Defining and Measuring AC Frequency Based on Symmetry Principles

Kempei Seki

***Abstract*—This paper reports new developments in applying symmetry principles to power systems. At first, it defines AC frequency associated with a rotation phase angle that will be achieved by combining spiral vector theory with group theory. Then, it first proposes the gauge voltage group and the gauge difference voltage group, next construct these two group's vector multiplication tables and real number multiplication tables, next discovers some invariants that are the frequency coefficients, the gauge voltage, and the gauge difference voltage, finally obtains AC frequency and the amplitude respectively. In addition, it proposes measuring the DC component of input signals with the gauge voltage group. Further, it proposes three approaches to reduce the influence of harmonic distortion. At last, it presents results from a numerical simulation and a field test. In general, it proposes an effective method for PMU.**

***Keywords*: amplitude measurement, frequency measurement, group theory, PMU, spiral vector theory, symmetry principles.**

I. Introduction

High-speed and high-precision tracking of AC frequency is required urgently for developing smart grid. Here AC frequency means real-time frequency of a power system. Commonly, the signal process of AC frequency is time-consuming and PMU is expensive. However, spiral vector theory that using spiral vector variables that are rotating counterclockwise in the complex plane was proposed by Dr. Yamamura [1]. In consequence of this, we had developed spiral vector methods for power systems [2]-[11]. On the other hand, since A. Einstein introduced symmetry into physics and developed relativity theory, symmetry principles had been applied not only to quantum mechanics, but also to quantum field theory, and all of them obtained great success [12]-[13]. Therefore, after happened to discover that spiral vectors have symmetry properties, we had started to develop symmetry principles for power systems by combing spiral vector theory with group theory [14]-[19]. This paper reports new developments and it is organized as follows: Section II shows defining AC frequency with rotation phase angle; section III proposes the gauge voltage group; section VI proposes the gauge difference voltage group; section V calculates the DC component; section VI deals with harmonic distortion; section VII implements numerical simulation; section VIII conducts field test, and last section IX is the conclusions.

Kempei Seki is with Protection & Control Systems Department, Mitsubishi Electric Corporation, 1-5-8, Takatsukadai, Nishi-ku, Kobe 651-2271, Japan (e-mail: Seki.Kempei@dc.MitsubishiElectric.co.jp).

Paper submitted to the International Conference on Power Systems Transients (IPST2013) in Vancouver, Canada July 18-20, 2013.

II. Defining AC Frequency with Rotation Phase Angle

As a matter of fact, according to IEEE Std. [20], for a sinusoidal signal as

$$x(t) = X_m \cos[\psi(t)] \tag{1}$$

where X_m is the amplitude, $\psi(t)$ is the synchrophasor associated with the rated frequency reference, the frequency is defined as

$$f(t) = \frac{1}{2\pi}\frac{d\psi(t)}{dt} \tag{2}$$

And the rate of change of frequency estimation is defined as

$$ROCOF(t) = \frac{df(t)}{dt} \tag{3}$$

The above definition treats every frequency all the same and AC frequency is limited to one-dimensional space. Then, one has to utilize DFT or similar methods to extract AC frequency from input signals [21]. On the contrary, we tried to discover new definition focusing on AC frequency and to abandon reference frames due to a power system can't operate in the point of the rated frequency. Referring to Fig. 1 in next page, we introduce a rotation phase angle to define AC frequency as [19]

$$\frac{f(t)}{f_S} = \frac{\alpha(t)}{2\pi} \tag{4}$$

where *f(t)* is AC frequency and f_S is the gauge sampling frequency. Then, AC frequency can be obtained as

$$f(t) = \frac{f_S}{2\pi}\alpha(t) = \frac{\alpha(t)}{2\pi T} \tag{5}$$

where *T* is the gauge sampling period and is expressed as

$$T = \frac{1}{f_S} \tag{6}$$

Next we define the rate change of frequency estimation as

$$ROCOF(t) = \frac{df(t)}{dt} = \frac{d\alpha(t)}{2\pi T} = \frac{\alpha(t) - \alpha(t-nT)}{2n\pi T^2} \tag{7}$$

where n is any integer. For focusing AC frequency and reducing the influence of harmonic distortion, the rotation phase angle should be set to a large value (e.g., larger than 90 degrees) by choosing suitable gauge sampling frequency. Then, AC frequency definition is extended to a two-dimensional space. Next we start symmetry processing.

III. Defining and Using Gauge Voltage Group

Here we define three voltage vectors in the complex plane in Fig. 1 as the gauge voltage group. Then three voltage vectors are expressed as [19]

$$\begin{cases} v_1(t) = Ve^{j(\omega t+\alpha)} \\ v_1(t-T) = Ve^{j\omega t} \\ v_1(t-2T) = Ve^{j(\omega t-\alpha)} \end{cases} \tag{8}$$

where α is the rotation phase angle, T is the gauge sampling period, V is the amplitude, ω is the angular velocity of the voltage vectors and it is expressed as

$$\omega = 2\pi f \tag{9}$$

where f is AC frequency.

Consider the group rotating through some degrees in the complex plane, the structure of the group will not change if AC frequency does not change in the rotation time. It is the rotation invariance according to group theory, thus we can discover invariants of the group. In Fig. 1, we find that $v_1(t)$ and $v_1(t-2T)$ are symmetrical about x axis with α, then these two elements, plus the middle element $v_1(t-T)$ will be used to calculate the frequency coefficient invariant.

In what follows, we construct the vector multiplication table of the gauge voltage group that is shown in Table I. Substituting (8) into the elements of Table I, we obtain

$$\begin{cases} v_1^2(t) = V^2 e^{j(2\omega t+2\alpha)} \\ v_1(t)v_1(t-T) = V^2 e^{j(2\omega t+\alpha)} \\ v_1(t)v_1(t-2T) = V^2 e^{j(2\omega t)} \\ v_1^2(t-T) = V^2 e^{j(2\omega t)} \\ v_1(t-T)v_1(t-2T) = V^2 e^{j(2\omega t-\alpha)} \\ v_1^2(t-2T) = V^2 e^{j(2\omega t-2\alpha)} \end{cases} \tag{10}$$

The above elements consist of the phase space of the gauge voltage group that is illustrated in Fig. 2. Thus we find that $v_1^2(t-T)$ and $v_1(t)v_1(t-2T)$ equal to each other, this pair will be used to calculate the gauge voltage invariant.

TABLE I
Vector Multiplication Table of Gauge Voltage Group

$\times$	$v_1(t)$	$v_1(t-T)$	$v_1(t-2T)$
$v_1(t)$	$v_1^2(t)$	$v_1(t)v_1(t-T)$	$v_1(t)v_1(t-2T)$
$v_1(t-T)$	$v_1(t)v_1(t-T)$	$v_1^2(t-T)$	$v_1(t-T)v_1(t-2T)$
$v_1(t-2T)$	$v_1(t)v_1(t-2T)$	$v_1(t-T)v_1(t-2T)$	$v_1^2(t-2T)$

TABLE II
Real Number Multiplication Table of Gauge Voltage Group

$\times$	v_{11}	v_{12}	v_{13}
v_{11}	v_{11}^2	$v_{11}v_{12}$	$v_{11}\,v_{13}$
v_{12}	$v_{11}\,v_{12}$	v_{12}^2	$v_{12}\,v_{13}$
v_{13}	$v_{11}\,v_{13}$	$v_{12}\,v_{13}$	v_{13}^2

Moreover, three instantaneous voltages of (8) are expressed as

$$\begin{cases} v_{11} = \mathrm{Re}[v_1(t)] = V\cos(\omega t+\alpha) \\ v_{12} = \mathrm{Re}[v_1(t-T)] = V\cos(\omega t) \\ v_{13} = \mathrm{Re}[v_1(t-2T)] = V\cos(\omega t-\alpha) \end{cases} \tag{11}$$

With the above data, we construct the real number multiplication table of the gauge group that is shown in Table II. Substituting (11) into the elements of Table II, we obtain

$$\begin{cases} v_{11}^2 = V^2\cos^2(\omega t+\alpha) \\ v_{11}v_{12} = V^2\cos(\omega t+\alpha)\cos(\omega t) \\ v_{11}v_{13} = V^2\cos(\omega t+\alpha)\cos(\omega t-\alpha) \\ v_{12}^2 = V^2\cos^2(\omega t) \\ v_{12}v_{13} = V^2\cos(\omega t)\cos(\omega t-\alpha) \\ v_{13}^2 = V^2\cos^2(\omega t-\alpha) \end{cases} \tag{12}$$

From the above investigations, firstly, we can discover the frequency coefficient as [19]

$$f_C = \frac{v_{11}+v_{13}}{2v_{12}} = \cos\alpha \tag{13}$$

Then, we can calculate the rotation phase angle as

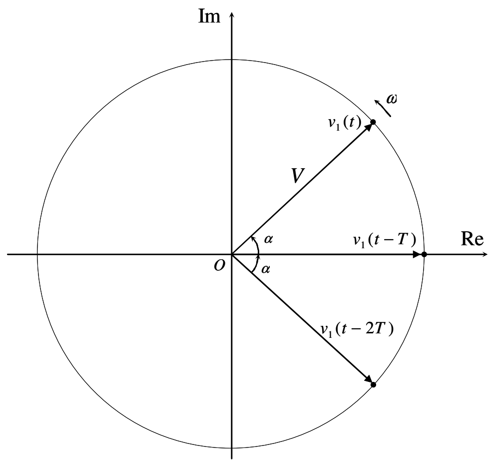

Fig. 1. A gauge voltage group is rotating with an angular speed ω in the complex plane

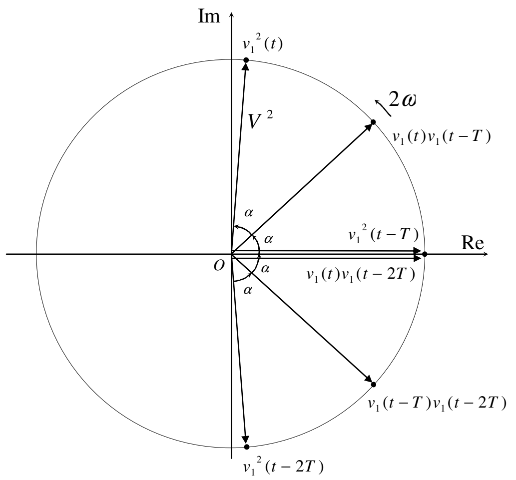

Fig. 2. The phase space of gauge voltage group is rotating with the angular speed 2ω in the complex plane

$$\alpha = \cos^{-1} f_C \tag{14}$$

Namely, from (5), we can calculate AC frequency as

$$f = \frac{\alpha}{2\pi} f_S \tag{15}$$

Secondly, we can discover the gauge voltage invariant as

$$V_g = \sqrt{v_{12}{}^2 - v_{13} v_{11}} = V \sin\alpha \tag{16}$$

In this fashion, we obtain the amplitude as

$$V = \frac{V_g}{\sin\alpha} = \frac{V_g}{\sqrt{1 - f_C{}^2}} \tag{17}$$

By the way, we can calculate the rms (root-mean-square) voltage as

$$V_{rms} = \frac{V}{\sqrt{2}} = \frac{V_g}{\sqrt{2(1 - f_C{}^2)}} \tag{18}$$

IV. Defining and Using Gauge Difference Voltage Group

In this section, we will present another symmetry group to calculate AC frequency and the amplitude.

Similarly, we define three difference voltage vectors in the complex plane in Fig.3 as the gauge difference voltage group. Then three difference voltage vectors are expressed as

$$\begin{cases} v_2(t) = Ve^{j(\omega t + \frac{3\alpha}{2})} - Ve^{j(\omega t + \frac{\alpha}{2})} \\ v_2(t-T) = Ve^{j(\omega t + \frac{\alpha}{2})} - Ve^{j(\omega t - \frac{\alpha}{2})} \\ v_2(t-2T) = Ve^{j(\omega t - \frac{\alpha}{2})} - Ve^{j(\omega t - \frac{3\alpha}{2})} \end{cases} \tag{19}$$

Consider the group rotating through some degrees in the complex plane, the structure of the group will not change if AC frequency does not change in the rotation time. It is the rotation invariance according to group theory, thus we can discover invariants of the group. In Fig. 3, we find that $v_2(t)$ and $v_2(t-2T)$ are symmetrical about x axis with α, then these two elements, plus the middle element $v_2(t-T)$ will be used to calculate the frequency coefficient invariant.

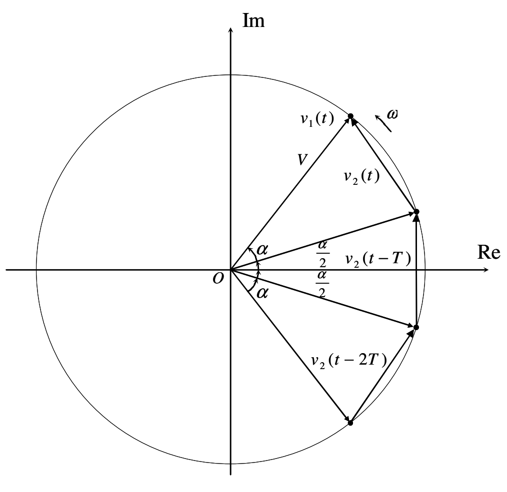

Fig. 3. A gauge difference voltage group is rotating with an angular speed ω in the complex plane

TABLE III

Vector Multiplication Table of Gauge Difference Voltage Group

$\times$	$v_2(t)$	$v_2(t-T)$	$v_2(t-2T)$
$v_2(t)$	$v_2^2(t)$	$v_2(t)v_2(t-T)$	$v_2(t)v_2(t-2T)$
$v_2(t-T)$	$v_2(t)v_2(t-T)$	$v_2^2(t-T)$	$v_2(t-T)v_2(t-2T)$
$v_2(t-2T)$	$v_2(t)v_2(t-2T)$	$v_2(t-T)v_2(t-2T)$	$v_2^2(t-2T)$

TABLE IV

Real Number Multiplication Table of Gauge Difference Voltage Group

$\times$	v_{21}	v_{22}	v_{23}
v_{21}	$v_{21}{}^2$	$v_{21}v_{22}$	$v_{21}\, v_{23}$
v_{22}	$v_{21}\, v_{22}$	$v_{22}{}^2$	$v_{22}\, v_{23}$
v_{23}	$v_{21}\, v_{23}$	$v_{22}\, v_{23}$	$v_{23}{}^2$

In what follows, we construct the vector multiplication table of the gauge difference voltage table that is shown in Table III. Substituting (19) into the elements of Table III, we obtain

$$\begin{cases} v_2{}^2(t) = 4V^2 \sin^2\frac{\alpha}{2} e^{j(2\omega t + 2\alpha - \pi)} \\ v_2(t)v_2(t-T) = 4V^2 \sin^2\frac{\alpha}{2} e^{j(2\omega t + \alpha - \pi)} \\ v_2(t)v_2(t-2T) = 4V^2 \sin^2\frac{\alpha}{2} e^{j(2\omega t - \pi)} \\ v_2{}^2(t-T) = 4V^2 \sin^2\frac{\alpha}{2} e^{j(2\omega t - \pi)} \\ v_2(t-T)v_2(t-2T) = 4V^2 \sin^2\frac{\alpha}{2} e^{j(2\omega t - \alpha - \pi)} \\ v_2{}^2(t-2T) = 4V^2 \sin^2\frac{\alpha}{2} e^{j(2\omega t - 2\alpha - \pi)} \end{cases} \tag{20}$$

The above elements consist of the phase space of the gauge difference voltage group that is illustrated in Fig. 4. Thus we find that $v_2^2(t-T)$ and $v_2(t)v_2(t-2T)$ equal to each other, this pair will be used to calculate the gauge difference voltage invariant.

Moreover, three instantaneous difference voltages of (19) are expressed as

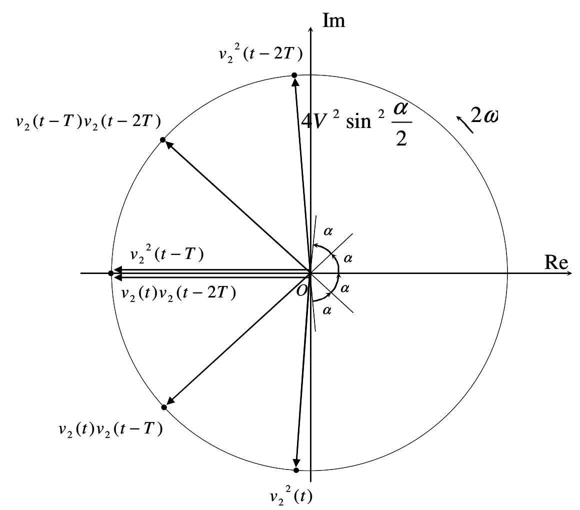

Fig. 4. The phase space of gauge difference voltage group is rotating with the angular speed 2ω in the complex plane

$$
\begin{cases}
v_{21} = \mathrm{Re}[v_2(t)] = -2V\sin\frac{\alpha}{2}\sin(\omega t+\alpha) \\
v_{22} = \mathrm{Re}[v_2(t-T)] = -2V\sin\frac{\alpha}{2}\sin(\omega t) \\
v_{23} = \mathrm{Re}[v_2(t-2T)] = -2V\sin\frac{\alpha}{2}\sin(\omega t-\alpha)
\end{cases}
\tag{21}
$$

With the above data, we construct the real number multiplication table of the gauge difference voltage group that is shown in Table IV. Substituting (21) into the elements of Table IV, we obtain

$$
\begin{cases}
{v_{21}}^2 = 4V^2\sin^2\frac{\alpha}{2}\sin^2(\omega t+\alpha) \\
v_{21}v_{22} = 4V^2\sin^2\frac{\alpha}{2}\sin(\omega t+\alpha)\sin(\omega t) \\
v_{21}v_{23} = 4V^2\sin^2\frac{\alpha}{2}\sin(\omega t+\alpha)\sin(\omega t-\alpha) \\
{v_{22}}^2 = 4V^2\sin^2\frac{\alpha}{2}\sin^2(\omega t) \\
v_{22}v_{23} = 4V^2\sin^2\frac{\alpha}{2}\sin(\omega t)\sin(\omega t-\alpha) \\
{v_{23}}^2 = 4V^2\sin^2\frac{\alpha}{2}\sin^2(\omega t-\alpha)
\end{cases}
\tag{22}
$$

From the above investigations, we firstly, we discover the frequency coefficient as

$$f_C = \frac{v_{21}+v_{23}}{2v_{22}} = \cos\alpha \tag{23}$$

Then we can obtain the rotation phase angle and AC frequency according to (14) and (15) respectively.

Secondly, we can discover the gauge difference voltage as

$$V_{gd} = \sqrt{{v_{22}}^2 - v_{23}v_{21}} = 2V\sin\alpha\sin\frac{\alpha}{2} \tag{24}$$

In this fashion, we obtain the amplitude as

$$V = \frac{V_{gd}}{2\sin\alpha\sin\frac{\alpha}{2}} = \frac{V_{gd}}{2(1-f_C)\sqrt{1+f_C}} \tag{25}$$

By the way, we can calculate the rms voltage as

$$V_{rms} = \frac{V}{\sqrt{2}} = \frac{\sqrt{2}V_{gd}}{4(1-f_C)\sqrt{1+f_C}} \tag{26}$$

Because all elements of the gauge difference group are difference voltages, the above results are little influence by the DC component. Furthermore, we will calculate the DC component directly with measured AC frequency and the gauge voltage group next.

V. Calculating DC Component of Input Signals

In Fig.5, we can express three voltage vectors contains the DC component as

$$
\begin{cases}
v_{11} = V\cos(\omega t+\alpha) + v_{DC} \\
v_{12} = V\cos(\omega t) + v_{DC} \\
v_{13} = V\cos(\omega t-\alpha) + v_{DC}
\end{cases}
\tag{27}
$$

where v_{DC} is the DC component. Then, we can find the following relationship from the above equations.

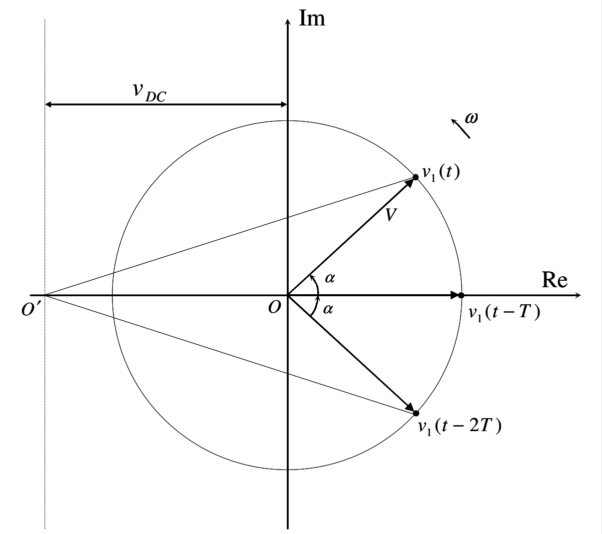

Fig. 5. A gauge voltage group containing DC component is rotating with an angular speed ω in the complex plane

$$f_C = \frac{v_{11}+v_{13}-2v_{DC}}{2(v_{12}-v_{DC})} = \cos\alpha \tag{28}$$

Solving the above equation, we can obtain the DC component as

$$v_{DC} = \frac{v_{11}+v_{13}-2v_{12}f_C}{2(1-f_C)} \tag{29}$$

Here f_C is obtained with the gauge difference voltage group.

Commonly, the DC component is treated as low frequency component that is difficult to calculate. In contrast to this, (29) is the much simple and fast solution.

VI. Dealing with Harmonic Distortion

The above solutions are only valid for the pure sinusoidal signal. But harmonics exist in power systems. Therefore we propose three approaches to reduce the influence of harmonic distortion next.

A. *Separating Gauge Sampling Frequency from Data Sampling Rate*

Firstly, we separate the gauge sampling frequency from the data sampling rate. We show the concept in Fig. 6. In this example, we set one gauge sampling period as four times to data sampling periods. In general, the relationship between the gauge sampling frequency and the data sampling rate can be expressed as

$$f_S = \frac{1}{T} = \frac{1}{nT_1} = \frac{1}{n}f_1 \tag{30}$$

where f_S is the gauge sampling frequency, T is the gauge

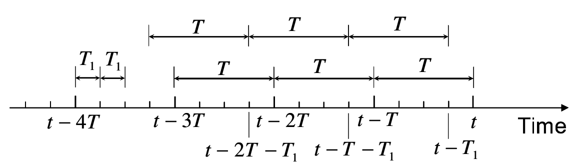

Fig. 6. Separation of gauge sampling frequency from data sampling rate

sampling period, f_1 is the data sampling rate, T_1 is the data sampling period, n is any integer respectively.

B. Establishing a Symmetry Broken Criterion

Secondly, we establish a symmetry broken criterion as

$$f_{CBRK} = \left| \frac{v_{21} + v_{23}}{2v_{22}} \right| > 1 \tag{31}$$

When the above criterion is satisfied, AC frequency is not measurable with the group and we latch previous measured value. So this criterion can eliminate the bad data effectively.

C. Implementing a Moving Average

Thirdly, we can reduce the influence of harmonic distortion by implementing a moving average. Here are the averaging equations of first the frequency coefficients, second the rotation phase angle, third AC frequency, last the amplitude respectively.

$$\begin{cases} f_{Cavg} = \dfrac{1}{M} \displaystyle\sum_{k=0}^{M-1} f_{Ck} \\ \alpha_{avg} = \dfrac{1}{M} \displaystyle\sum_{k=0}^{M-1} \alpha_k \\ f_{avg} = \dfrac{1}{M} \displaystyle\sum_{k=0}^{M-1} f_k \\ V_{avg} = \dfrac{1}{M} \displaystyle\sum_{k=0}^{M-1} V_k \end{cases} \tag{32}$$

where M is the number points of moving average process. Then the time of moving average can be calculated as

$$T_{avg} = M \times T_1 \tag{33}$$

D. Flow Chart of the Measuring Process

To this end, we present the flow chart of AC frequency measuring process in Fig.7. In addition to this, we can obtain real-time active and reactive power with measured AC frequency [9]. In this connection, we can stamp measured AC frequency and real-time power with UTC time flag and send them to PDC of wide-area monitoring and control system.

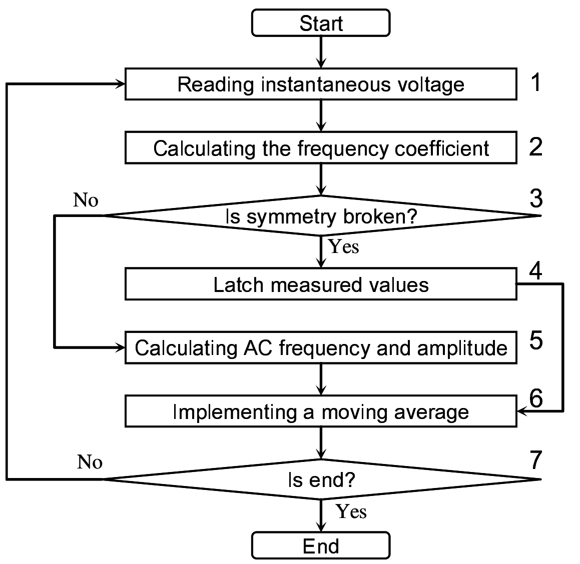

Fig. 7. Flow chart of the measuring process

VII. Performing Numerical Simulation

In this section, we present a numerical simulation. The parameters are shown in Table V. Then, the input signal can be expressed as

$$v(t) = \begin{cases} 1.5 + \cos(373.9t) & for\ t < 0.05, \\ 1.5 + \cos[2\pi(373.9 + df)t + \phi_C] & for\ t \geq 0.05. \end{cases} \tag{34}$$

where ϕ_C is the phase angle at the point of starting frequency ramp. Therefore in this case, one is that the DC component 1.5V is larger than AC amplitude 1V, another is that steady AC frequency 59.51Hz is not the rated frequency 60Hz, still another frequency ramp speed is 2.5 Hz/s. All these conditions will cause large TVE (total vector error) according to [20]. Then, the gauge sampling period is obtained

$$T = \frac{1}{f_S} = 0.005(s) \tag{35}$$

And we define the rate change of frequency estimation as

$$ROCOF(t) = \frac{\alpha(t) - \alpha(t - T)}{2\pi T^2} \tag{36}$$

In simulation, we use the gauge difference voltage group to calculate AC frequency and the amplitude. Figure 8 illustrates the results of the numerical simulation.

In Fig. 8a, the summation of measured AC amplitude and the DC component indicate that not only AC amplitude, but also the DC component are correctly measured even the latter is larger than the former.

In Fig. 8b, the measured frequencies are linear before the starting point indicates that AC frequency is measured correctly even it is the shifted frequency 59.51 Hz that has large TVE. After the starting point, some measured instantaneous frequencies are pulse shapes because frequency ramping changes invariants of symmetry groups. After implemented the moving average, the measured average frequencies become approximately linear but still track the theoretical frequency fast.

In Fig. 8c, the measured ROCOF according to (36) is shown. It indicates that the result is oscillated around the

TABLE V
PARAMETERS OF NUMERICAL SIMULATION

Symbol	Quantity	Setting data
f_0	Rated frequency	60 Hz ($1/T_0$)
f_S	Gauge sampling frequency	200 Hz
f_1	Data sampling rate	4000 Hz ($20 f_S$)
f	AC frequency	59.51 Hz
df	ROCOF	2.5 Hz/S
V	Voltage amplitude	1V
v_{DC}	DC component	1.5V
V_{deg}	Voltage initial phase angle	0 Degree
T_{avg}	Time of moving average	0.01667 Second (T_0)
T_{total}	Time of data recording	0.40 Second

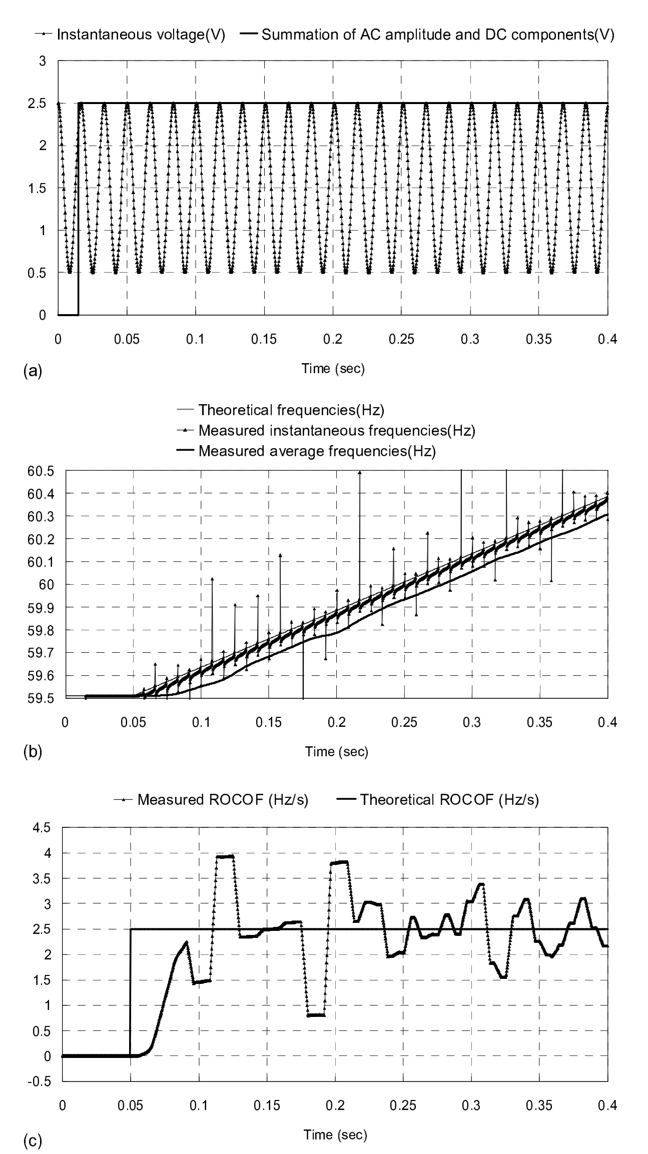

Fig. 8. The measured results of the numerical simulation.
(a) The instantaneous voltage and the summation of AC amplitude and the DC component.
(b) The theoretical, the measured instantaneous, and the measured average frequency
(c) The theoretical and the measured ROCOF

theoretical ROCOF 2.5 Hz/s.

By the way, because the measured results of the gauge difference voltage group is little influence by the DC component, we can use the gauge difference active power group (instead of voltage with active power) to measure low frequency oscillations in power systems.

VIII. Conducting Field Test

In this section, we present a field test. The parameters are shown in Table VI. Then, the gauge sampling period is obtained as

$$T = \frac{1}{f_S} = 0.004(s) \tag{37}$$

And we define the rate change of frequency estimation as

TABLE VI
PARAMETERS OF FIELD TEST

Symbol	Quantity	System data
f_0	Rated frequency	60 Hz ($1/T_0$)
f_S	Gauge sampling frequency	250 Hz
f_1	Data sampling rate	4000 Hz (16 f_S)
T_{avg}	Time for moving average	0.03333 Second ($2T_0$)
T_{total}	Time of data recording	0.40 Second

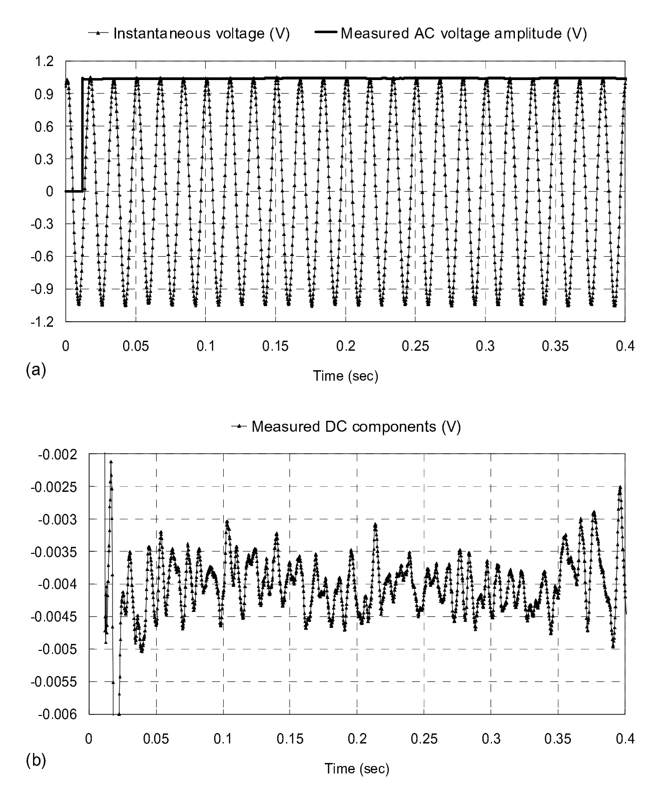

Fig. 9. The measured amplitude of the field test.
(a) The instantaneous voltage and the measured AC amplitude.
(b) The measured DC component.

$$ROCOF(t) = \frac{\alpha(t) - \alpha(t-4T)}{8\pi T^2} \tag{38}$$

Because input signals containing DC component, we use the gauge difference voltage group in field test. Figure 9 and 10 illustrates the results of field test.

In Fig. 9a, it indicates that we measured the amplitude properly though input signals contain harmonics.

In Fig. 9b, the measured DC components oscillate around -0.004pu that is the DC offset of the device. This implies that we can shift the DC offset with proposed measuring method.

In Fig. 10a, the measured instantaneous frequencies oscillate around the rated frequency 60Hz because input signals contain harmonics. At some points, the measured instantaneous frequency even equal to zero because of at that point symmetry is broken. This result shows that the power system contains a large amount of harmonics.

In Fig. 10b, after made moving average, the measured

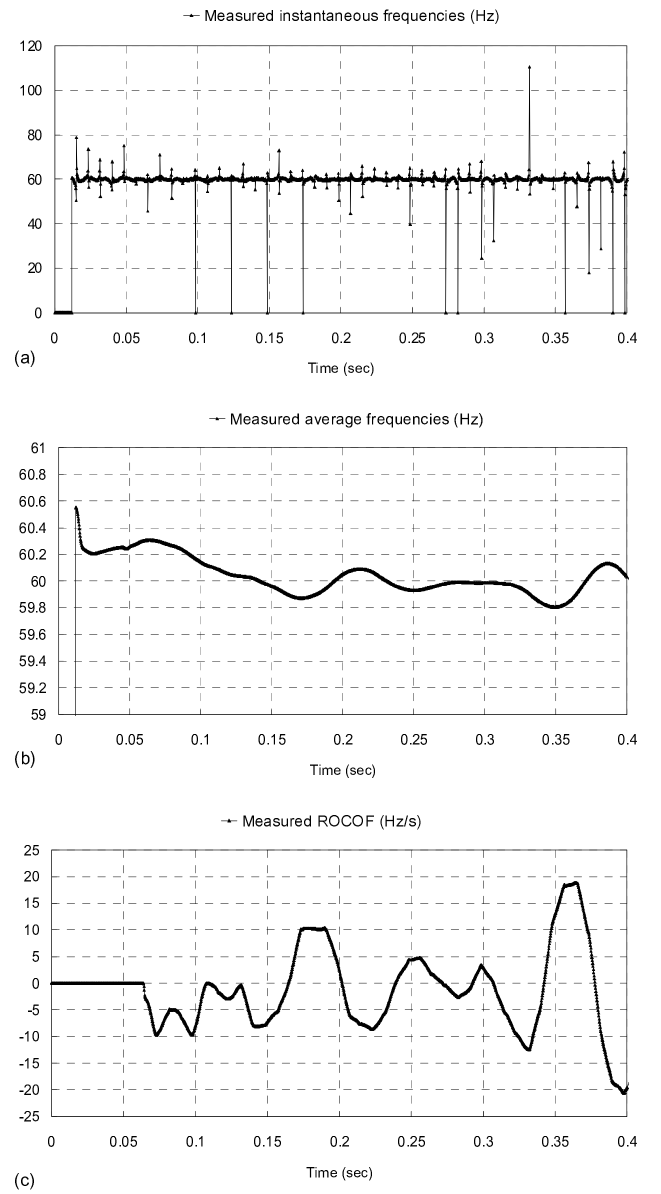

Fig. 10. The measured frequencies of the field test
(a) The measured instantaneous frequency.
(b) The measured average frequency.
(c) The measured ROCOF

average frequencies oscillate around the rated frequency 60Hz and variation scale becomes much smaller. Therefore, for real control and protection systems, we should select suitable length of moving average according to their objects.

In Fig. 10c, the measured ROCOF oscillations are large because of the influence of harmonic distortion.

IX. CONCLUSIONS

In summary, we defined and measured AC frequency by combing spiral vector theory with group theory. Moreover, we dealt with harmonic distortion with three approaches and these ideas are also from symmetry principles. Besides, the numerical simulation and the field test show that the proposed method is valid. As we all know, conventional methods first measure synchrophasor in the reference frame, second measure AC frequency. In contrast to this, the proposed method first measure AC frequency, second measure synchrophasors. Because it is easy to improve accuracy of time measurement, we can develop low-cost and high-speed and high-precision PMUs for smart grid. At last, we will also report new results how to define and measure synchrophasors based on symmetry principles in the near future.

X. REFERENCES

[1] S. Yamamura, Spiral Vector Theory of AC Circuits and Machines, Oxford University Press, 1992.

[2] K. Seki. "Analysis of an AC LC Ladder Circuit with Spiral Vector Theory," in *Power Conversion Conference, 2002, PCC-Osaka 2002,* Osaka, Japan, pp 749-754 vol.2.

[3] K. Seki, "Frequency measuring device," U.S. Patent 6985824, Jan. 10, 2006.

[4] K. Seki, "Synchronous vector measuring device," U.S. Patent 7006935, Feb. 28, 2006.

[5] K. Seki. "Development of Active Filters with Spiral Vector Theory," in *Power Conversion Conference – Nagoya, 2007,* PCC'07, Nagoya, Japan, pp 526-533

[6] K. Seki, "Rotation phase angel measuring device, and control-protecting devices of electric power system using the same," U.S. Patent 7504818, Mar. 17, 2009.

[7] K. Seki, "Active filter," U.S. Patent 7701090, Apr. 20, 2010.

[8] K. Seki, "Frequency measurement apparatus," U.S. Patent 8190387, May 29, 2012.

[9] K. Seki, "AC electric quantity measuring device," U.S. Patent 8129980, Mar. 6, 2012.

[10] K. Seki, "Synchrophasor measuring device and inter-bus-line phase angle difference measurement unit using the same," U.S. Patent 8198904, Jun. 12, 2012.

[11] K. Seki. "Development of Synchrophasors Measuring Method for Power Systems," in *International Conference on Power Systems Transients 2009 (IPST2009),* Kyoto, Japan.

[12] H. Weyl, Symmetry, Princeton University Press, 1952.

[13] Ian Stewart, Why Beauty Is Truth: The History of Symmetry, Basic Books, 2007

[14] K. Seki. "Measuring AC Quantities with Rotation Symmetry Groups," in *CPEM 2010, Conference on Precision Electromagnetic Measurement 2010 Conf. Digest,* pp. 199-200.

[15] K. Seki. "Developing an Active Filter Based on Least Square Method," in *IEEE Proc. Power Electronics Conference (IPEC),* pp191-196, Sapporo, Japan, 2010.

[16] K. Seki. "Measuring AC Power Based on Symmetry Principles," in *Proc. 2011 IEEE International Instrumentation and Measurement Technology Conference (I^2MTC 2011),* Hangzhou, China, May 2011, pp. 1749-1754.

[17] K. Seki. "Defining and Measuring Synchrophasors Based on Symmetry Principles," in *International Conference on Power Systems Transients 2011 (IPST2011),* Delft, The Netherlands.

[18] K. Seki. "Defining and Measuring AC Quantities Based on Symmetry Principles," in *TENCON 2011 – 2011 IEEE Region 10 Conference,* Bali, Indonesia, Nov. 2011, pp828-832.

[19] K. Seki. "Measuring AC Frequency and Amplitude with Gauge Voltage Group," in *2012 International Conference on Power System Technology,* Auckland, New Zealand, Oct.30–Nov.2, 2012, pp1-6.

[20] IEEE Standard for Synchrophasor Measurements for Power Systems, IEEE Std C37.118.1™-2011.

[21] A. G. Phadke, J. S. Thorp, and M. G. Adamiak, "A New Measurement Technique for Tracking Voltage Phasors, Local System Frequency, and Rate of Change of Frequency," *IEEE Trans. Power Apparatus and Systems*, vol. PAS-102, no.5, pp. 1025-1038, May 1983.