A Novel Method for the Optimal Parameter Selection of Discrete-Time Switch Model

R. Razzaghi, C. Foti, M. Paolone and F. Rachidi

***Abstract* – The paper proposes a novel method for the optimal parameter selection of the discrete-time switch model used in circuit solvers that adopt the Fixed Admittance Matrix Nodal Method (FAMNM) approach. As known, FAMNM-based circuit solvers allow to reach efficient computation times since they do not need the inversion of the circuit nodal admittance matrix. However, these solvers need to optimally tune the so-called discrete switch conductance, since this parameter might largely affect the simulations accuracy. Within this context, the method proposed in the paper minimizes the distance between the eigenvalues of the original circuit's nodal admittance matrix with those associated with the presence of the discrete-time switches. The method is proven to provide values of the discrete-time switch conductance that maximize the simulation accuracy and minimize the losses on this artificial parameter. The performances of the proposed method are finally validated by making reference to two test cases: (i) a circuit composed of RLC elements, (ii) a network model that includes a single-phase transmission line.**

***Keywords*: Discrete-time switch model, modified nodal analysis, fixed admittance matrix nodal method, real-time simulation.**

I. INTRODUCTION

Accurate and computationally-efficient time-domain simulation of power systems including switches (e.g., traditional circuit breakers or power electronic devices) is a challenging subject since the tradeoff between accuracy and computation time depends on the adopted models of the switching devices. These aspects play an important role when power systems models need to account for a large number of switches and, also, when real-time simulation constraints have to be considered.

Detailed switch models reproducing their physical properties are used when studying phenomena such as switching losses, arcing times and electromagnetic transients associated with switching arc extinction. However, in many power system applications, these sophisticated models cannot be used because of their required computational efforts and complexity of deployment. As a result, simplified switch models have been proposed in the literature (e.g. [1,2]).

In addition to the ideal switch model, one of the most popular methods consists in representing this device as a lumped electrical component (e.g., the two-valued resistor model) with a value associated to each switch state. In particular, a typical representation consists in replacing the switch by means of a resistor characterized by a "small" value of resistance for the "closed-state" and a "large" value for the "open-state". However, in both the ideal switch and the two-valued resistor models, the system's admittance matrix needs to be updated and re-factorized after each switching state change (e.g. [3,4]).

Within the context of real-time simulations, updating the admittance matrix imposes additional computational burden to solution algorithms that need to be executed within a determined time window. Another critical example refers to the off-line simulation of power electronics converters characterized by a large number of switches (e.g., Modular Multilevel Converters (MMC) used in HVDC systems). Indeed, high switching frequencies, combined with the high number of switching devices, result in prohibitive computational times [5]. As a consequence, in both of the above-mentioned cases, the admittance matrix re-factorization represents a major obstacle.

A possible approach to circumvent this problem is the use of modeling techniques that keep the system admittance matrix constant (e.g., [6,7]). To this end, discrete circuit models for switching devices were proposed [1,8,9]. The basic idea is that the switch could be represented by an inductance when its state is 'closed' and by a capacitance when its state is 'open'. As a consequence, the discrete switch model is represented by an equivalent conductance (G_s) in parallel with a current source controlled by the so-called history term (e.g., [4]). The consequence of such a representation is an approach for the circuit solution called Fixed Admittance Matrix Nodal Method (FAMNM) [10]. In this category of solvers, the discrete-time switch conductance G_s is kept constant during switches state-transitions, the change of the switches state affects only the value of the current source which does not appear in the circuit admittance matrix. On the other hand, such a switch representation introduces artificial resonance frequencies that produce unreal transients [2,6].

Solutions to solve this problem have been proposed in the literature. In particular, a damping resistance can be added in series to the discrete-time switch model [11]. However, this approach increases the model complexity and, also, poses the problem of the optimal choice for the value of such a damping

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resistance.

Another more consistent approach to solve this problem is the optimal selection of the G_s parameter. One possibility is to consider an a priori value for G_s and, then, find the corresponding optimal value by comparing the simulation results with benchmark ones (i.e., obtained by off-line simulations where the switches are represented by ideal models) in order to minimize the relevant errors [2]. However, such an approach provides solutions that require specific tunings and, also, is impracticable for networks including a large number of switches, for which the number of possible permutations becomes prohibitively large.

Within this context, this paper presents a new method to find the optimal G_s value. The proposed method is based on the minimization of the Euclidian distance between the eigenvalues of three nodal admittance matrices. The first matrix refers to the circuit where discrete-time switch models are accounted for. The second and third matrices are associated with the circuit with ideal switches in 'open' and 'closed' states, respectively.

The structure of the paper is as follows. Section II presents the mathematical model describing the FAMNM. Section III illustrates the proposed method to find the optimum value for the discrete-time switch conductance. Section IV presents a validation of the proposed method by making reference to two test cases. Finally, Section V concludes the paper with the final remarks and potential deployment of the proposed method.

II. FAMNM REPRESENTATION OF THE SWITCH

The idea of FAMNM is the discrete-time representation of the switch with a constant impedance model [1,8-10]. Such a model assumes that the equivalent model of the ideal switch is piecewise linear and could be represented by a capacitance when it is open and an inductance when it is closed. The inductance and capacitance are represented, in a discrete form, by a conductance in parallel with a current source. In order to set the value of the conductance for both switch states, in case the backward-Euler numerical integration method is used, the following constraint should be satisfied:

$$G_s = \frac{C_s}{\Delta t} = \frac{\Delta t}{L_s} \tag{1}$$

where C_s and L_s are the discrete-time switch capacitance and inductance respectively, and Δt is the simulation time-step. For other numerical integration methods, a similar approach could be applied to find the G_s value.

As a consequence of this representation, the relevant model is composed of a constant conductance and a current source (see Fig. 1). As a function of the switch on/off state, the value of the current source is updated at each time-step based on the switch current/voltage. The advantage of this method is that the value for the switch conductance G_s is fixed irrespective of the switch on/off state. As a result, the nodal admittance matrix will remain unchanged during switching operations as the switch state only affects the value of the shunt current source. The current source associated with the switch at the simulation step $n+1$ is defined as [1]:

$$J_s^{n+1} = \begin{cases} -i_s^n & \text{for the 'on' state} \\ G_s v_s^n & \text{for the 'off' state} \end{cases} \tag{2}$$

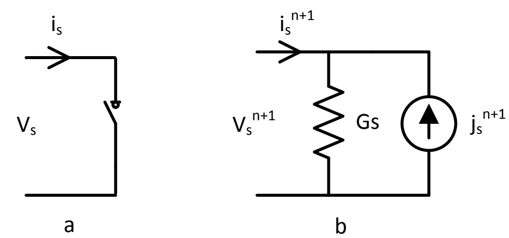

Fig. 1. Ideal switch (a) and corresponding discrete-time model (b).

An important issue regarding the FAMNM is the appropriate selection of G_s since its value affects the accuracy of the approximate switch model and, as a consequence, it affects the overall model accuracy [1,2]. According to [1], one approach for determining G_s is to select C_s and L_s equal to the corresponding real switch parameters. Then, the values of G_s and Δt could be determined based on (1). However, the main drawback of this approach is that the required simulation time step might become extremely small resulting in an increased computational time [1].

As already stated in the introduction, a different procedure refers to the assessment of an optimal G_s value by means of a trial-and-error process where benchmark results are obtained by means of off-line simulations carried out by adopting ideal switches. However, for cases characterized by a large number of switches, such an approach requires non-negligible pre-computation efforts.

III. PROPOSED METHOD

A. *Network Modeling*

Generally, there are two main types of solution methods currently used in the field of power system, power electronics and electronic circuit simulations [12]: (i) modified nodal analysis (MNA) and (ii) State-Space (SS) approach. In this study, in order to formulate the network equations, MNA is used. Compared with the state-space method, MNA provides a more straightforward way of formulating the network equations [12]. The MNA formulation is expressed as

$$[A_n][x_n] = [b_n] \tag{3}$$

where matrix $[A_n]$, in the discrete time domain, is formed by the discrete representation of the network elements; $[x_n]$ is the vector of unknown network's node voltages and branch currents; and $[b_n]$ is a vector composed of the independent sources and current history terms related to the network components. In each iteration, the unknown vector $[x_n]$ is calculated and, then, the vector $[b_n]$ is updated. It is worth noting that representing switches with FAMNM allows to keep $[A_n]$ fixed during switching transitions.

In order to solve equation (3) in discrete form, a numerical integration method should be used. Several numerical integration methods can be applied to solve differential equations. Among them, forward Euler, backward Euler and the trapezoidal methods are the most common in power system applications [13]. The trapezoidal rule can cause numerical oscillations under certain conditions due to the neglected terms [14]. In comparison to the trapezoidal method, backward Euler rule gives better damping to numerical oscillations introduced by switches [1]. In this paper, we have adopted the backward Euler method.

In view of the use of the MNA, all the relevant network components should be discretized to form the so-called nodal equations [14,15]. While the behavior of power system variables is continuous in time, this approach introduces the concept of "discrete solution". Namely, the solution of the electrical circuits' differential equations is obtained in discrete time steps. Thus, models of system elements should be formulated by means of their discrete representation. To this end, lumped elements (RLC) are represented by their discrete companion models composed of an equivalent resistance in parallel with a current source indicating the history terms [15,16]. The values of the equivalent resistance and the current source are determined based on the applied numerical integration method.

Concerning the case of transmission lines, one of the most popular solutions is given by the so-called Bergeron model [17]. It allows a straightforward representation of constant (frequency-independent) transmission line models [4] and, with some adaptations, it can also be applied to the case of frequency-dependent transmission lines [18]. As well known, this approach is based on a circuit representation of the telegraphers' equations where each line termination is replaced by means of a lumped impedance in parallel with a controlled current or voltage source.

B. Proposed method for the optimal assessment of discrete-time switches conductance

The proposed method is based on the minimization of the changes of nodal admittance matrix eigenvalues $[A_n]$ introduced by the discrete-time switches. In particular, it is based on the minimization of the Euclidian distance between the eigenvalues of the network admittance matrix based on FAMNM and those associated with the admittance matrices of two reference networks corresponding to the two states of the switches (on and off). To this end, the following procedure is applied (note that, for the sake of clarity, we are making reference to a circuit with a single switch; the procedure can be straightforwardly extended to multiple switches).

First, the eigenvalues of the nodal admittance matrices, assumed all of rank n, corresponding to (i) FAMNM method, (ii) ideal switch in 'on' state, and (iii) ideal switch in 'off', are calculated. Namely,

$$\lambda_i^p\left(G_s\right) = eig\left\{\left[A_n^p\right]\right\}, i = 1,2,...,n \tag{4}$$

$$\lambda_i^c = eig\left\{\left[A_n^c\right]\right\}, i = 1,2,...,n \tag{5}$$

$$\lambda_i^o = eig\left\{\left[A_n^o\right]\right\}, i = 1,2,...,n \tag{6}$$

where $[A_n^p]$, $[A_n^c]$, and $[A_n^o]$ are the nodal admittance matrices when the switch is represented by FAMNM, ideal switch in 'on' state, and ideal switch in 'off' state, respectively. $\lambda_i^p\left(G_s\right)$, λ_i^c, λ_i^o indicate respectively the corresponding eigenvalues for each nodal admittance matrix. It is worth noting that $\lambda_i^p\left(G_s\right)$ is a function of the value of G_s whereas, λ_i^c and λ_i^o are fixed.

After sorting the eigenvalues order, the squared Euclidian distances for each eigenvalue are calculated as:

$$\begin{aligned}\gamma_i^c\left(G_s\right) = &\left\{\mathrm{Re}\left[\lambda_i^p\left(G_s\right)\right] - \mathrm{Re}\left[\lambda_i^c\right]\right\}^2 + \\ &+\left\{\mathrm{Im}\left[\lambda_i^p\left(G_s\right)\right] - \mathrm{Im}\left[\lambda_i^c\right]\right\}^2\end{aligned} \tag{7}$$

$$\begin{aligned}\gamma_i^o\left(G_s\right) = &\left\{\mathrm{Re}\left[\lambda_i^p\left(G_s\right)\right] - \mathrm{Re}\left[\lambda_i^o\right]\right\}^2 + \\ &+\left\{\mathrm{Im}\left[\lambda_i^p\left(G_s\right)\right] - \mathrm{Im}\left[\lambda_i^o\right]\right\}^2\end{aligned} \tag{8}$$

In these equations, $\gamma_i^c\left(G_s\right)$ and $\gamma_i^o\left(G_s\right)$ indicate, as a function of G_s, the squared Euclidian distances between the i^{th} eigenvalue of $[A_n^p]$ and the corresponding one of $[A_n^c]$ and $[A_n^o]$, respectively. Then, the overall distance is calculated as:

$$\eta_i\left(G_s\right) = \gamma_i^o\left(G_s\right) + \gamma_i^c\left(G_s\right) \tag{9}$$

Therefore, it is possible to define an objective function as the sum of all normalized distances (η_{i,G_s}):

$$\Gamma\left(G_s\right) = \sum_{i=1}^{n}\left(\frac{\eta_i\left(G_s\right)}{\max\left\{\eta_i\left(G_s\right)\right\}}\right) \tag{10}$$

Note that the normalization is done in order to give the equal weight to all eigenvalues taking part in the objective function. The optimum G_s value is defined as the one that minimizes the objective function (10), namely:

$$G_s^* = \arg\Big|_{G_s} \min\left\{\Gamma\left(G_s\right)\right\} . \tag{11}$$

In order to verify the correspondence of the minimum of the objective function (10) with the best accuracy of the circuit simulation, an error function has been defined. It includes time-domain switch voltage and current waveforms subsequent to switch state transitions (in particular, subsequent to pairs of 'on'-'off' transitions). Indeed, as it is stated in [1], switch current error in 'off' state is proportional to G_s, whereas, switch voltage error in 'on' state is inversely proportional to G_s. This specific property has been exploited to define the error function. In particular, the following procedure has been adopted. The switch-current error is calculated as the difference between the instantaneous values of the switch current given by the FAMNM solver and the current provided by a reference simulation performed, in our case, using the EMTP-RV simulation environment [15,19-20] where the switch is considered as an ideal device. It is worth noting that this error is calculated in the time window when the switch is 'off'. The same procedure is considered to calculate the switch voltage error in the period when the switch is 'on'. The switch current and voltage errors, $E^I(G_s)$ and $E^V(G_s)$, are then given by

$$E^I(G_s) = \sum_{k=0}^{m} \left[i(G_s)_k - i_k^* \right]^2, m = \frac{T}{\Delta t} \tag{12}$$

$$E^V(G_s) = \sum_{k=0}^{m} \left[v(G_s)_k - v_k^* \right]^2, m = \frac{T}{\Delta t} \tag{13}$$

where T is the time window where the switch-state changes are observed. Discrete variables $i(G_s)_k$ and $v(G_s)_k$ correspond to the discretized instantaneous values of switch current and voltage when the switch is represented by its approximate model and, thus, they are function of G_s. Discrete variables i_k^* and v_k^* are the corresponding discretized instantaneous switch current and voltage obtained from references simulations where the switch are represented as ideal devices. As a consequence, it is possible to define an overall error function as:

$$\mathrm{E}(G_s) = \left\{ \frac{E^I(G_s)}{\max\left\{E^I(G_s)\right\}} + \frac{E^V(G_s)}{\max\left\{E^V(G_s)\right\}} \right\} \tag{14}$$

An additional way to justify the proposed objective function is to consider switch losses when they are represented by using the FAMNM approach. To this end, the switch losses in 'off' and 'on' states can be straightforwardly calculated as follows:

$$P^O(G_s) = \sum_{k=0}^{m} \left(v^o(G_s)_k \cdot i^o(G_s)_k \right), m = \frac{T}{\Delta t} \tag{15}$$

$$P^C(G_s) = \sum_{k=0}^{m} \left(v^c(G_s)_k \cdot i^c(G_s)_k \right), m = \frac{T}{\Delta t} \ . \tag{16}$$

where T is the time window where the switch-state changes are observed. Discrete variables $i^o(G_s)_k$ and $v^o(G_s)_k$ correspond to the discretized instantaneous values of switch current and voltage when the switch is represented by its approximate model and its state is 'off'. $i^c(G_s)_k$ and $v^c(G_s)_k$ are switch current and voltage when the switch state is 'on'.

The total switch losses as a function of G_s are calculated by adding the normalized values of the 'on' and 'off' states as formulated in (17). It is worth observing that, the best G_s value is the one that minimizes this losses function.

$$P(G_s) = \left\{ \frac{P^O(G_s)}{\max\left\{P^O(G_s)\right\}} + \frac{P^C(G_s)}{\max\left\{P^C(G_s)\right\}} \right\} \tag{17}.$$

In the next section, we will show that the optimal G_s value provided by (11) corresponds to the minimum of the error function (14) and losses function (17), proving that the proposed approach satisfies these two criteria at the same time.

IV. Validation

In order to verify our proposed method, two simulation cases are considered. The first simulation refers to an electric circuit composed of RLC elements and a switch. The schematic diagram of the considered circuit is shown is Fig. 2. This circuit is simulated within the EMTP-RV simulation environment and the obtained results are considered as the benchmark ones. In addition, the circuit equations are also formulated and numerically solved by the procedure mentioned in Section III-A, and implemented in MATLAB. The numerical integration method and the relevant integration time-step have been chosen to be the same in both EMTP-RV and FAMNM numerical simulations (i.e., backward-Euler and Δt=4 μs for the integration method and time step respectively).

A. Simple RLC circuit test case

For the circuit shown in Fig. 2, the nodal admittance matrices for the cases where the switch is represented by FAMNM, ideal switch in 'on' case, and ideal switch in 'off' case are formed. Then, according to the proposed method, the objective function is calculated. In order to calculate the error and losses functions, the following switching transition is considered: the switch is in open position and it is closed at t=10 ms. Then, it is opened again at t=25 ms (see Fig. 3). For all values of G_s (i.e., $0 \le G_s \le 1$), equations (10), (14) and (17) are calculated. The objective function together with corresponding error and losses functions are shown in Fig. 4. As it can be clearly observed, all the three functions have their minimum when G_s is equal to 0.28.

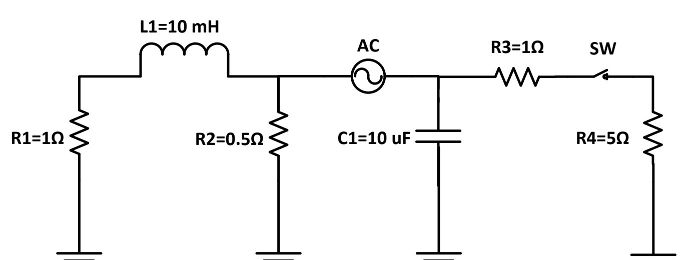

Fig. 2. Schematic representation of first test case composed of RLC elements and a switch.

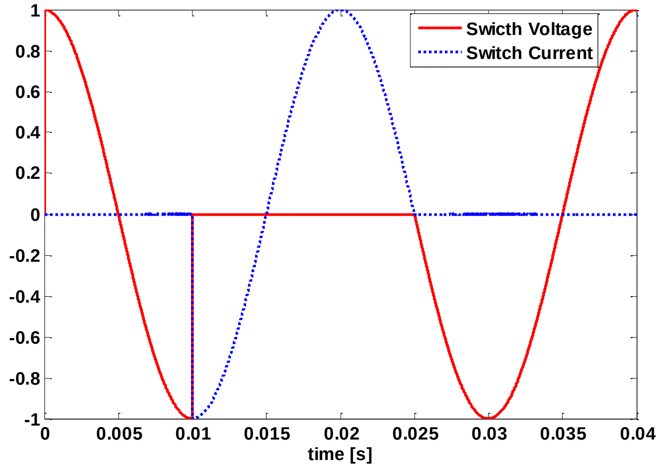

Fig. 3. Reference waveforms of switch current and voltage for two switching transitions: open-close-open.

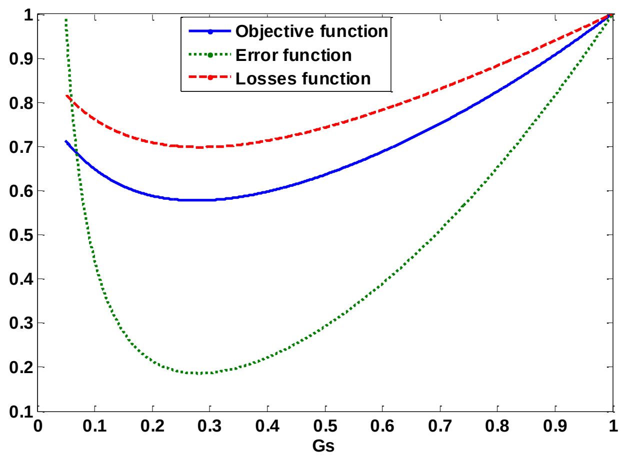

Fig. 4. Objective, error, and switch-losses functions for the case of the RLC test case (Fig. 2) with variable G_s.

B. Transmission line test case

The second simulation example refers to a network composed of a single-conductor transmission line which is represented by using a constant-parameter model [4]. Such a model has been chosen in order to show the robustness of the proposed method even for simulations involving propagation along transmission lines (indeed, the presence of the G_s is able to largely affect the consequent electromagnetics transients). The line ends are terminated on high resistances. The network schematic is shown in Fig. 5.

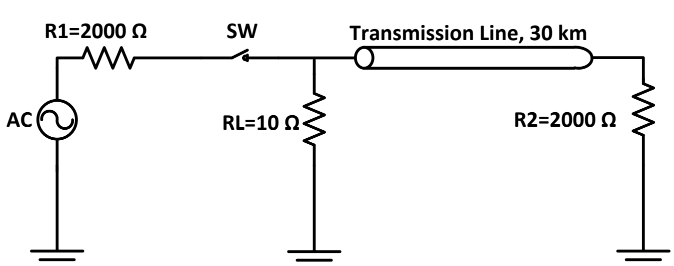

Fig. 5. Schematic representation of the second test case composed of a single-conductor transmission line and a switch.

By applying the same procedure for $0 \le G_s \le 1$, the objective, error, and losses functions are calculated (see Fig. 6). As it is shown on Fig. 6, these three functions exhibit the same behavior as for the previous case, with a minimum occurring for a value of G_s equal to 0.055.

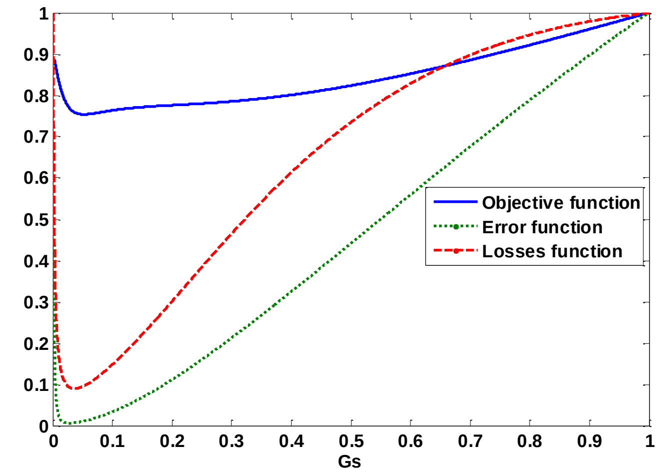

Fig. 6. Objective, error, and switch-losses functions for the case of transmission line test case with variable G_s.

With reference to the second test case, Figs. 7 and 8 illustrate the time-domain simulations of the voltage at the end of the line and the switch current at the beginning of the line for different values of G_s including the optimal one previously identified (G_s^*=0.055). The following observations can be made: (i) the value of G_s affects the accuracy of the simulated model drastically (ii) the best match between reference and FAMNM simulations corresponds to the optimal G_s^*.

Fig. 4 and Fig. 6 show that the proposed objective function could be utilized as an efficient tool to find the optimum G_s value without performing the off-line benchmark simulations as the effect of G_s on the adopted system model could be precisely predicted. This method could also be generalized to networks where several switches are placed.

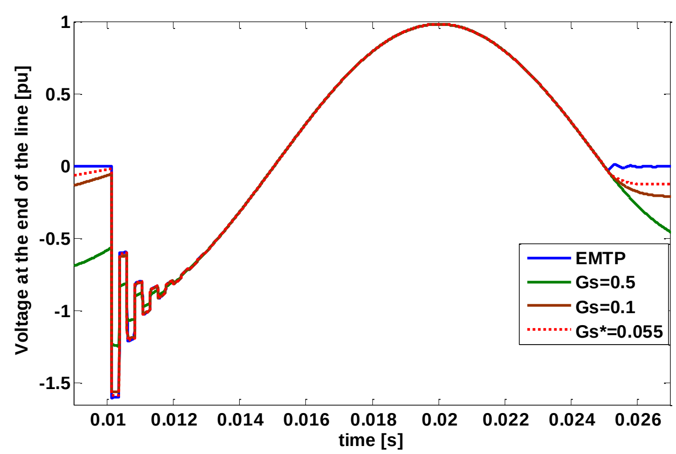

Fig. 7. Time-domain simulated waveforms for voltage at the end of the transmission line (second test case) for different values of G_s.

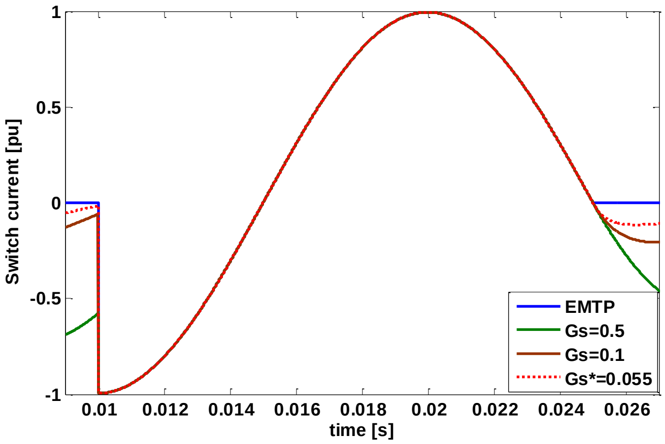

Fig. 8. Time-domain simulated waveforms for the switch current at the beginning of the transmission line (second test case) for different values of G_s.

V. THE EXTENSION TO MULTIPLE SWITCHES

The proposed method could be generalized for the case of networks with multiple switches.

For the case of a network with N switches, the number of possible switching permutations is 2^N. Therefore, there are 2^N set of eigenvalues of the nodal admittance matrix associated to ideal switches representations, namely:

$$\lambda_i^x = eig\left\{\left[A_n^x\right]\right\}, \begin{cases} i = 1,2,\ldots,n \\ x \in 2^N \end{cases} \tag{18}$$

where x is the one of possible switches permutations. Additionally, the eigenvalues of nodal admittance matrix associated to the FAMNM switch representation are function of several switches conductances:

$$\lambda_i^p\left(G_{s1},G_{s2},\ldots\right) = eig\left\{\left[A_n^p\right]\right\}, i = 1,2,\ldots,n \tag{19}$$

The squared Euclidian distances associated with all eigenvalues have to be calculated for all the possible permutations considering the various conductance values as reported below:

$$\begin{aligned}\eta_i\left(G_{s1},G_{s2},\ldots\right) = \sum_x\Big(&\left\{\mathrm{Re}\left(G_{s1},G_{s2},\ldots\right) - \mathrm{Re}\left[\lambda_i^x\right]\right\}^2 + \\ &+\left\{\mathrm{Im}\left[\lambda_i^p\left(G_{s1},G_{s2},\ldots\right)\right] - \mathrm{Im}\left[\lambda_i^x\right]\right\}^2\Big)\end{aligned} \tag{20}$$

The proposed objective function is then defined based on the calculated Euclidian distances as:

$$\Gamma\left(G_{s1},G_{s2},\ldots\right) = \sum_{i=1}^{n}\left(\frac{\eta_i\left(G_{s1},G_{s2},\ldots\right)}{\max\left\{\eta_i\left(G_{s1},G_{s2},\ldots\right)\right\}}\right) \tag{21}$$

The optimum values of switches conductance are found as follows:

$$G_{s1}^*,G_{s2}^*,\ldots = \arg\Big|_{G_{s1},G_{s2},\ldots} \min\left\{\Gamma\left(G_{s1},G_{s2},\ldots\right)\right\} \tag{22}$$

An approach to reduce the computational complexity of the optimal problem stated by (22) is to group the switches with identical G_s. This is the case, for instance, of the switches representing the three poles of a three phase breaker or to switches located in topological proximity.

However, it is worth noting that further investigations are needed in this respect in order to reduce the computational complexity of the problem formulated by (22).

VI. CONCLUSION

In this paper, a new method to find the optimum value of discrete-time switch conductance has been proposed. As known, the value of this parameter should be chosen in a way to minimize the errors introduced by the approximate representation of the discrete-time switch model.

The method is based on the minimization of the Euclidian distance between the eigenvalues of the network admittance matrix based on FAMNM and those associated with the admittance matrices of two reference networks corresponding to the two possible states of the switches. To prove the correctness the proposed method, a comparison between the considered Euclidian distance and other error functions is analyzed and discussed. By making reference to two different test cases (i.e., a simple RLC circuit and a system that includes a transmission line), the proposed method has been proven to be robust in identifying the optimal conductance value of the discrete-time switch model that minimizes: (i) the differences with reference-model current/voltage waveforms, and (ii) losses on the discrete-time switch conductance.

The proposed method can be therefore used as an efficient tool to find optimal G_s values without performing off-line benchmark simulations as the effect of G_s on a given system could be precisely predicted.

It is finally worth noting that the method proposed in the paper could also be generalized to networks with multiple switches by extending the proposed objective function to an arbitrary number of switches.

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