# A method for the evaluation of the Sea Return Impedance and application of the Stochastic Collocation Technique to take into account the uncertainty of input parameters

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*Abstract*-- **We present a method for the evaluation of the sea return impedance term to be employed in the construction of the per-unit-length parameter matrix which is required to describe submarine cables in Electromagnetic Transient software.**

**We evaluate the sea return impedance using the quasi Monte Carlo technique, and an adapted sequence of pseudo-random variables to get a faster convergence. In the calculation of the impedance integral we employ a change of variable in order to reduce its oscillatory behavior and allow it to converge.**

**We also introduce the stochastic collocation technique in order to analyze the impact of uncertainties on the knowledge of the input parameters. This technique is well adapted to numerical calculations; it gives results similar to those obtained with a standard Monte Carlo method, but requires a minor number of computations, some units, instead of the thousands needed with the Monte Carlo method; this is an advantage because the computation time with the latter method would be too long. Furthermore this technique is non intrusive and thus does not necessitate a modification of the function.**

**We show that for the evaluation of the sea return impedance a certain inaccuracy in the knowledge of the cables laying configuration and of the conductivities of the soil can be tolerated in some cases.**

## I. INTRODUCTION

The computation of electromagnetic transients with the aid of EMT-like software programs [*1*], requires accurate models for the propagation along lines and cables.

When the Transmission Line Approach is used, the ground conductor normally acts as the reference. As it is widely known, it is necessary in transient simulations to represent correctly the soil, as this has a strong impact on the propagation of current/voltage waveforms, even more so in underground cables.

In some cases it is sufficient to consider the soil as consisting of a single layer, so that, for underground cables, Pollaczek integral [*2*] and its approximated solutions [*3*] [*4*] [*5*] [*6*] can be utilized.

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In the case of submarine cables, though, the difference in the conductivity between the sea and the seabed is so big that the single layer approximation is not acceptable and a two-layer model needs to be used. Tsiamitros et Al. [*7*] and Lucca [*8*] propose a method, which fulfills this condition, although it is based on the calculation of a highly oscillatory integral, difficult to evaluate numerically. In this paper we show how to use for the calculation of that integral the quasi Monte Carlo method [*9*] which offers the advantage to be programmed easily in an EMT-like software program.

## II. THE SEA RETURN IMPEDANCE

### *A. Description of the integral*

For describing the usually called ground (or earth) return impedance we shall employ here the term "sea return impedance".

The following situation is assumed: two cables are laid at the interface sea/seabed at a depth of h, their mutual distance is $x_{12}$, their external radius R, the electromagnetic parameters $\mu_i$, $\varepsilon_i$, $\sigma_i$ (magnetic permeability, electric permittivity and conductivity) of each layer are as shown in the figure 1 below.

We use the formulation described in [*7*] and solved in [*8*] for a two-layer soil. The integral derived for the per unit length (p.u.l.) mutual impedance is the following:

$$Z'_{ground-mut} = \frac{j\omega\mu_0}{\pi}\int_0^{+\infty} f_{mut}(u)\,du, \qquad (1)$$

$$f_{mut}(u) = \frac{\alpha_1+\alpha_0+(\alpha_1-\alpha_0)e^{-2\alpha_1 h}}{(\alpha_1+\alpha_0)(\alpha_1+\alpha_2)-(\alpha_1-\alpha_0)(\alpha_1-\alpha_2)e^{-2\alpha_1 h}}\cos(ux_{12})$$

(2)

where $\alpha_i = \sqrt{u^2+\gamma_i^2}$ and $\gamma_i^2 = j\omega\mu_0(\sigma_i + j\omega\varepsilon_i)$ .

We have shown here a configuration (Fig.1) that is of interest for submarine cables, but the solution described in the next section could be also applied to different soil configurations.

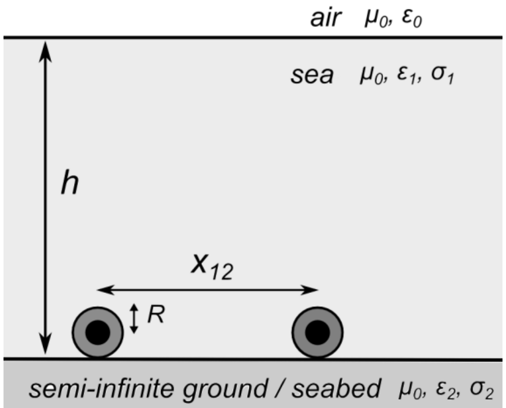

Fig. 1 - The two-layered "ground" formed by the sea water and the seabed

### B. The quasi Monte Carlo as a solution method

The quasi Monte Carlo method, described accurately in [*9*], starts from the following equation in order to solve an integral:

$$\int_0^1 f(x)dx \approx \frac{1}{N}\sum_{n=1}^{N} f(\lambda_n) \tag{3}$$

where $N$ is a large integer number and $\lambda_n$ are independent pseudo-random variables.

The use of pseudo-random variables, and more specifically here the van der Corput suite [*10*], allows for a faster convergence of the numerical sum. That said, we need to transform (1) into an integral bounded between 0 and 1; in [*9*] the integral was split in two integrals and the second, bound between *1* and $+\infty$, was transformed using the transformation $u=1/v$.

Due to the different oscillating nature of the sea return integral we chose instead

$$v=\exp(-u)\ \ ,\ u=-\log(v)\ ,\ du=-\frac{1}{v}dv. \tag{4}$$

This different change of variable acts as a filter on integral (1) allowing for the convergence of (3).

The mutual impedance is then obtained numerically as:

$$Z'_{ground-mut}=\frac{j\omega\mu_0}{\pi}\frac{1}{N}\sum_{i=1}^{n} f_{mut}(u=-\log(\lambda_i))\frac{1}{\lambda_i}. \tag{5}$$

The results given by this formula have been successfully compared with those obtained using the commercial software MathCad [*11*].

Furthermore, if the integral is calculated using the same electrical parameters for sea and seabed layers, the results obtained well agree with the Pollaczek formulation solved as in [*9*].

The two-layer formulation, and for that matter the present work, derive their justification from the fact that, even when the two layers have very different conductivities (as is the case of sea water and soil), the classical Pollaczek formulation which ignores the lesser conductive layer, when compared with the one proposed here, results in considerable errors in the evaluation of both components of the overall impedance, as can be seen from the example in Fig.2.

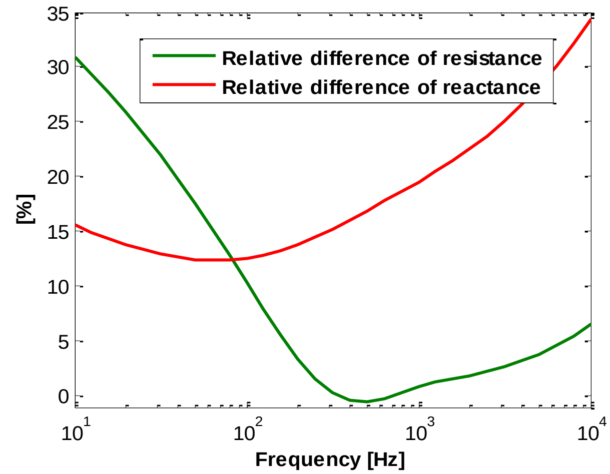

Fig. 2 – Relative differences resulting in the configuration as in Fig.1 with $h$=25 m and $x_{12}$=1 *m* from the use of Pollaczek integral with $\sigma$ =3 S/m, when compared with the two-layer formulation with $\sigma_1$=3 S/m and $\sigma_2$= 0.005 S/m.

## III. The Stochastic Collocation Method

In order to take into account the variability of the configuration parameters, we propose to use the stochastic collocation (SC) method, previously validated on electromagnetism problems [*12*] [*13*]. The choice of this method to solve this problem is due to the following reasons. First, it is non intrusive whence the advantage to use deterministic software. Further, as we shall see later in the paper, it is rather simple and rapid providing good results when compared with the Monte Carlo one [*14*]. The aim here is to determine the first statistical moments (average and standard deviation) of sea return impedance. In this section, the general SC method is detailed for a single random variable but the formalism can be extended to the case of multiple random variables.

### A. Case of a single random variable

The uncertain depth $h$ of the cable can be considered, for instance, as the following function of a random variable (RV):

$$h=h^0(1+ax) \tag{6}$$

where $h^0$ is the central (mean) value of the depth, $x$ corresponds to a RV following a given statistical law (for example, an uniform law on [-1,1]) and $a$ represents a parameter that fixes the randomness intensity. $h^0 \in D_h$ where $D_h$ is the domain of definition of h.

The originality of the SC method lays in the choice of the polynomial approximation used to represent an observable Z (in our case, the sea return impedance) which is a function of the considered RV.

First, for a given $h^0$, the function $x \rightarrow Z(h^0;x)$ is developed on a Lagrange polynomial basis of order n

$$Z(h^0;x)\simeq\sum_{i=0}^{N} Z_i(h^0)L_i(x) \tag{7}$$

where the expression of the Lagrange polynomial is given by

$$L_i(x)=\prod_{\substack{j=0\\ j\neq i}}^{n}\frac{x-x_j}{x_i-x_j} \tag{8}$$

It is straightforward to show that we have

$$Z_i(h^0) = Z(h^0; x_i) \qquad (9)$$

The key of the SC method relies on the selection of the collocation points $x_i$ that define the Lagrange polynomial. These points are chosen to match those of the Gauss quadrature integration rule:

$$I = \int_D p(x) f(x) dx \simeq \sum_{i=0}^{n} \omega_i f(x_i) \qquad (10)$$

where *p(x)* is the probability density function of the RV x. The positive real numbers $\omega_i$ are called integration weights.

By definition, the mean value of $Z$ is given by:

$$Z_{mean} = \int_{D_h} Z(h^0; x) p(x) dx. \qquad (11)$$

By replacing $Z\left(h^0, x\right)$ with its polynomial approximation (7)

$$\begin{aligned} Z_{mean} &\simeq \int_{D_h} \sum_{i=0}^{n} Z_i(h^0) L_i(x) p(x) dx \\ &\simeq \sum_{i=0}^{n} Z_i(h^0) \int_{D_h} L_i(x) p(x) dx. \end{aligned} \qquad (12)$$

Using (10) and the well-known property of the Lagrange polynomial, $L_i\left(x_j\right) = \delta_{ij}$, $\delta$ being the Kronecker symbol, we can show

$$\int_{D_h} L_i(x) p(x) dx = \omega_i \qquad (13)$$

therefore the mean value is simply given by

$$Z_{mean} = \sum_{i=0}^{n} \omega_i Z_i(h^0). \qquad (14)$$

In a similar way, we can obtain the variance

$$\mathrm{var}(Z) = \sum_{i=0}^{n} \omega_i Z_i^2(h^0) - Z_{mean}^2. \qquad (15)$$

The application of the SC method consists in evaluating the deterministic problem (given here by equation 5) for each one of the collocation points. Since no theoretical results exist, the convergence of the method is obtained by comparing the results obtained by increasing the number of collocation points. In general, as we shall see in the section III, a few number of collocation points is needed to evaluate the mean and variance of the output and this makes the SC very interesting as compared to the classic Monte Carlo method.

### B. *Case of multiple random variables*

The previous technique can be generalized to the case of multiple RVs. As an illustration, we consider two RVs associated with the parameters $\sigma_1$ and $\sigma_2$ (conductivity values of the sea water and seabed layers) of the function Z. We adopt the same notations

$$\sigma_1 = \sigma_1^0(1 + ax) \qquad (16)$$

$$\sigma_2 = \sigma_2^0(1 + ay) \qquad (17)$$

with $\sigma_1^0 \in D_{\sigma_1}$ et $\sigma_2^0 \in D_{\sigma_2}$ , $D_{\sigma_1}$ and $D_{\sigma_2}$ being the domain of definition of $\sigma_1^0$ and $\sigma_2^0$ respectively.

In the previous expressions, if the same parameter $a$ has been chosen for the two RVs, it is possible to introduce two different parameter values.

The function $\left(x, y\right) \to Z(\sigma_1^0, \sigma_2^0; x, y)$ is developed on a Lagrange polynomial basis

$$Z(\sigma_1^0, \sigma_2^0; x, y) \simeq \sum_{i=0}^{n} \sum_{j=0}^{n} Z_{ij}(\sigma_1^0, \sigma_2^0) L_i(x) L_j(y) \qquad (18)$$

with

$$Z_{ij}(\sigma_1^0, \sigma_2^0) = Z(\sigma_1^0, \sigma_2^0; x_i, y_j) \qquad (19)$$

In the same way as previously, we can show that the mean and variance of Z can be written as:

$$Z_{mean} \simeq \sum_{i=0}^{n} \sum_{j=0}^{n} \omega_i \omega_j Z_{ij}(\sigma_1^0, \sigma_2^0) \qquad (20)$$

$$\mathrm{var}(Z) \simeq \sum_{i=0}^{n} \sum_{j=0}^{n} \omega_i \omega_j Z_{ij}^2(\sigma_1^0, \sigma_2^0) - Z_{mean}^2 \qquad (21)$$

It is very important to point out that the SC method is non-intrusive (for one or several RVs) and does not require modifications of the deterministic solver used (analytic, numerical homemade or commercial). Moreover, the proposed method takes into account in a natural process the probability law of the RV.

## IV. Application of the method to the sea return impedance

In order to show how the variation of the input parameters of the formula of the sea-return impedance impacts on its values we have considered separately two variations:

- The variation of the size of the first layer (or the laying depth of the cables).
- The variation of the conductivity of both sea and seabed layers (case of two RVs).

The standard deviation is defined from the previous formulas as

$$stdev(Z) = \sqrt{\mathrm{var}(Z)} \qquad (22)$$

Table 1 gives the values of the collocation points and of the weights when the RV h follows a uniform law.

Table 1 - Collocation points and weights used for the 1 RV case

| $x_i$ (depth $h$) | $\omega_i$ |
|---|---|
| 13,38 | 0,2778 |
| 25 | 0,4445 |
| 36,61 | 0,2778 |

The convergence of the method is such that only 3 collocation points have been employed here. With an increase of their number (for instance using 5 points) the results do not vary.

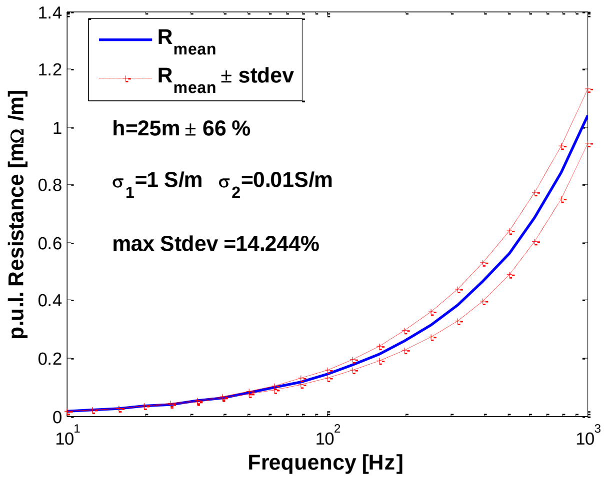


Fig. 3 – Mean mutual sea return resistance, when depth h has 66% randomness

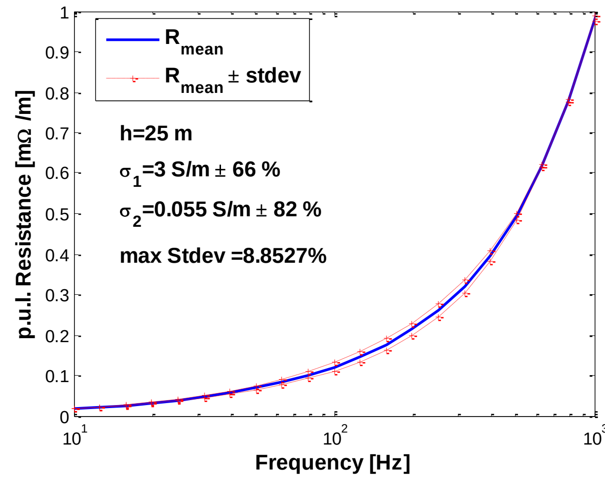


Fig. 5 – Mean mutual sea return resistance, when conductivities have 66% and 82 % randomness respectively

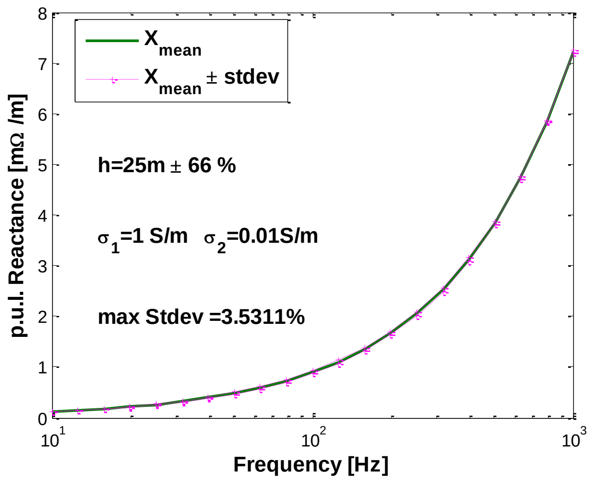


Fig. 4 – Mean mutual sea return reactance, when depth h has 66% randomness

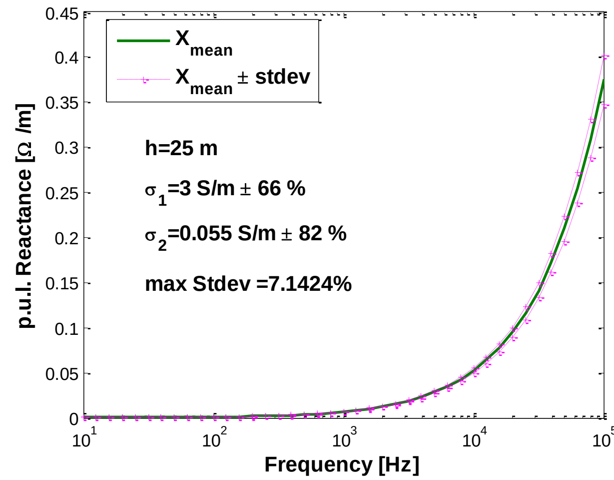


Fig. 6 – Mean mutual sea return resistance, when conductivities have 66% and 82 % randomness respectively

Table 2 - Collocation points and weights used for the 2 RV case

| $x_i$ ($\sigma_1$) | $y_j$ ($\sigma_2$) | $\omega_i * \omega_j$ |
|---|---|---|
| 1,450 | 0,0550 | 0,1234 |
| 1,450 | 0,0201 | 0,0771 |
| 1,450 | 0,0898 | 0,0771 |
| 3,000 | 0,0201 | 0,1234 |
| 3,000 | 0,0550 | 0,1975 |
| 3,000 | 0,0898 | 0,1234 |
| 4,549 | 0,0201 | 0,0771 |
| 4,549 | 0,0550 | 0,1234 |
| 4,549 | 0,0898 | 0,0771 |

In the case of two RVs again 3 points for each one of the variables were sufficient, but as can be seen from equations (20-21) 9 computations were necessary. It is therefore evident that this technique can be only applied when dealing with a limited number of RVs.

It is reasonable to ascertain that, since this variation is limited, one can use the mean of the value of the impedance as the entry value for the cable model, even when relatively significant uncertainties are present in the input parameters. And, since cables laid on the sea bed have variable laying depths, it is of great interest that the stochastic collocation permits to calculate the mean of the impedance very simply.

## V. CONCLUSIONS

In this article we have presented a method for the calculation of the sea-return impedance of submarine cables. A precise integral accounting for the different electrical characteristics of the two layers (sea and seabed) is solved using a quasi Monte Carlo method. This method is numerical, and can be easily implemented and included in cable parameters calculations.

The results it gives have been compared with those obtained with a commercial software. The main advantage of proposing a closed form solution to the integral is that this can be included in deterministic calculations of per-unit-length parameters.

We have also presented the stochastic collocation method, a non intrusive technique that allows evaluating the mean and variance of a function in a way similar to the Monte Carlo method and requires only very few computations instead of thousands.

Applying the stochastic collocation to the evaluation of the sea return impedance we have then shown how the variation of the input parameters (conductivities and cables' laying depth) affects the mean value of the impedance.

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