Derivation of Theoretical Formulas of Sequence Currents on Underground Cable System

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Abstract-- A reliable operation of a cable system necessitates an accurate calculation of sequence impedances of the system. It is recently even more important as longer cable lines are being constructed and planned. It has been a common practice that these sequence impedances or currents are measured after the installation and it is difficult to predict these values beforehand with good accuracy. This paper derives theoretical formulas of the sequence currents for a cross-bonded cable and a normalbonded cable. The formulas give an important advantage in setting up transient overvoltage studies as well as planning studies. The accuracy of the proposed formulas is verified through a comparison with EMTP simulations.

Keywords: cable, sequence current, sequence impedance.

I. INTRODUCTION

In recent years, many cable systems are under construction and are planned for construction. For example, 400 kV underground systems whose total length will reach 400 km will be constructed in Denmark [1]. Since 1995, all new transmission and distribution lines in Denmark at 100 kV or below have been mandated to be undergrounded. In November, 2008, the Danish Parliament additionally decided to underground all new transmission lines up to 400 kV. Total circuit length undergrounded at 132 kV or higher will be expected to reach 3000 km. In Italy, 38 km 380 kV cable from the main land to Sicily is under construction [2][3].

Power flow calculations, transient stability analyses, and relay setting studies are performed for the planning and operation of the network. These analyses and studies require sequence impedances or currents as input data. The sequence impedance / current calculation of overhead lines is well known and introduced in textbooks [4]. For underground cables, theoretical formulas are proposed for the cable itself [5-8]. In order to derive accurate theoretical formulas, however, it is necessary to consider the whole cable system, including sheath bonding, since the return current of an underground cable flows through both metallic sheath and ground. Until now, there has existed no formula of the sequence impedances / currents which can consider sheath bonding and sheath grounding resistance at substations and normal joints. As a result, it has been a common practice that those sequence impedances or currents are measured after the installation, and it is considered difficult to predict those values beforehand.

This paper derives theoretical formulas of the sequence currents for a cross-bonded cable and a normal-bonded cable. The formulas are, of course, useful for planning studies. In addition, they will help improve the initial condition setup for transient overvoltage studies. The accuracy of the proposed formulas is verified through a comparison with EMTP simulations.

II. DERIVATION OF THEORETICAL FORMULAS OF SEQUENCE CURRENTS

For underground cable systems which are longer than about 2 km, it is a common practice to cross-bond the metallic sheaths of three phase cables to reduce sheath currents and to suppress sheath voltages at the same time [9]. Due to the increase of off-shore wind farms and cross-border transactions, submarine cables, which are generally normal-bonded, are now becoming a popular type of cable.

Therefore, this paper derives theoretical formulas of the sequence currents for a majority of underground cable systems, that is, a cross-bonded cable which has more than a couple of major sections. It also derives theoretical formulas for a normal-bonded cable, considering the increased use of submarine cables.

A. Cross-bonded Cable

1) 6×6 impedance matrix

One cable system corresponds to 6 conductor system composed of 3 cores and 3 metallic sheaths. The 6×6 impedance matrix of the cable system is given by the following equation [4].

$$\begin{bmatrix} Z \end{bmatrix} = \begin{bmatrix} Zc \\ [Zm]^{t} & [Zs] \end{bmatrix} = \begin{bmatrix} Zc \\ [Zm] & [Zs] \end{bmatrix}$$

$$\begin{bmatrix} Zaa \ Zab \ Zac \end{bmatrix}^{c} \qquad \begin{bmatrix} Zaa \ Zab \ Zac \end{bmatrix}^{s}$$
(1)

$$\begin{bmatrix} Zc \end{bmatrix} = \begin{bmatrix} Zab & Zbb & Zab \\ Zac & Zab & Zcc \end{bmatrix}, \begin{bmatrix} Zs \end{bmatrix} = \begin{bmatrix} Zab & Zbb & Zab \\ Zac & Zab & Zcc \end{bmatrix}$$
$$\begin{bmatrix} Zm \end{bmatrix} = \begin{bmatrix} Zaa & Zab & Zac \\ Zab & Zbb & Zab \\ Zac & Zab & Zcc \end{bmatrix}^{m}$$

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Fig. 1 Cross-bonded cable and its equivalent model.

where *c*: core, *s*: sheath, *m*: mutual coupling between core and sheath, *t*: transpose

In (1), cable phase a is assumed to be laid symmetrical to phase c against phase b. The flat configuration and the trefoil configuration, which are typically adopted, satisfy this assumption.

2) 4×4 reduced impedance matrix [10][11]

The lengths of minor sections can have imbalances due to the constraint on the location of joints. The imbalances are designed to be as small as possible since they increases sheath currents and raises sheath voltages. When a cable system has multiple major sections, the overall balance is considered to minimize sheath currents. As a result, when a cable system has more than a couple of major sections, sheath currents are generally balanced among 3 conductors, which allows us to reduce 3 metallic sheaths to one conductor.

Reducing the sheath conductors, the 6 conductor system is reduced to the 4 conductor system composed of 3 cores and 1 equivalent metallic sheath as shown in Fig. 1. The 4×4 reduced impedance matrix can be expressed as

$$\begin{bmatrix} Z' \end{bmatrix} = \begin{bmatrix} Zaa & Zab & Zac & Zsa \\ Zab & Zbb & Zab & Zsb \\ Zac & Zab & Zaa & Zsa \\ Zsa & Zsb & Zsa & Zss \end{bmatrix}$$
(2)

Here, Z'(4, j) = Z'(j, 4) can be calculated from the 6×6 impedance matrix *Z* as

$$Z'(4,j) = \frac{1}{3} \sum_{i=4}^{6} Z(i,j); \quad j = 1 \cdots 4$$
(3)

3) Zero sequence current

The following equations are derived from Fig. 2. Here, the sheath grounding at normal joints are ignored.

$$(V_1) = [Z'](I_1)$$
where $(V_1) = (E \ E \ V_S)^t, (I_1) = (Ia \ Ib \ Ic \ Is)^t$ (5)

Fig. 2(a) shows zero-sequence current measurement circuit for a cross-bonded cable.



Fig. 2 Setup for measuring sequence currents for a cross-bonded cable.

Assuming grounding resistance at substations Rg, the sheath voltage Vs can be found by

$$Vs = -2RgIs \tag{6}$$

Since Zsa = Zsc stands in the flat configuration and the trefoil configuration, the following equations can be obtained by solving (4) to (6).

$$Ia = Ic = (Z_{22} - Z_{12})E/\Delta_0$$

$$Ib = (Z_{11} - Z_{21})E/\Delta_0$$
where
(7)

$$\Delta_0 = Z_{11}Z_{22} - Z_{12}Z_{21}$$

$$Z_{11} = Zaa + Zac - 2Zsa^2 / Z'ss$$

$$Z_{22} = Zbb - Zsb^2 / Z'ss$$

$$Z_{12} = Zab - Zsa Zsb / Z'ss, \quad Z_{21} = 2Z_{12}$$

$$Z'ss = Zss + 2Rg$$

Zero-sequence current can be found from (7) by the following equation. $I_0 = (2Ia + Ib)/3$

$$=\frac{E}{3\Delta_0}(Z_{11}+2Z_{22}-2Z_{12}-Z_{21})$$
(8)

When three phase cables are laid symmetrical to each other, the following equations are satisfied.

$$Z_{aa}^{c} = Z_{bb}^{c} = Zc, \quad Z_{aa}^{s} = Z_{bb}^{s} = Zs$$

$$Z_{ab}^{c} = Z_{ac}^{c} = Zm, \quad Zsa = Zsb = Zn$$
(9)

Using symmetrical impedances Zc, Zm, and Zn in (9), Z_{11} , Z_{22} , and Z_{12} can be expressed as

$$Z_{11} = Zc + Zm - 2Zn^2 / Z'ss$$

$$Z_{22} = Zc - Zn^2 / Z'ss$$
(10)

 $Z_{12} = Zm - Zn^2 / Z'ss$

Substituting Z_{11} , Z_{22} , and Z_{12} in (7) and (8) by symmetrical impedances,

$$Ia = Ib = Ic \approx E / \Delta_1 , \quad Is \approx -3ZnE / Z'ss\Delta_1$$

$$I_0 \approx E / \Delta_1$$
(11)

where $\Delta_1 = Zc + 2Zm - 3Zn^2 / Z'ss$

4) Positive sequence current

In Fig. 2(b), the equation Isa + Isb + Isc = 0 is satisfied at the end of the cable line. The following equations are obtained since Vs = 0.

$$:: \begin{pmatrix} Ia \\ Ib \\ Ic \end{pmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{12} & Z_{22} & Z_{12} \\ Z_{13} & Z_{12} & Z_{11} \end{bmatrix}^{-1} \begin{pmatrix} E \\ \alpha^{2}E \\ \alpha E \end{pmatrix}$$

$$= \frac{1}{\Delta} \begin{bmatrix} Z_{11}Z_{22} - Z_{12}^{2} & Z_{12}(Z_{13} - Z_{11}) & Z_{12}^{2} - Z_{13}Z_{22} \\ Z_{12}(Z_{13} - Z_{11}) & Z_{11}^{2} - Z_{13}^{2} & Z_{12}(Z_{13} - Z_{11}) \\ Z_{12}^{2} - Z_{13}Z_{22} & Z_{12}(Z_{13} - Z_{11}) & Z_{11}Z_{22} - Z_{12}^{2} \end{bmatrix} \begin{pmatrix} E \\ \alpha^{2}E \\ \alpha E \end{pmatrix}$$
(13) Here.

$$Z_{11} = Zaa - Zsa^{2} / Zss$$
$$Z_{22} = Zbb - Zsb^{2} / Zss$$
$$Z_{12} = Zab - Zsa Zsb / Zss$$
$$Z_{13} = Zab - Zsa Zsc / Zss$$

Positive sequence current is derived from (13).

$$I_{1} = \frac{1}{3}(Ia + \alpha Ib + \alpha^{2}Ic)$$

$$= \frac{E}{3\Delta_{2}}\{(Z_{11} - Z_{13})(Z_{11} + Z_{13} + 2Z_{12})$$

$$+ Z_{22}(2Z_{11} + Z_{13}) - 3Z_{12}^{2}\}$$
where $\Delta_{2} = (Z_{11} - Z_{13})\{Z_{22}(Z_{11} + Z_{13}) - 2Z_{12}^{2}\}$
(14)

When three phase cables are laid symmetrical to each other, (14) can be simplified using (9).

$$I_1 = \frac{E}{Zc - Zm} \tag{15}$$

B. Normal-bonded cable

1) 6×6 impedance matrix

Fig. 3 shows sequence current measurement circuit for a normal-bonded cable. The following equations are given from the 6×6 impedance matrix in (1) and Fig. 3.

$$(E) = [Zc](I) + [Zm](Is)$$
⁽¹⁶⁾

$$(V_s) = [Zm](I) + [Z_s](I_s) = -2[R_g](I_s)$$
(17)

Here,
$$(I) = (Ia \quad Ib \quad Ia)^t$$
: core current
 $(Is) = (Isa \quad Isb \quad Isa)^t$: sheath current

$$\begin{bmatrix} Rg \end{bmatrix} = Rg \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

From (17), sheath current (Is) is found by

$$(Is) = -([Zs] + 2[Rg])^{-1}[Zm](I)$$
⁽¹⁸⁾

Eliminating sheath current (Is) in (16), core current (I)can be derived as

$$(I) = ([Z_c] - [Z_m]([Z_s] + 2[R_g])^{-1} [Z_m])^{-1} (E)$$
(19)



Fig. 3 Setup for measuring sequence currents for a normal-bonded cable.

2) Zero-sequence current

From Fig. 3(a), (E) and (I) are expressed as

 $(E) = [E \ E \ E]^t, \quad (I) = [Ia \ Ib \ Ia]^t$ (20) Core current (I) is obtained from (19) and (20), and then

zero sequence current is calculated as $I_0 = (Ia + Ib + Ic)/3$.

Since the relationship $[Zn] \approx [Zs]$ stands for a cable with a general circular shape, (16) and (17) can be simplified to (21) using (9) when three phase cables are laid symmetrical to each other and resistance of the cables are small.

$$((E) - (Vs)) = ([Zc] - [Zs])(I)$$

$$= (Zc - Zs)[U](I)$$
where $[U]: 3 \times 3$ unit (identity) matrix
$$(21)$$

Hence.

1

$$I_0 = Ia = Ib = Ic$$

$$=\frac{1}{Zc-Zs}(E-Vs)$$
(22)

(22)

Using (22), core current (I) in (17) can be eliminated, which yields (23).

$$(V_s) = \frac{1}{Z_c - Z_s} [Z_m]((E) - (V_s)) - [Z_m](I_s)$$
(23)

Adding all three rows in (23),

$$3Vs = 3\frac{Zs + 2Zm}{Zc - Zs} (E - Vs) - \frac{Zs + 2Zm}{2Rg} Vs$$
⁽²⁴⁾

Solving (24) for *Vs* and eliminating *Vs* from (22), zero sequence current is found as

$$I_{0} = \frac{6Rg + Zs + 2Zm}{6Rg(Zc + 2Zm) + (Zc - Zs)(Zs + 2Zm)}E$$
(25)

3) Positive-sequence current

From Fig. 3(b), (E) and (I) are expressed as

$$(E) = \begin{bmatrix} E \ \alpha^2 E \ \alpha E \end{bmatrix}^t, \quad (I) = \begin{bmatrix} Ia \ Ib \ Ia \end{bmatrix}^t$$
(26)
Core current (I) is obtained from (19) and (26). Once core

core current (1) is obtained from (19) and (20). Once core current is found, positive sequence current can be calculated as $I_1 = (Ia + \alpha Ib + \alpha^2 Ic)/3$.

The theoretical formula of positive sequence current can also be simplified using (21).

$$I_{1} = \frac{1}{3(Zc - Zs)} \left\{ (E - Vs) + \alpha \left(\alpha^{2} E - Vs \right) + \alpha^{2} (\alpha E - Vs) \right\}$$
$$= \frac{E}{Zc - Zs}$$
(27)

Equation (27) shows that positive sequence current can be approximated by coaxial mode current. It also shows that, similarly to a cross-bonded cable, positive sequence current is not affected by substation grounding resistance R_g .

III. COMPARISON WITH EMTP SIMULATIONS

A comparison with EMTP simulations are conducted in order to verify the accuracy of theoretical formulas derived in the previous chapter.

Fig. 4 shows physical and electrical data of the 400 kV cable used for the comparison. An existence of semiconducting layers introduces an error in the charging capacity of the cable. Although deriving admittance is not a focus of this paper,, relative permittivity of the insulation (XLPE) is converted from 2.4 to 2.729 according to (28) in order to correct the error and have a reasonable cable model.



$$\begin{split} R2 &= 3.26 \text{cm}, R3 = 6.14 \text{cm}, R4 = 6.26 \text{cm}, R5 = 6.73 \text{cm} \\ \text{Core inner radius: 0.0 \text{cm}, Core resistivity: } 1.724 \times 10^{-8} \Omega \text{m}, \\ \text{Metallic sheath resistivity: } 2.840 \times 10^{-8} \Omega \text{m}, \\ \text{Relative permittivity (XLPE, PE): } 2.4 \end{split}$$

Fig. 4 Physical and electrical data of the cable.

$$\varepsilon_{\rm r}' = \frac{\ln(R3/R2)}{\ln(Rso/Rsi)} \varepsilon_{\rm r} = \frac{\ln(61.40/32.60)}{\ln(59.50/34.10)} \cdot 2.4 = 2.729$$
(28)

where Rsi: inner radius of the insulation, Rso: outer radius of the insulation

Fig. 5 shows the layout of the cables. It is assumed that the cables are directly buried at the depth of 1.3 m with the separation of 0.5 m between phases.



Fig. 5 Layout of the cable.

The lengths of a minor section and a major section are respectively set to 400 m and 1200 m. The total length of the cable is set as 12 km with 10 major sections.

Table I shows zero and positive sequence currents derived by proposed formulas and EMTP simulations. The 6×6 impedance matrix Z is found by CABLE CONSTANTS in both cases [12-14]. Calculation process in case of a crossbonded cable using proposed formulas is shown in Appendix.

In the calculations, the applied voltage is set to $E = 1 \text{ kV} / \sqrt{3}$ (angle: 0 degree) and the source impedance is not considered. In this paper, sequence currents are derived in accordance with the setups for measuring sequence currents in Fig. 2 and Fig. 3. The assumptions on the applied voltage and the source impedance match a condition in actual setups for measuring sequence currents since testing sets are generally used in the measurements. Grounding resistances at substations and normal joints are set to 1Ω and 10Ω , respectively.

TABLE I COMPARISON OF PROPOSED FORMULAS WITH EMTP SIMULATIONS (a) Cross-bonded cable

	Zero Sequence		Positive Sequence		
	Amplitude	Angle	Amplitude	Angle	
	[A]	[deg]	[A]	[deg]	
EMTP	133.8	-21.42	356.4	-86.35	
Simulation					
Proposed					
formulas, eq.	124.1	-21.23	356.7	-86.79	
(8)/(14)					
(b) Normal-bonded cable					
	Zero Sequence		Positive Sequence		
	Amplitude	Angle	Amplitude	Angle	
	[A]	[deg]	[A]	[deg]	
EMTP	121.6	-21.80	694.9	-50.40	
Simulation					
Proposed					
formulas, eq.	124.8	-22.50	722.7	-49.08	
(19), (20)/(26)					

From the results in Table I, it is confirmed that the proposed formulas have satisfactory accuracy for planning and implementation studies, compared to the results of EMTP simulations. An error of 7 % is observed in zero sequence current of a cross-bonded cable. It is caused by the impedance matrix reduction [10][11] discussed in Section II. Due to the matrix reduction, unbalanced sheath currents that flow into earth at normal joints cannot be considered in proposed formulas. Thus, the error can be increased to 14 % when the grounding resistance at normal joints is decreased to 5 ohm. In contrast, the error becomes negligible when the cables are laid in a trefoil formation.

Table I shows that positive sequence impedance is smaller for a normal-bonded cable than for a cross-bonded cable as positive sequence current is larger for a normal-bonded cable. This is because return current flows only through metallic sheath of the same cable and earth in a normal-bonded cable whereas return current flows through metallic sheath of all three phase cables in a cross-bonded cable (Zc - Zm > Zc - Zs).

Phase angle of zero sequence current in Table I demonstrates that zero sequence current is highly affected by grounding resistance at substations in both cross-bonded and normal-bonded cables. As a result, there is little difference in zero sequence impedance between a cross-bonded cable and a normal-bonded cable. The result shows an importance of obtaining accurate grounding resistance at substations to derive accurate zero sequence impedances of cable systems.

IV. CONCLUSION

This paper has derived theoretical formulas of sequence currents of a cross-bonded cable and a normal-bonded cable. These formulas are obtained by solving equations which are derived from the setups for measuring sequence currents of cross-bonded and normal-bonded cables. For a cross-bonded cable, the equations are solved utilizing the known impedance matrix reduction technique. The derived formulas consider the cable as a cable system; they can thus consider sheath bonding and sheath grounding resistance.

In Section III, an accuracy of proposed formulas is verified through a comparison with EMTP simulation results. The verified accuracy of the proposed formulas shows sequence impedance / current can now be obtained before the installation without making measurements for a majority of cables. This gives an important advantage in setting up transient overvoltage studies as well as planning studies.

V. APPENDIX

First, the 4×4 reduced impedance matrix is found from the 6×6 impedance matrix *Z* by (2) and (3).

[Z'] (upper: I	R, lower: X, unit: 9	Ω)	
0.71646353	0.59136589	0.59136589	0.5913705
8.44986724	6.28523815	5.76261762	6.63566685
0.59136589	0.71646353	0.59136589	0.5913705
6.28523815	8.44986724	6.28523815	6.80987369
0.59136589	0.59136589	0.71646353	0.5913705
5.76261762	6.28523815	8.44986724	6.63566685
0.5913705	0.5913705	0.5913705	0.83438185
6.63566685	6.80987369	6.63566685	6.63485268

Zero Sequence Current

$\Delta_0 = -3.9605900 + j12.489448$
$Z_{11} = 4.0641578 + j 2.2224336$
$Z_{22} = 2.1959039 + j 2.1450759$
$Z_{12} = 2.0193420 + j 0.1372637$
$Z_{21} = 4.0386840 + j 0.2745275$
I_0 (rms) = 81.814700 - j31.778479

Positive Sequence Current

$$\begin{split} &\Delta_2 = -4.8574998 \text{ - } j1.8394591 \\ &Z_{11} = 0.3670612 \text{ + } j1.8221556 \\ &Z_{22} = 0.3801952 \text{ + } j1.4707768 \\ &Z_{12} = 0.2482475 \text{ - } j0.5159115 \\ &Z_{13} = 0.2419636 \text{ - } j0.8650940 \\ &I_1(\text{rms}) = 14.118637 \text{ - } j251.86277 \end{split}$$

VI. ACKNOWLEDGMENT

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