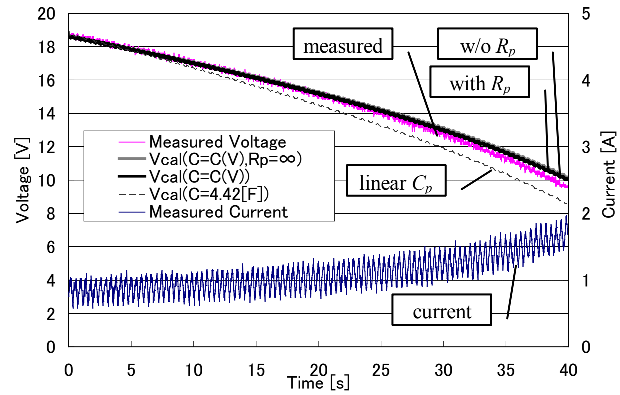
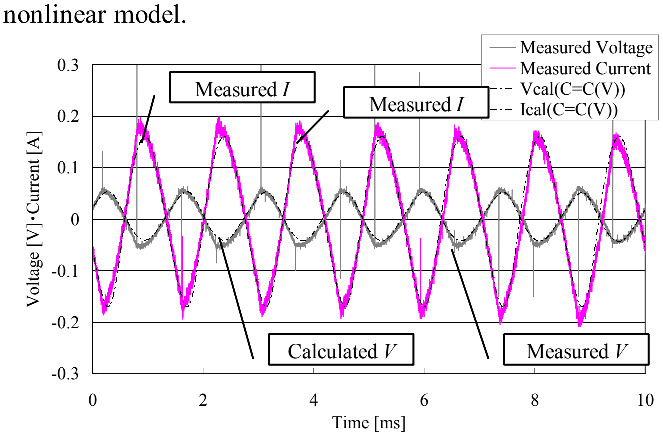
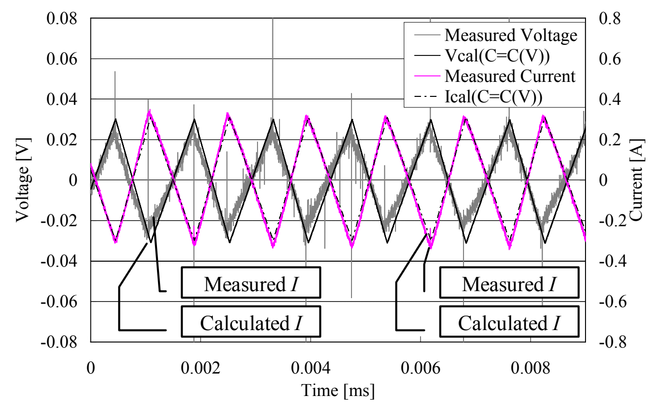


Fig. 11 Calculated EDLC voltages for an electrical wheel chair.

Fig. 12 illustrates the voltage and current fluctuation of the EDLC unit for the electrical wheel chair. The voltage ripple is reproduced with enough accuracy by the proposed nonlinear model.



(a) 10-series connected EDLC unit (50 F×10)



(b) high power-density EDLC unit (64 V, 34.5 F)

Fig. 12 Voltage fluctuation of an EDLC unit for an electrical wheel chair.

V. Conclusions

This paper has proposed a nonlinear equivalent circuit of EDLCs for the EMTP and its parameter estimating method. The proposed model is applicable to a high-power density

EDLCs as well as a small capacity EDLCs. The nonlinear characteristic is based on the *Q*-*V* curve instead of a *C*-*V* curve which is employed by the existing nonlinear model. The measurement of the *Q*-*V* curve is more stable than that of the *C*-*V* curve, because the measurement based on integration rather than differentiation. The accuracy of the proposed model is confirmed to be reasonable compared with measured results.

VI. References

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