Swift Detection of Power Transformer Inrush Current Based on the Windowed-Adaptive Linear Combiner Estimation Algorithm

Arash Tavighi Soudabe Soleymani José Martí Hamed Abdollahzadeh

***Abstract*-- In the initial few cycles of energizing a power transformer, a large inrush current is passed through the primary winding as a result of saturation. The differential protection is inherently unable to discriminate between inrush current and internal fault, and hence it leads to unwanted outage of transformer while it is being inserted into the network. As known, inrush current spectrum contains a remarkable DC component and 2nd order harmonic. This characteristic makes it possible to discriminate energization from a short circuit which mostly includes a high amplitude current of fundamental frequency. Among various signal processing methods, Adaptive Linear Combiner (Adaline) has been known as a fast algorithm in detecting harmonics of voltage and current of a power system. This paper addresses a novel structure for the conventional Adaline algorithm introduced as Windowed-Adaline (W-Adaline). In the proposed algorithm, an estimation window is defined with variable number of samples, which are being updated as time goes by. In addition, the accuracy and transient response of W-Adaline have been remarkably improved when compared to the conventional Adaline. This paper presents the application of W-Adaline in detecting a test-case signal of an inrush current in MATLAB/Simulink. In this case, an input signal containing a damped DC component in addition to 1st, 2nd, and 3rd order harmonics of particular frequency, amplitude, and phase angle is created. The defined signal is then applied to the proposed W-Adaline, the conventional Adaline, and the FFT to form the estimation of the amplitude and phase angle of harmonics and the value of the DC component. Furthermore, a relative error is specified to evaluate the accuracy and response of the mentioned estimation algorithms. The performance of W-Adaline is also assessed in a more realistic situation by estimating the inrush current of a transformer in PSCAD/EMTDC.**

***Keywords*: Inrush current, Differential protection, Estimation of harmonic components, W-Adaline.**

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I. Introduction

Power transformers are known as one of the most expensive equipments of power systems. To protect them against internal faults to ground, current differential protection is normally used. Figure (1) shows a schematic diagram of the differential protection. In this protection, 3-phase primary and secondary currents are initially sampled. Then, by taking into consideration the turns ratio and primary and secondary configuration of the windings, a difference between primary and secondary currents is considered as an internal fault. Protective relays then operate and command the circuit breakers (CB) to open [1].

When energizing a transformer, the inrush current flows through the primary winding of a transformer (the side connected to the source). If this current is not discriminated from the internal fault, the differential relay will operate and disconnect the transformer [2].

A simple approach to prevent this problem is a temporarily deactivation of the relays for a short duration after the switching in order to delay the action of the circuit breakers. However, this procedure would ignore a fault caused by insulation failure due to sub-transient over-voltages, which are quite likely to happen at the moment of energizing the transformer [3].

Another solution is to compare the waveforms of internal fault current and inrush current. Regardless of the transformer type, inrush currents usually have a large DC component as well as particular harmonic components. However, internal fault currents mostly include a fundamental harmonic with high amplitude. In fact, inrush current can be readily distinguished from the internal fault current through the precise detection of the specific components [4]. Conventional RLC filters can be used to track the harmonics of a system; however, insufficient accuracy and low response are two major drawbacks of these circuits [5].

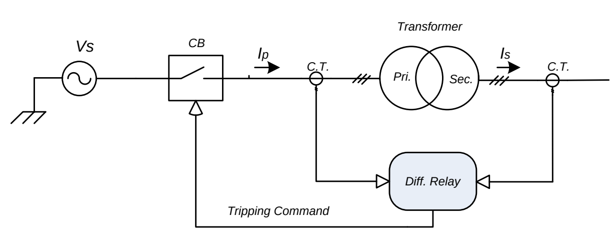

Fig. 1. Differential protection in a transformer

Nowadays, the application of Digital Signal Processing (DSP) algorithms, as for example, Discrete Fourier Transform (DFT), Fast Fourier Transform (FFT), Kalman Filter, and Adaptive Linear Combiner (Adaline) are of renewed interest in modern digital relays. Among the mentioned methods, Adaline is known as an algorithm with fast and accurate response in estimating dynamic changes in voltages/currents of a power system. Adaline is a Multi-Inputs Single-Output (MISO) neural network whose operating principle is based on the weighted addition of the *n* inputs and one bias. In contrast with other neural networks which are trained offline, Adaline can be trained online [6]-[8].

This paper presents a novel structure for the Adaline algorithm called "Windowed-Adaline (W-Adaline)". The conventional Adaline algorithm is reformulated based on the DC component and particular harmonic components. Moreover, by defining an estimation window of adjustable length, Adaline is extended into a Multi-Inputs Multi-Outputs (MIMO) algorithm. With the proposed extensions, the estimation response and accuracy of the algorithm are effectively improved. The proposed W-Adaline can be used for instant discrimination between the inrush currents and the internal fault currents.

II. Inrush Current of a Transformer

In a transformer, the voltage applied to the primary side $v(t)$, the voltage induced in the winding $e(t)$, and magnetizing current $i_\varphi(t)$ are expressed in (1)-(3),

$$v(t) = V_m \sin(\omega t + \theta) \quad (1)$$

$$e(t) = L\frac{di_\varphi(t)}{dt} \quad (2)$$

$$i_\varphi(t) = L^{-1}\int e(t)dt \quad (3)$$

Where, θ is a variable that indicates the switching moment, and L is the inductance of the winding. Ignoring the drop in the resistance, the derivative of the flux passing through the core equals the applied voltage. In (4), (5),

$$v(t) = N\frac{d\varphi(t)}{dt} \quad (4)$$

$$d\varphi(t) = \frac{v(t)dt}{N} \quad (5)$$

Where, N is the turn ratio of the transformer. By taking the integral of both sides in (5), the flux is given by (6) as

$$\varphi(t) = \frac{V_m}{N}\int \sin(\omega t + \theta)dt = \frac{-V_m}{N\omega}\cos(\omega t + \theta) + C \quad (6)$$

C is a constant of integration and is determined from the initial condition. Assuming that hysteresis flux at the moment of energizing of transformer (t=0) is φ_R, C can be calculated in (7) as

$$C = \frac{V_m}{N\omega}\cos(\theta) + \varphi_R \quad (7)$$

Replacing (7) in (6), $\varphi(t)$ is given in (8) as

$$\varphi(t) = \frac{-V_m}{N\omega}\cos(\omega t + \theta) + \frac{V_m}{N\omega}\cos(\theta) + \varphi_R \quad (8)$$

Considering the peak flux as $\varphi_p = V_m/N\omega$, (8) is rewritten in (9).

$$\varphi(t) = -\varphi_p\cos(\omega t + \theta) + \varphi_p\cos(\theta) + \varphi_R \quad (9)$$

Figure (2) shows the instantaneous flux calculated in (9). As seen in Fig. 2, when the applied voltage crosses zero (θ = 0), the flux will have its peak value of $2\varphi_p + \varphi_R$. In other words, the flux has an increase of $2\varphi_p$ with respect to the hysteresis flux φ_R at the moment of energizing of transformer. Such an increase in the flux will bring the operating point of the transformer into the saturation zone. Since the inductance of winding is negligible in this zone (Small value of L in (3)), a large non-sinusoidal magnetizing current will pass through the primary side of the transformer during the few initial cycles of energization (Fig. 3). The amplitude of this current ("inrush current") can reach to about five times the full-load current.

Figure (4) shows the particular harmonic spectrum of an inrush current and the range of variation in amplitude [9]. As can be seen in Fig. 4, the harmonic spectrum of the inrush current contains a high DC value and a 2nd order harmonic. However, these two components (specifically 2nd order harmonic) do not usually appear in the other types of short circuit currents. By identifying these components, inrush current can be effectively distinguished from the inrush current. Providing differential relays with a DC component and a 2nd order harmonic detector will prevent false tripping during transformer energization [4].

Figure (5) shows the conceptual configuration of a modern differential relay with a harmonic detector. The purpose of this structure is to identify internal faults of the transformer based on the 1st order harmonic of the current by means of W-Adaline. In what follows, the mathematical implementation of W-Adaline algorithm is discussed.

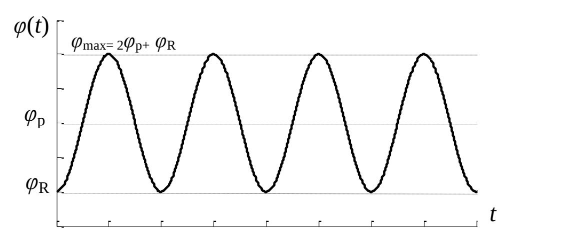

Fig. 2. Instantaneous flux of the transformer at the moment of energizing

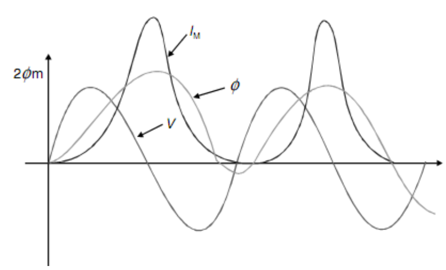

Fig. 3. Inrush current of a transformer

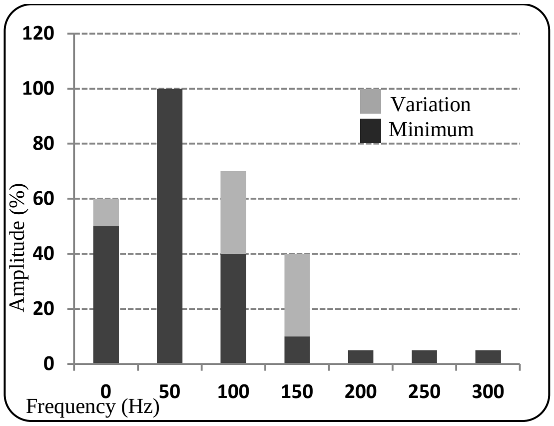

Fig. 4. Harmonic spectrum of an inrush current

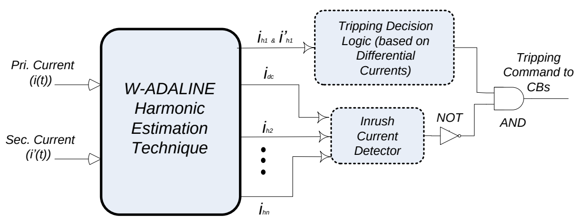

Fig. 5. Logical structure to identify the inrush current of a transformer

III. Mathematical Implementation of W-Adaline

Assume first that an input discrete single-phase current signal is formed as in (10), which includes damped DC component and different order of harmonics.

$$i(k)=\sum_{n=1}^{N}\left(|i_n|\sin(\omega_n t_k+\varphi_n)+A_{DC}e^{-\beta t_k}\right)$$
$$\omega_n=n\omega_1\ ,\ t_k=k\Delta t=k/f_s \tag{10}$$

Where, each individual parameter is defined as follows:

- n: Order of harmonic,
- N: Highest order of harmonic,
- k: Sample number,
- $|i_n|$: Current amplitude of the harmonic order n,
- φ_n: Current phase angle of the harmonic order of n,
- A_{DC}: Amplitude of the damped DC component,
- β: Damping factor of the DC component,
- ω_1: Fundamental frequency,
- Δt: Sampling interval,
- f_s: Sampling frequency.

The damping factor of DC component (β) of the inrush current for transformers up to 100 kVA is less than one second, and for those up to a few 100 MVA is about one second [10], [11].

Expression (10) can be written according to the M consecutive samples of the current ($i(k{=}1)$ to $i(k{=}M)$). Then, by extending the resultant expression, based on the trigonometric term of $\sin(\alpha+\beta)=\sin(\alpha).\cos(\beta)+\cos(\alpha).\sin(\beta)$, and re-arranging the equations in the form of a matrix, the estimated current is formed in (11).

$$\hat{I}(k)=A(k)\times W(k) \tag{11}$$

Where:

$$\hat{I}(k)=\left[i(1)\quad i(2)\quad \dots\quad i(N)\right] \tag{12}$$

$A(k)$ is the input vector at each sampling time of k, calculated as

$$A(k)=\left[a_1\quad a_2\quad \dots\quad a_n\quad \dots\quad a_N\quad e^{-\beta t_k}\right] \tag{13}$$

$$a_n=\begin{bmatrix}\sin(\omega_n(t_k-\Delta t)) & \dots & \sin(\omega_n(t_k-M\Delta t))\\ \cos(\omega_n(t_k-\Delta t)) & \dots & \cos(\omega_n(t_k-M\Delta t))\end{bmatrix}^T \tag{14}$$

As shown in (14), the elements of matrix of A are dependent on the instantaneous value of $\omega_n t_k$. For the estimation, A is synchronized to the network through a Phase-Locked Loop (PLL). $W(k)$ is considered as weight vector at each sampling time of k, which is calculated in (15).

$$W(k)=\left[W_1\quad W_2\quad \dots\ W_n\ \dots\quad W_{2N+1}\quad W_{2N+2}\right] \tag{15}$$

Where:

$$\begin{cases}W(2n)=|i_{2n}|\cos(\varphi_{2n})\\ W(2n+1)=|i_{2n+1}|\sin(\varphi_{2n+1})\\ W(2n+2)=A_{DC}\end{cases} \tag{16}$$

As observed in (16), odd and even elements of W include sinusoidal and co-sinusoidal Fourier coefficients of the signal of (10) respectively. In fact, the objective in estimating a signal is to determine the weight vector corresponding to it. In this way, the amplitude and phase angle of different harmonic orders of a signal can be achieved. Figure (6) shows the block diagram of the proposed W-Adaline estimation algorithm.

As shown in Fig. 6, at first the initial condition is applied to the weight vector W=$W(0)$. Then, according to (11), an initial estimation of the signal in the first sampling time of t_1 is obtained through the multiplication of the corresponding input vector by the weight vector. Afterwards, error vector of the initial estimation is calculated in (17),

$$E(k=1)=I_m(k=1)-A(k=1)\times W(k=1) \tag{17}$$

Where, I_m is the vector with the dimension of M which is obtained through the measurement of the estimated signal and is injected to W-Adaline.

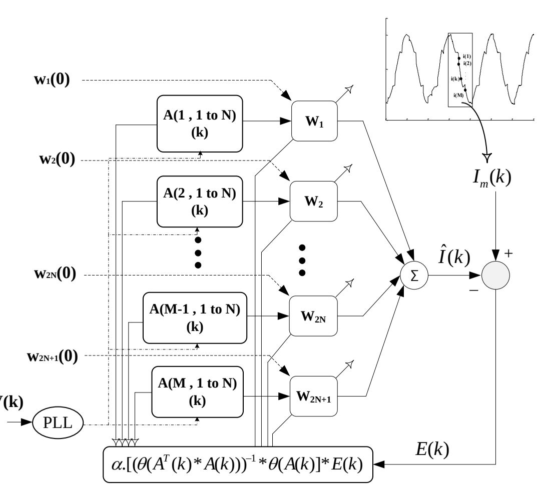

Fig. 6. Block diagram of the proposed W-Adaline algorithm

This vector is considered a moving estimation window which is formed by the previous M-1 samples and the current sample. According to the obtained estimation error, the weight vector of W-Adaline is updated at each sampling time based on the Widrow-Hoff Rule. Application of this rule is based on minimizing the squared roots of the error in the next iteration as shown in (18) [12]:

$$W(k+1) = W(k) + \Delta W(k) \tag{18}$$

Where:

$$\Delta W(k) = \alpha . \left[\left(\theta\left(A^T(k)\right) \times A(k) \right)^{-1} \times \theta\left(A(k)\right) \right] \times E(k) \tag{19}$$

In (19), k is the sample number and α is considered a learning factor of the neural network. $\theta(A(k))$ is a non-linear function applied to increase the convergence of the algorithm. This function is defined in (20),

$$\theta\left(A(k)\right) = \begin{bmatrix} SGN(a_{11}) & \dots & SGN(a_{1N}) \\ \vdots & \ddots & \vdots \\ SGN(a_{M1}) & \dots & SGN(a_{MN}) \end{bmatrix} \tag{20}$$

Where:

$$SGN(a_{ij}) = \begin{cases} -1, a_{ij} < 0 \\ +1, a_{ij} \geq 0 \end{cases} \tag{21}$$

$$i = 1, 2, \dots, M \ \& \ j = 1, 2, \dots, N$$

By following the above procedure, an updated weight vector W=W(1) is obtained for the next iteration. In the second iteration, similarly to the first step, a recent estimation of the current is made by multiplying the updated weight vector by the input vector A(k=2). Then, the error of the estimation is formed by subtracting the recent estimated vector from the estimation window which has already moved ahead by one sample. This error is then applied to update the next iteration. This procedure takes place for each time sample, and in each iteration the estimation error is reduced with respect to the previous iteration. Eventually, at the iteration corresponding to the time sample k_f, the estimation error would almost become zero, and the W-Adaline is said to be converged.

$$E(k = k_f) \cong 0 \tag{22}$$

$$\hat{I}(k = k_f) \cong I_m(k = k_f) \tag{23}$$

In this case, the weight vector defines the Fourier coefficients of the estimated signal. If vector of $W_f(k)$ is considered as the vector of the Fourier coefficient after the convergence, the amplitude and phase angle of the harmonic component of the order n can be calculated as in (24), (25) respectively.

$$\left|\hat{i}_n\right| = \sqrt{W_f^2(2n) + W_f^2(2n-1)} \tag{24}$$

$$\hat{\varphi}_n = \tan^{-1}\left[\frac{W_f(2n-1)}{W_f(2n)}\right] \tag{25}$$

Application of W-Adaline depends on choosing three parameters: sampling frequency (f_s), length of the estimation window (M), and learning factor of neural network (α). Based on the Nyquist criterion, the sampling frequency is chosen as twice the highest harmonic order [13]. However, there is no analytical approach to determine the length of the window of the estimation algorithm. The learning factor is generally a number less than the unity, and it highly influences the rate of damping the oscillations and the convergency of the algorithm. It is important to notice that each individual input parameter of W-Adaline is extremely dependent on the other two ones. To select the input parameters optimally, a comprehensive sensitivity analysis is required to determine the effect of each input parameter on the response of W-Adaline. This could be performed by means of adaptive fuzzy logic techniques. However, in this paper parameters have been selected based on trial and error to meet the suitable response.

IV. Simulation of W-Adaline in Matlab/Simulink

In this analysis, a mathematical case-study of a signal containing a damped DC component in addition to the 1st, 2nd, and 3rd order harmonics is created in (26).

$$\begin{aligned} i(t) = \sin(100\pi t + 190^\circ) + \\ 0.55\sin(200\pi t + 265^\circ) + \\ 0.2\sin(300\pi t + 0^\circ) + 0.8e^{-5t}; t > 0.023 \end{aligned} \tag{26}$$

The case-study signal is shown in Fig. 7 which is applied to W-Adaline, conventional Adaline, and FFT algorithms in order to estimate the amplitude and phase angle of each harmonics and the value of the DC component.

The sampling frequency and the learning factor of both algorithms are taken as f_s = 2.5 kHz and α = 0.25 respectively. The estimation window of W-Adaline includes 12 samples. Results of the estimation of the amplitude and phase angle of the 1st, 2nd, and 3rd order harmonics and the value of the DC component are compared in Fig. 8-11.

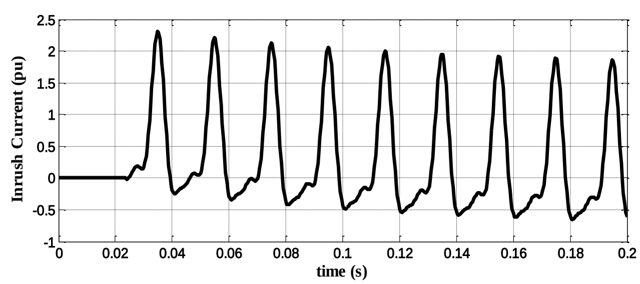

Fig. 7. Case-study signal of an inrush current

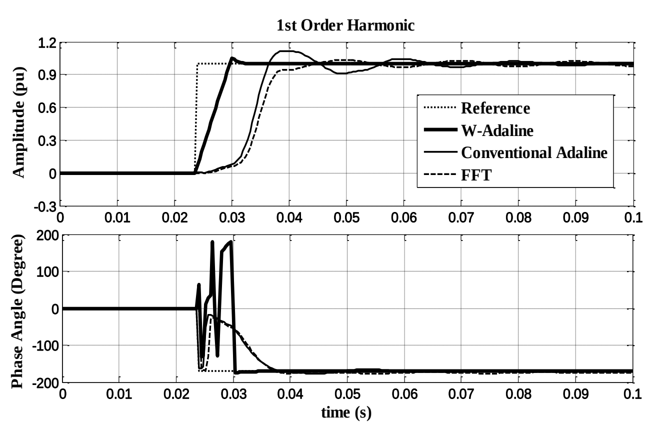

Fig. 8. Estimation of amplitude and phase angle of 1st order harmonic

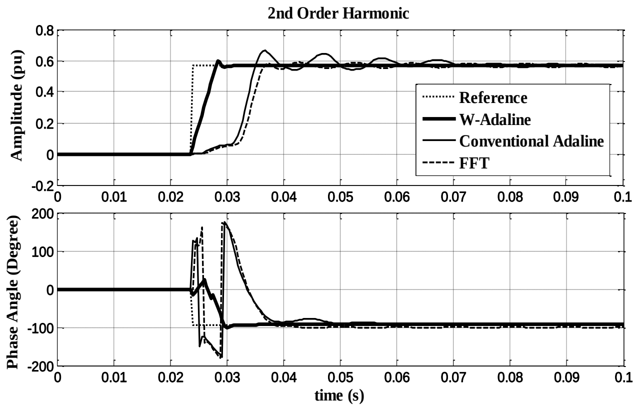

Fig. 9. Estimation of amplitude and phase angle of 2nd order harmonic

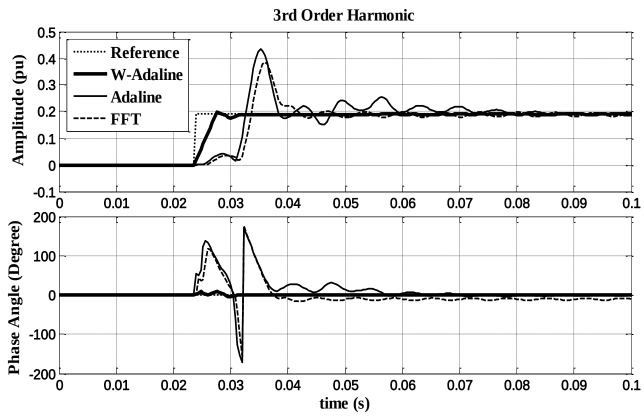

Fig. 10. Estimation of amplitude and phase angle of 3rd order harmonic

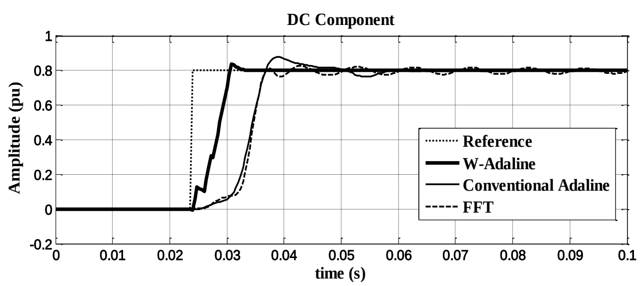

Fig. 11. Estimation of DC component

As shown in the results of Fig. 8-11, the response of W-Adaline is about 5 ms faster than the conventional Adaline and the FFT, while not including fluctuations. The accuracy of the mentioned estimation algorithms can be quantitatively analyzed by defining a relative error as expressed in (27).

$$Er = \log\left\{ \left| X_{Estimated} - X_{Reference} \right| \times \Delta t \right\} \quad (27)$$

This error represents the surface formed by the deviation of the estimated signal X (either amplitude or phase angle) from its reference value at each sampling time (0.4 ms).

The error is specified in a log scale, so that it can properly represent small error values. Following (27), the accuracy of the mentioned algorithms in estimating the amplitude and phase angle is shown in the bar chart of Fig. 12, 13 respectively. As observed in the results of Fig. 12, the accuracy of W-Adaline is much higher than the conventional Adaline and FFT in estimating the amplitude of the harmonics and the DC component.

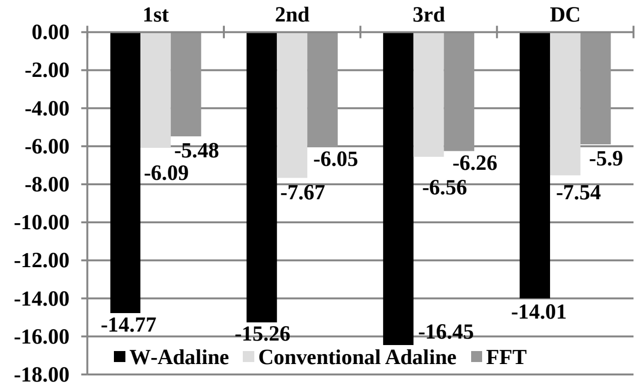

Fig. 12. Relative error of W-Adaline and conventional Adaline in estimating the amplitude

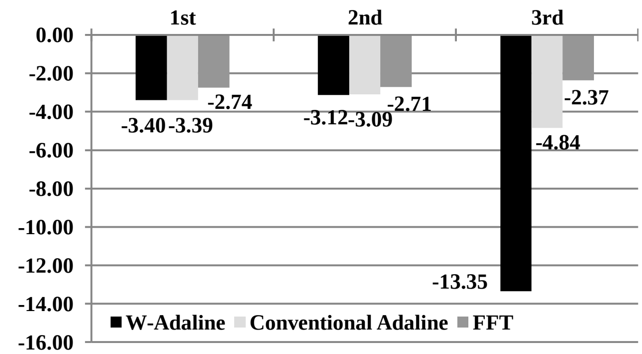

Fig. 13. Relative error of W-Adaline and conventional Adaline in estimating phase angle

The results of Fig. 13 show that relative error of the phase angle estimation is remarkably higher than the amplitude estimation. However, even in this case the accuracy of W-Adaline is relatively higher than the other two algorithms.

V. Energizing of a No-load Transformer in PSCAD/EMTDC

This section investigates the performance of W-Adaline, conventional Adaline, and FFT in a quasi-real situation. For this purpose, a no-load power transformer with the parameters given in the appendix table [14] is simulated in PSCAD/EMTDC. The schematic diagram of the transformer is shown in Fig. 13. Waveforms of the source voltage and inrush current are shown in Fig. 14.

The sampling frequency, learning factor, and damping factor of W-Adaline and conventional Adaline are taken as f_s = 2.5 kHz, α = 0.4, and β = 1 respectively. The estimation window of W-Adaline includes 50 samples. Figures 15, 16 show the result of the estimation algorithms.

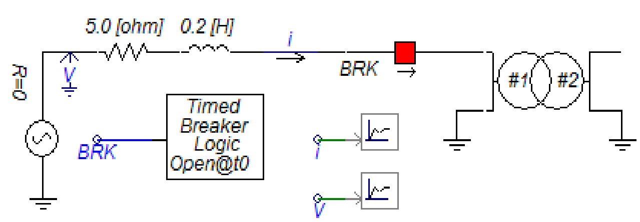

Fig. 13. Single line diagram of an energized transformer

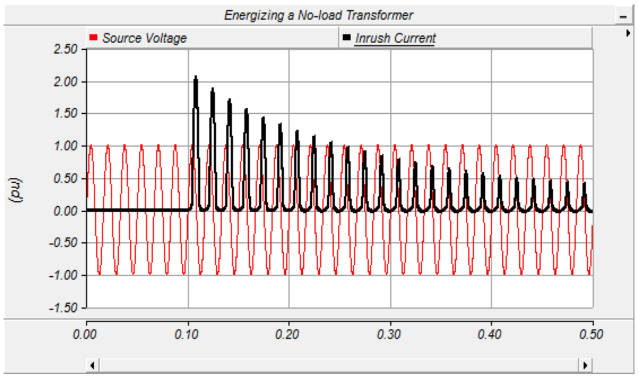

Fig. 14. Source voltage and inrush current of a case-study transformer

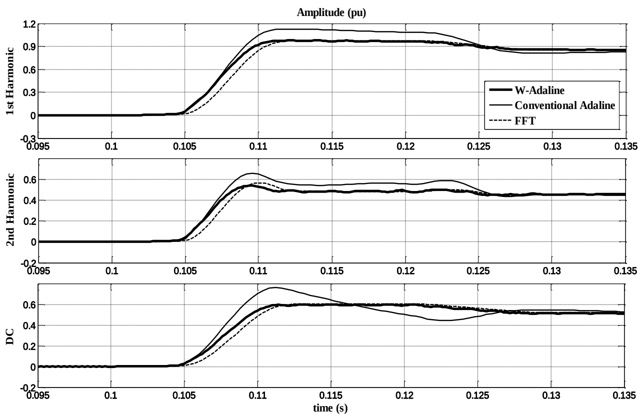

Fig. 15. Estimation of the amplitude of inrush current
(1st and 2nd order harmonic and DC component)

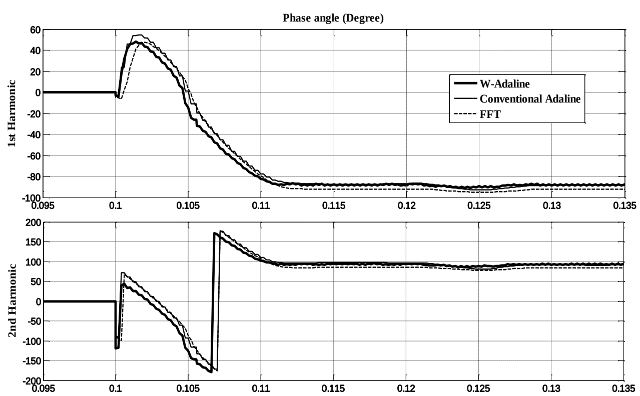

Fig. 16. Estimation of the phase angle of inrush current
(1st and 2nd order harmonic)

The results of Fig. 15, 16 show that the performance of W-Adaline is superior, in accuracy and response time, to the conventional Adaline and FFT.

VI. Conclusions

In this paper, the erroneous operation of differential relays in distinguishing inrush current during transformer energization was discussed. Since the inrush current contains specific components such as DC and 2nd order harmonic, it can be discriminated from internal short circuit faults. Furthermore, an algorithm is proposed, W-Adaline, to estimate these components and prevent false differential relay tripping. W-Adaline is based on a moving estimation window. The algorithm was tested using Matlab/Simulink with excellent results in terms of accuracy and speed of response. The capability of the proposed W-Adaline was also verified in a more realistic simulation environment by energizing an unloaded transformer in PSCAD/EMTDC.

VII. Appendix Table

Parameters	Values
Rated Power	100 (MVA)
Nominal voltage	230/132 (kV)
Frequency	60 (Hz)
No load current	5% (of nominal current)
Leakage reactance	15%
No load loss	160 (kW)
Short circuit loss	1150 (kW)
Residual flux	0.5 (pu)

VIII. References

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