Extension of a Capacitance Model for Inductive Medium Voltage Transformers

C. Buchhagen, H. Däumling, L. Hofmann

Abstract-- **Due to the rising amount of harmonic distortion, especially in the medium voltage grid, it becomes more important to monitor harmonics and interharmonics. Additionally, the monitoring of transients is also essential. Inductive voltage transformers may have natural frequencies at a few thousand hertz that influence the measurement accuracy of frequency components at those frequencies to a great extent. To improve the frequency transmission range, it is necessary to be able to calculate the first resonance. Therefore, proper input variables are essential. One important variable is the layer capacitance. In this paper, the model for this capacitance [1] is significantly improved compared to a previous one by using a finite element program. With the extended model it is possible to calculate the layer capacitances for various wires and insulation materials and, therefore the accuracy of the calculated resonance is improved.**

Keywords: **Capacitance, Capacitance measurement, Frequency response, Instrument transformers, Voltage transformers.**

I. INTRODUCTION

DUE to the change in the electric energy supply, more decentralized generation systems are built. They often have frequency converters which are a source for harmonics and interharmonics. Additionally, the number of non-linear loads such as switch mode power supplies is rising. These devices are mostly connected to the low or medium voltage grid. It becomes more important to monitor the power quality including transients in the future. The total harmonic distortion (THD) determines inter alia the power quality and includes all frequency harmonics up to 2.5 kHz. Different standards such as [2] and [3] limit the harmonics. However, power electronics often cause frequency components above 2.5 kHz. Therefore, it is considered to monitor the frequency range up to 9 kHz.

Voltage measurement in the electric grid is mostly done by using inductive voltage transformers to transform the voltage proportionally to a level that can be measured directly. However, current standards for such devices only define rated frequencies of the electrical grids like, e.g. 16.7 Hz, 50 Hz, 60 Hz. Thus, the behavior at higher frequencies is not standardized. But it is not in question that inductive voltage transformers have natural frequencies at a few thousand hertz [4]-[7]. Up till now resistive-capacitive dividers would be suitable to monitor transients and high order harmonics. Inductive voltage transformers are preferred by system operators because they have some advantages. Most important is the galvanic separation between the primary and secondary side and that they are maintenance free. Due to this, it is necessary to develop an inductive voltage transformer with extended frequency transmission range. Thereby, a basic milestone is the ability to calculate its natural frequency. One important input variable is the layer capacitance which has to be known. The model determined in [1] for an exemplary voltage transformer is precise if the voltage transformer of interest has similar design parameters. However, the geometrical variables could be different and it is necessary to extend the developed capacitance model.

II. INDUCTIVE MEDIUM VOLTAGE TRANSFORMERS

Common inductive voltage transformers consist of at least an iron core, a primary coil and a secondary coil. The coils are manufactured as cylindrical windings with continuous winding layers [4], [5]. Thereby, the first layer is wound and an insulation paper is integrated to guarantee electrical strength. Then, the wire is wound back to the start position. In principle, two different wire configurations are possible. If every layer has the same geometrical length it is called a steep coil. If every outer layer is shorter than the inner layers it looks like a trapezoid in cross section and is named like this (Fig. 1).

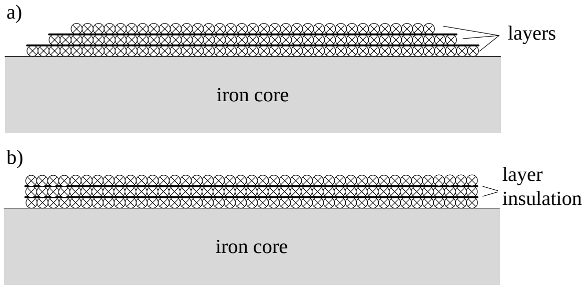

Fig. 1. Construction of coils of inductive voltage transformers [1].
a) trapezoid coil
b) steep coil

For a more detailed view, Fig. 2 illustrates a section of a winding layer and specifies the dimension parameters of the enameled copper wire and the insulation paper.

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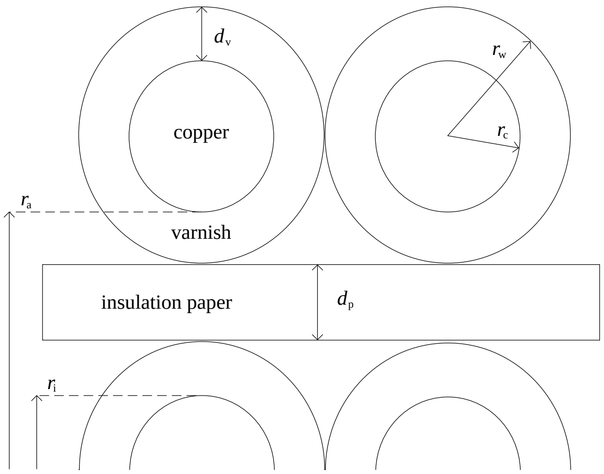


Fig. 2. Detailed section of a winding layer.

A. Modeling of Inductive Voltage Transformers

The modeling of inductive medium voltage transformers was done using a practical related equivalent circuit. Therefore, all elements of a winding layer are merged into constant elements (Fig. 3). A layer equivalent circuit consists of the copper resistance *R*, the leakage inductance L_σ and the main inductance L_h .

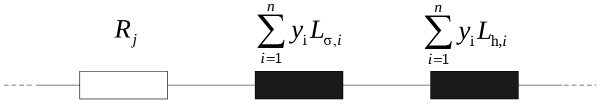


Fig. 3. Equivalent circuit of a single winding layer [1].

The inductances are coupled over the whole transformer and have to be multiplied with the related factor y_i which represents the ratio between the current of the individual layer $\underline{I}_j$ and the current of the corresponding inductance $\underline{I}_i$:

$$y_i = \frac{\underline{I}_i}{\underline{I}_j} \tag{1}$$

A series connection of several equivalent circuits of single winding layers is the first step for determination of the equivalent circuit of an inductive voltage transformer. For the primary coil, the number of series connections has to be equivalent to the number of winding their layers n_1 . Additionally, a layer capacitance $C_L/2$ has to be integrated between each double layer. The secondary coil with n_2 winding layers is modeled in the same way. Both coils are coupled over a coupling capacitance C_{Kl} (Fig. 4).

The ground and front part capacitances, which are present in a real voltage transformer, only have a small influence on the first resonance [6], [7]. So the models developed in [1] are sufficient and do not need to be improved.

The evaluation of the simplified equivalent circuit provides the equation for the first natural frequency [1]. Thereby, all resistances are neglected because of their small impact on the natural frequency. Additionally, the inductances of the single layers are assumed with the same values because they do not differ significantly. The layer capacitances are set to the average layer capacitance $C_{L,m}$ and L_σ represents the leakage inductance of the voltage transformer. Thus, the first natural frequency can be calculated with:

$$\omega_e = \frac{1}{\sqrt{\frac{C_{L,m}}{2} L_\sigma \frac{1}{n_1}}} \tag{2}$$

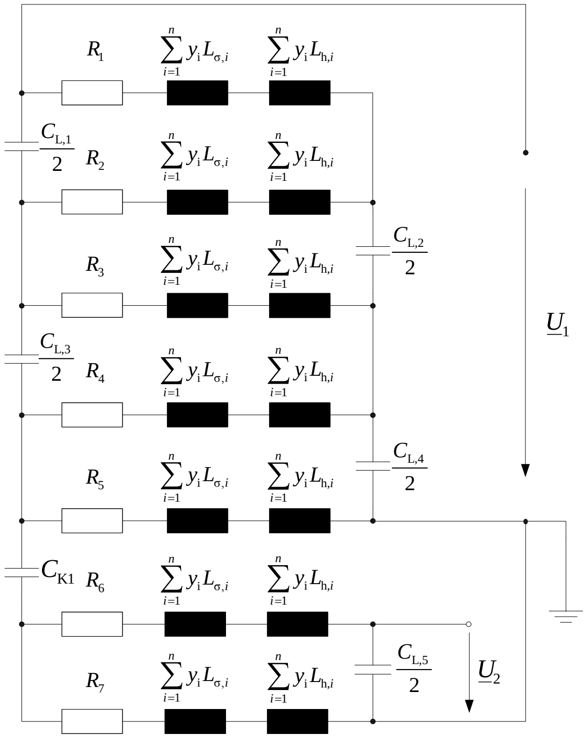


Fig. 4. Simplified equivalent circuit of a voltage transformer [7].

Thus, it is possible to calculate the first resonance. However, precise input variables are needed, so it is important to have an exact model for the leakage inductance and layer capacitances.

III. EXISTING LAYER CAPACITANCE MODEL

A. Capacitance Model

In [1] a capacitance model was determined on the basis of measurements. Therefore, all layer capacitances were measured at a special inductive voltage transformer with winding taps (Fig. 5). Thereby, two different measurement systems were used to ensure that a measurement error is unlikely.

Based on the equation for a cylindrical capacitor, an equation was determined to calculate the layer capacitances. Due to the geometric characteristics of inductive voltage transformers, the introduction of two correction factors was necessary. First, a double layer winding does not consist of a flat surface. It has several single wires which reduce the active electrical length. So the correction factor k_1 is necessary which was determined by measurement on coil models with only three winding layers [1].

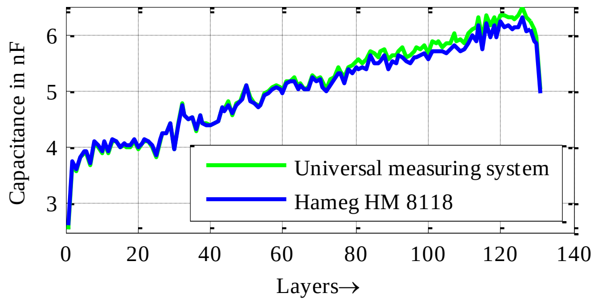


Fig. 5. Measured layer capacitances of a 20 kV voltage transformer [1].

A second correction factor k_r is required to represent the small decrease in the distance between two winding layers (Fig. 6) when going from inner layers to outer layers (pressure). This factor depends on the winding layer x. The correction factor k_r results in $k_r = xk_x$ with $k_x = 0.53‰$ [1]. So the layer capacitances can be calculated with (3) if the resulting relative permittivity, which is described later and the design parameters are very similar to those of the inductive voltage transformer which was analyzed in [1]. Thereby, l represents the length of a winding layer. The results are shown in Fig. 6.

$$\frac{C_L}{2} = \pi\varepsilon_{res}\frac{l\,k_l}{\ln\frac{r_a(1-k_r)}{r_i}} \qquad (3)$$

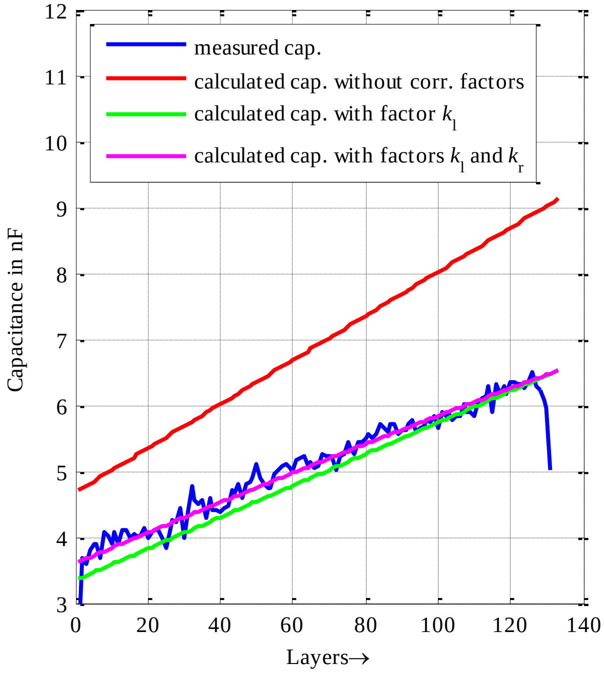


Fig. 6. Comparison of measured and calculated layer capacitances [1].

The inner radius r_i is the distance from the middle of the iron core to the outer surface of the copper of the corresponding layer. Therefore, the outer radius r_a results from the inner radius by adding the thickness of the insulation paper d_p and twice the thickness of the insulation varnish d_v (cp. Fig. 2).

B. Limits of the existing Capacitance Model

As the capacitance model was determined on a single voltage transformer, it is obvious that there are deviations in the calculated capacitance values of voltage transformers with different design parameters.

1) Wire Diameter

For example, the wire diameter has a big influence on the capacitance value. As shown in Fig. 7, the calculated capacitance value by the simple model and those values calculated by the finite element program “Comsol Multiphysics” differ a lot. Thereby, a winding layer was much longer than the distance between two layers and edge effects could be neglected. As the wire radius is not considered in the simple model, it cannot be used for calculating layer capacitances with other geometrical parameters. Only if the radius corresponds to that one which was used in [1], the result is the same.

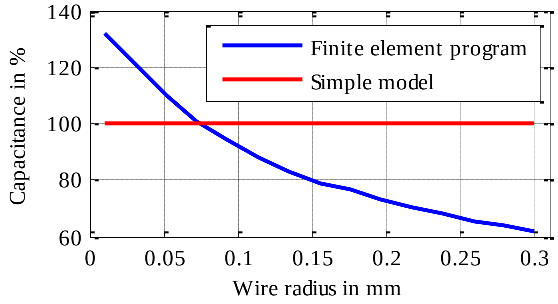


Fig. 7. Difference between the capacitance calculation with the finite element program and the simple model in relation to the wire radius.

2) Insulation Varnish

Another change in the capacitance value is caused by the insulation varnish of the copper wire. In contrast to the previously analyzed influence, the capacitance value of the simple model is not constant, because the distance between the layers was already taken into account. Fig. 8 illustrates that there is nearly no difference between both calculation methods.

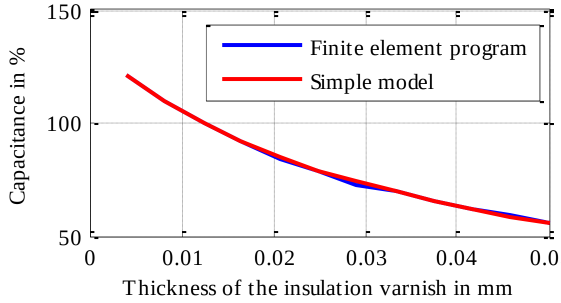


Fig. 8. Difference between the capacitance calculation with the finite element program and the simple model in relation to the thickness of the varnish.

3) Insulation Paper

Furthermore, the thickness of the insulation paper could be varied. Thereby, the influence of the curvature of the wire is changed. As the simple model does not consider this, there are some deviations which are shown in Fig. 9.

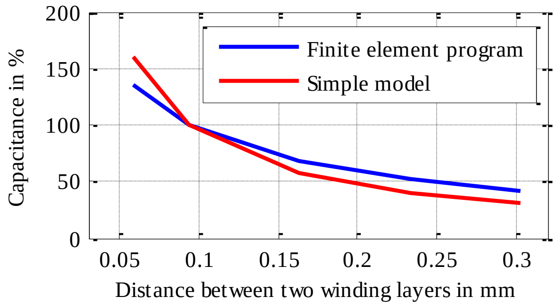


Fig. 9. Difference between the capacitance calculation with the finite element program and the simple model in relation to the thickness of the insulation paper.

4) *Relative Permittivity*

Finally, the value of the relative permittivity changes the curvature of the electric field lines. In Fig. 10, both the insulation paper and the varnish have the same relative permittivity. This already worsens the calculation results from the simple model compared with the finite element computations. However, the relative permittivities for both materials can vary from each other, which have to be considered in a more complex model.

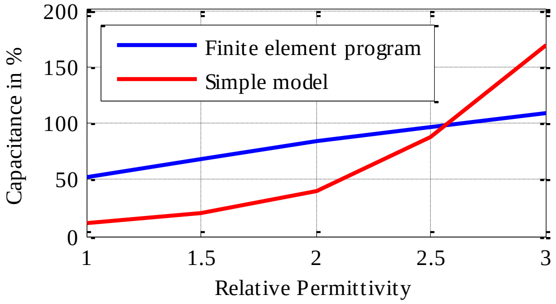


Fig. 10. Difference between the capacitance calculation with the finite element program and the simple model in relation to the relative permittivity.

Due to the partially big deviations between the calculation results from the simple model and the finite element program, it is necessary to improve the capacitance model.

IV. EXTENSION OF THE CAPACITANCE MODEL

A. *Influence of various Geometrical Parameters*

As the curvature of the field lines is influenced by the radius of the wire, the thickness of the insulation varnish, and the thickness of the insulation paper, those influencing variables cannot be considered separately. In addition, the relative permittivities of the varnish and the insulation paper which are considered in the next subsection have also a big effect. For determination of a new approximation model, capacitance values for various geometrical arrangements were calculated with a finite element program. Thereby, the wire radius and the thickness of the insulation varnish and insulation paper were varied.

As already shown in Fig. 7, the layer capacitance strongly depends on the radius of the wire. The calculated capacitance values increase with decreasing wire radius due to the increasing effective surface. A variation of the thickness of the insulation varnish influences the distance between the two layers as well as the distance between the copper of two single wires. Both effects increase the capacitance value if the varnish is thinner (Fig. 8). The last geometrical parameter which was not considered in the simple model is the thickness of the insulation paper between two winding layers. Previous calculations showed that the influence of the varnish is much smaller than the influence of the other variables. Therefore, Fig. 11 displays the capacitance value only in relation to the wire radius r_w and the distance between two winding layers, including the thickness of the insulation paper d_p and twice the thickness of the insulation varnish d_v .

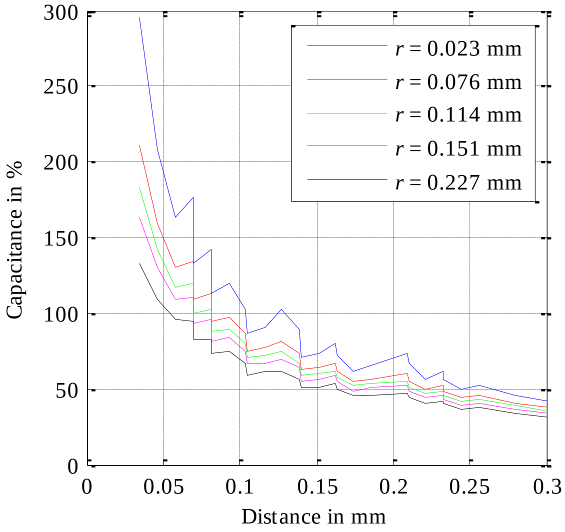


Fig. 11. Layer capacitances in relation to the distance between two layers and the wire radius.

As the displayed series of curves (Fig. 11) are not very smooth, the actual distance between two layers was replaced by an effective distance which is calculated by:

$$d_\mathrm{e} = d_\mathrm{p} + 3d_\mathrm{v} \tag{4}$$

This leads to curves which are a lot smoother (cp. Fig. 12). After that, a regression analysis is done to identify an equation which describes the correction factor k_l with a low deviation. As fitting target, the lowest sum of squared absolute errors was chosen. As one result, (5) shows a good correlation:

$$k_\mathrm{l} = a\, r_\mathrm{w}^{\mathrm{b}}\, d_\mathrm{e}^{\mathrm{c}} + o \tag{5}$$

For this equation, the following coefficients were identified:

$$\begin{aligned} a &= \ \ 2.17946 \cdot 10^{3} & c &= \ \ 9.12146 \cdot 10^{-5} \\ b &= -7.71199 \cdot 10^{-5} & o &= -2.17851 \cdot 10^{3} \end{aligned} \tag{6}$$

If the capacitance values calculated by the finite element program and by the new determined equation for the correction factor k_l are compared, both series of curves match (Fig. 12).

Only the values at the beginning of the curves show a bigger deviation. So, the validation of the determined equation should be limited to the inner values. Those values can only be determined by calculation with 10% uncertainty. A histogram of the devitations between both calculation methods (i.e. FEM and model) is shown in Fig. 13. Most deviations between both calculation methods are below ±4%. Only a few values reach ±6%.

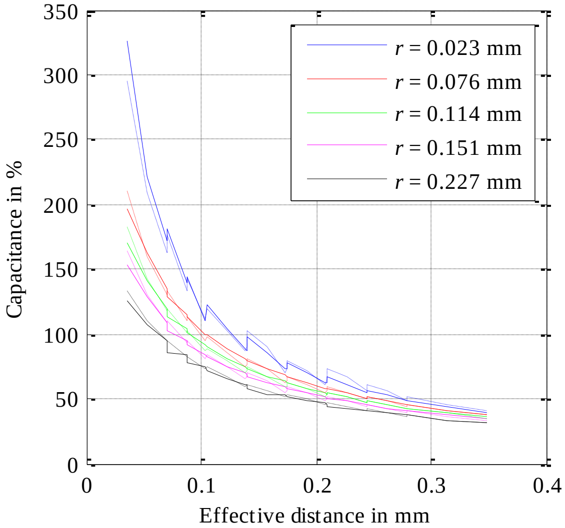


Fig. 12. Comparison of both calculated capacitance values.

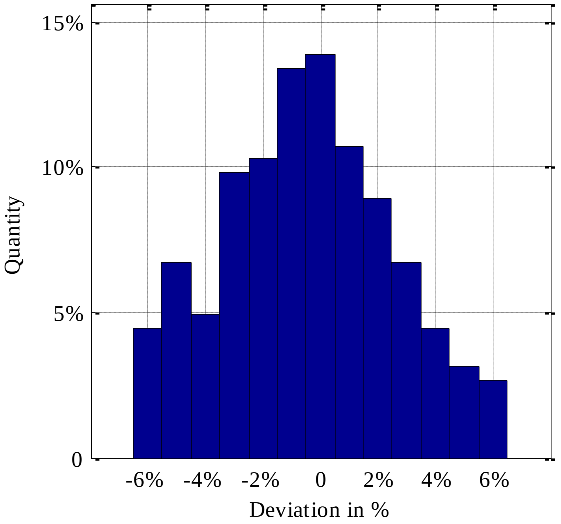


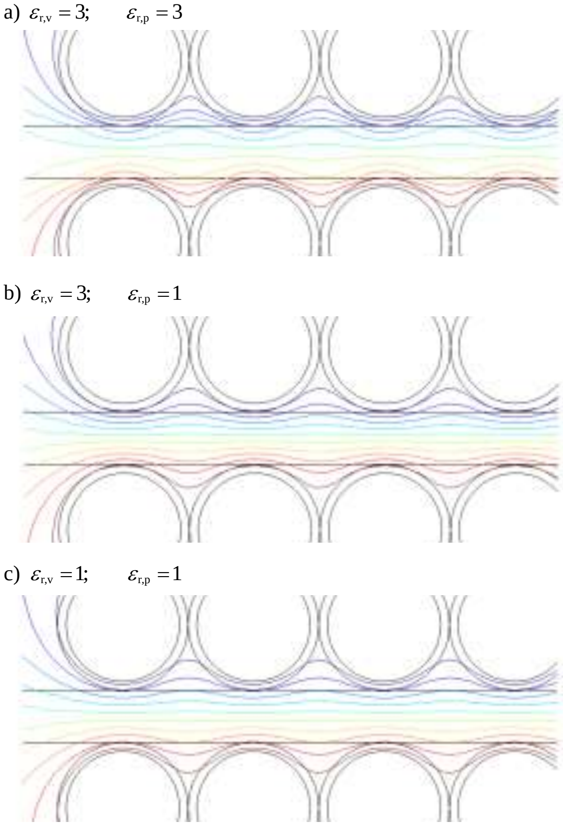
Fig. 13. Histogram of the deviations between both calculation methods.

B. *Influence of various relative Permittivities*

In addition to the geometrically influenced variables, the relative permittivities of the varnish and of the insulation paper have also a significant impact on the capacitance value. Various relative permittivities lead to different curvatures of lines of equal potential (Fig. 14) and to changed capacitance values.

Therefore, it is necessary to calculate a resulting relative permittivity. It can be determined from the relative permittivities of the layer insulation $\varepsilon_{\mathrm{r,p}}$ and the varnish $\varepsilon_{\mathrm{r,v}}$ equivalent to a capacitor with a multi-layer dielectric:

$$\varepsilon_{\mathrm{r,res}} = k_{\varepsilon} \frac{(d_{\mathrm{p}} + 2d_{\mathrm{v}})\,\varepsilon_{\mathrm{r,v}}\varepsilon_{\mathrm{r,p}}}{2d_{\mathrm{v}}\varepsilon_{\mathrm{r,p}} + d_{\mathrm{p}}\varepsilon_{\mathrm{r,v}}} \tag{7}$$


a) $\varepsilon_{\mathrm{r,v}} = 3; \quad \varepsilon_{\mathrm{r,p}} = 3$

b) $\varepsilon_{\mathrm{r,v}} = 3; \quad \varepsilon_{\mathrm{r,p}} = 1$

c) $\varepsilon_{\mathrm{r,v}} = 1; \quad \varepsilon_{\mathrm{r,p}} = 1$

Fig. 14. Electrical field simulations of various relative permittivities.

As the resulting permittivity depends on the geometrically influenced variables, a correction factor k_{ε} is necessary. In contrast to the factor k_{l} it is not easy to specify an equation for its calculation. However, it is possible to display a surface or a table for this factor respectively. It is calculated with the layer capacitance $C_{\mathrm{L,FEM}}$ resulting from the finite element program and the capacitance $C_{\mathrm{L,M}}$ which is computed with the extended model.

$$k_{\varepsilon} = \frac{C_{\mathrm{L,FEM}}}{C_{\mathrm{L,M}}} \tag{8}$$

Unfortunately, it is required to compute a surface for every wire diameter, if good calculation results are requested. A surface for a specific wire is given in Fig. 15. The figures for other wire diameters are mostly analog but differ a bit. The arrangements for the various relative permittivities in Fig. 15 are described in Table I.

TABLE I
ARRANGEMENTS FOR THE VARIOUS RELATIVE PERMITTIVITIES IN FIG. 15

Arrangement	1	2	3	4	5	6	7	8	9
Rel. permittivity ε_{p}	1	1	1	2	2	2	3	3	3
Rel. permittivity ε_{v}	1	2	3	1	2	3	1	2	3

Accordingly, the extended capacitance model consists of (3) with the improved correction factor k_{l} and the resulting relative permittivity. The factor k_r was not analyzed further in this work as the influence is not significant and k_r has a strong dependence on the manufacturing technology.

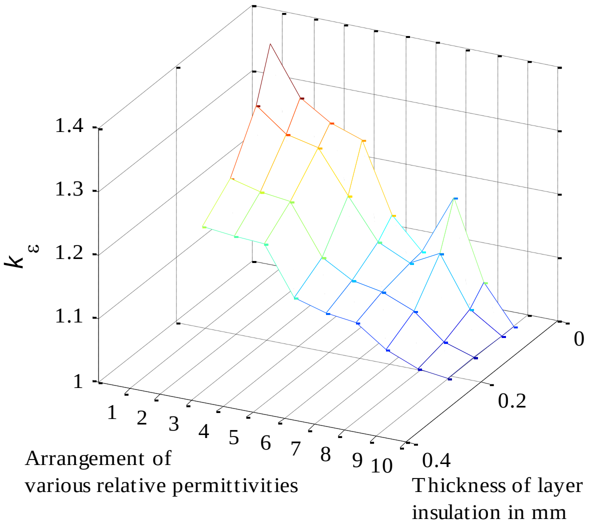


Fig. 15. Diagram for the correction factor of the relative permittivity for a wire with a radius of 0.0757 mm.

V. FURTHER INFLUENCING VARIABLES

Of course, there are lots of further geometrical variables which influence the layer capacitance. First of all it is conceivable that a layer is not fully wound. Instead of this, there is free space between single windings. This leads also to a curvature of field lines and to lower capacitance value. Several orienting simulations were performed to illustrate the influence of a not fully wound winding layer (Fig. 16). It is shown that the calculation error is growing with increasing distance. These deviations depend also on the wire diameter as the series of curves displays in Fig. 16.

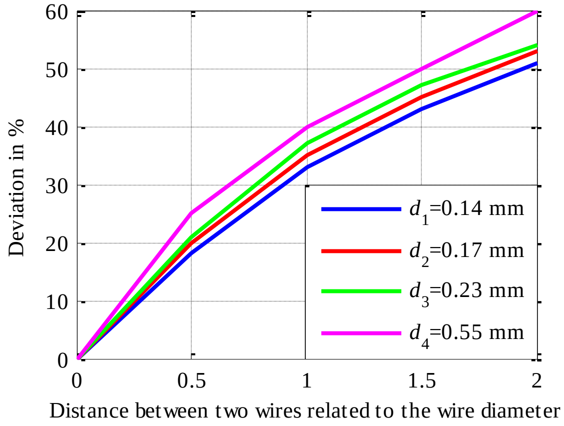


Fig. 16. Influence of the distance between two singles wires on the layer capacitance.

In addition, the capacitance values will be influenced by the coil shape. If a trapezoid winding is used, it is expected that the capacitance will be marginally smaller. Furthermore, the value will be changed if a flat wire is used instead of a round type. The effective electrical surface will be bigger and this leads to an increased capacitance value. Finally, there are more influencing variables thinkable. However, they are not integrated into the presented capacitance model because the layer capacitance is described well with the analyzed parameters.

VI. VERIFICATION OF THE CAPACITANCE MODEL

The model for the layer capacitances was improved with several analyzes using a field simulation program. Therefore, it was possible to determine the influences of the geometrical variables. Several equations were set up to calculate the needed correction factors. However, it is necessary to verify the capacitance model by simulating wires which are typically used in inductive voltage transformers. The calculated capacitance values which result from the field simulation have to be compared with those ones which result from the improved capacitance model. Only if the deviations are small, the model can be used for calculation of layer capacitances.

Therefore, five different wire types were applied during the simulations. To identify deviations, the number of varied parameters is rising in different scenarios. First, only the wire radius and the thickness of the insulation varnish were changed. As TABLE II shows, the difference between a field simulation and the determined capacitance model is below 4.3%. Thus, the capacitance model is sufficiently accurate if only wire parameters are changed.

TABLE II.
VERIFICATION OF THE CAPACITANCE MODEL BY VARYING WIRE PARAMETERS

No.	**Radius (mm)**	**Thickness of the varnish**	**Calculated capacitance (equation)**	**Calculated capacitance (FEM)**	**Deviation**
1	0.061	0.0103 mm	3.46 nF	3.50 nF	-1.04%
2	0.071	0.0113 mm	3.27 nF	3.36 nF	-2.56%
3	0.117	0.0172 mm	2.59 nF	2.71 nF	-4.32%
4	0.161	0.0200 mm	2.29 nF	2.31 nF	-1.07%
5	0.288	0.0279 mm	1.74 nF	1.71 nF	1.64%

Second, the thickness of the layer insulation is changed also between 50% and 300% of a commonly used material. The deviations between the determined model and the field calculations are displayed in Fig. 17.

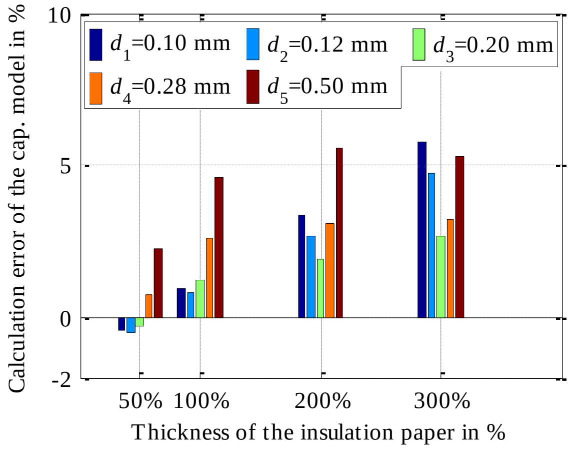


Fig. 17. Calculation error of the capacitance model in dependence on the thickness of the layer insulation paper.

It can be seen that the error is mostly below ±5%. Only at very thick layer insulations it reaches a maximum of about +5.7%.

Finally, these representative wires were used for further simulations with various thicknesses of the insulation paper and relative permittivities. For each wire two simulations were performed. One was done with doubled layer insulation and the other with a triple one. The relative permittivities of the insulation varnish and the paper were randomly chosen between one and three. Again, the capacitances were calculated by using the layer capacitance model and finite element field calculations. The results can be found in Table III and they show that the deviations between an exact field calculation and the determined model are often very low but can reach up to ±6.5%. Hence, it has to be taken into account that the calculation of the natural frequencies of inductive medium voltage transformers has already this impreciseness from the calculation of the layer capacitance.

TABLE III.
VERFIFICATION OF THE CAPACITANCE MODEL BY VARYING ALL PARAMETERS

No.	Thickness of the layer insulation	Relative permittivity of the varnish	Relative Permittivity of the paper	Deviation
1	200%	1	2	4.73%
1	300%	3	2	6.30%
2	200%	1	2	4.50%
2	300%	1	3	4.32%
3	200%	2	1	-0.08%
3	300%	3	2	1.78%
4	200%	2	2	0.54%
4	300%	3	1	0.83%
5	200%	3	1	3.14%
5	300%	1	3	4.88%

VII. CONCLUSION

The extension of the layer capacitance model was an important step for the calculation of the natural frequencies of inductive medium voltage transformers. Capacitance values calculated with the previous model differ by several 10% from those ones calculated with the finite element program. This is caused by the limitations of the simple model.

The new model has an accuracy of approximately ±6.5% and therefore, the accuracy for calculating the natural frequencies of inductive medium voltage transformers increases a lot. Up till now it was difficult to calculate them with sufficient accuracy for inductive voltage transformers. Due to the extended capacitance model, they can be calculated more precisely.

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