On the Inclusion of Nonlinear Conditions in Numerical Laplace Transform Analysis

C. Villanueva, P. Moreno, A. Ramírez, P. Gómez, J.L. Naredo

***Abstract*--In this paper, a description and comparison of two methods for the inclusion of nonlinear conditions in frequency domain transients analysis are presented. These methods are the piece-wise linear method and the polynomial method. The first is based on the superposition principle and is applied to model elements with nonlinear V-I characteristics represented in a piecewise-linear manner. The second is a Newton-type methodology. The basic idea of this latter method is to decompose the complete network into linear and nonlinear subnetworks. The nonlinear load is represented by the instantaneous current/voltage relation through a polynomial of order *p*. An example involving nonlinear loads in a network is presented for illustration of the procedures.**

***Keywords*: Transient Analysis, Frequency Domain Analysis, Nonlinear elements, Numerical Laplace Transform.**

I. INTRODUCTION

PROCEDURES for computing electromagnetic transients can be classified in time domain (TD) and frequency domain (FD) methods. TD methods are preferred due to their ability to take into account network topology changes and to include nonlinear elements. In contrast FD methods are traditionally used for analyzing linear time invariant systems with parameters highly dependent on frequency. However FD methods are also able to handle, although not directly, time variant and nonlinear elements.

Nonlinear elements can be handled in the FD using Spectral balance type methods [1]-[3], superposition procedures [4]-[8] or Newton type methods [9], each one of them with different advantages and limitations. In this work a comparison only between the two last methods is presented. The reason for this is that both methods do not require pre-selection of the harmonic content and can handle any number of nonlinear elements.

In the superposition procedures time varying conditions such as switching maneuvers and faults are treated as initial condition problems [4]-[6]. The complete response due to a switching operation is obtained by the addition of the system response before switching (initial condition) to that resulting from applying a current source that performs the switch maneuver. After this time domain waveforms are obtained by an inverse transformation procedure. In particular in this work the Numerical Laplace Transform (NLT) is employed. On the other hand, the inclusion of nonlinear elements such as arresters and saturable transformer nuclei are handled by means of a piecewise-linear approximation of the V-I or the flux-current characteristics [7]-[8]. The V-I linear approximation represents a cascade connection of series circuits formed by a resistance, a voltage source and a switch. Once this representation is obtained, the procedure is reduced to a series of sequential switching maneuvers.

Alternatively to the above, FD transient simulation of a network including nonlinear elements can be performed using Newton-type methods [9]. Basically, in this kind of methods the linear and nonlinear parts of the network are treated separately and a current mismatch at the node joining both sub-networks is defined. A nonlinear load is represented by the instantaneous current/voltage relation through a polynomial of order *p*. Current entering the linear subnetwork should be equal to that entering the non-linear part. The currents sum equation at the joining node is linearized and a recursive procedure is developed to find the node voltages. The Jacobian of the linear part is a known matrix whereas the Jacobian corresponding to a nonlinear element is calculated numerically via small perturbations of the input voltage at the connecting node. The corresponding nonlinear current is calculated via NLT operations.

An example involving nonlinear loads in a network is presented for illustration and comparison of the above procedures. The example consists of an energization of a 3-phase transmission cable with surge arresters connected at the receiving end.

II. PIECE-WISE LINEAR METHOD

A. Switching Modeling

Switching operations produce changes in network topology

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that turn the network into a time variant system precluding, apparently, the use of frequency domain methods. However the Superposition Principle can still be applied to overcome these problems [4]-[6].

In the case of a closure maneuver the initial condition of the switch is that of an open circuit. Invoking the Substitution Theorem this condition is equivalent to the connection of a voltage source equal to the potential difference between the switch terminals. Switch closure is performed by means of the series connection of another voltage source with equal magnitude but opposite polarity, as shown in Fig. 1(a). Considering a closure time $t_c > 0$ the voltage source V_{swC} required to close the switch is given by the numerical Laplace transform [8]

$$V_{swC} = NLT\left\{-v_{sw}(t)\, u(t-t_c)\right\} \tag{1}$$

where $v_{sw}(t)$ is the time domain voltage between the switch terminals, supposing it open for all the simulation period.

On the other hand, for the switch opening maneuver a short circuit initial condition is considered, such that the current flowing through its terminals can be represented by a current source. Opening of the switch is then performed by a shunt connection of another current source of equal magnitude but opposite direction as shown in Fig. 1(b). Considering the first zero-crossing time t_{zc} following a specified opening time, the current source I_{swO} required to open the switch is given by

$$I_{swO} = NLT\left\{-i_{sw}(t)\;u(t-t_{zc})\right\} \tag{2}$$

where $i_{sw}(t)$ is the time domain waveform of the current flowing through the closed switch for the whole observation time.

When using the nodal method to deal with networks ideal voltage sources can not be employed to simulate switch closures. To overcome this restriction the injection of voltage V_{swC} must be accomplished by means of a Norton equivalent with current source given by

$$I_{swC} = \frac{V_{swC}}{R_x} = G_x V_{swC} \tag{3}$$

where R_x (G_x) is the resistance (conductance) needed to perform the source transformation; R_x (G_x) must be small (large) to approximate an ideal source or it can take some particular value for representing a contact condition.

The switch model suitable for simulating closures and openings using the nodal method is shown in Fig. 2. The corresponding Norton injection current J_{sw} and conductance are given by

$$J_{sw} = \begin{cases} I_{swC}, & \text{closure} \\ I_{swO}, & \text{opening} \end{cases} \tag{4a}$$

$$G_{sw} = \begin{cases} 1/R_x, & \text{closure} \\ -1/R_x, & \text{opening} \end{cases} \tag{4b}$$

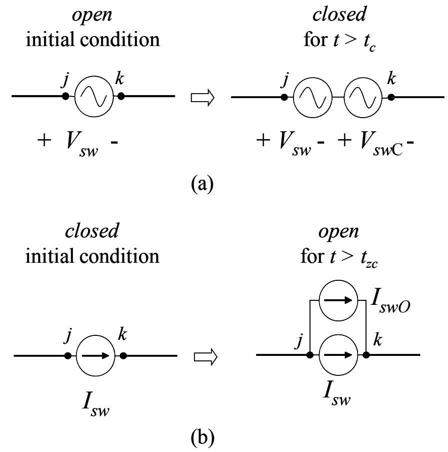


Fig. 1. Switching: (a) closure, (b) opening.

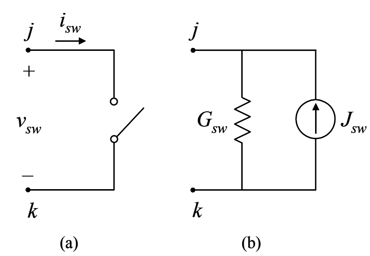


Fig. 2. Switch model: (a) ideal, (b) Norton equivalent.

After the network is solved for a set of N complex frequencies $s_n = c + j\omega_n$, each node voltage in the time domain is obtained through a numerical inverse Laplace transformation [7],[8],[10]:

$$v_i = INLT\left\{V_i\right\} \tag{5}$$

where $V_i = [\ V_{i,1}\ \ V_{i,2}\ \dots\ V_{i,N}\ ]$ is the voltage at the ith node sampled at the complex frequencies $s_1, s_2, \dots, s_N$; v_i is the time domain voltage sampled at time values $t_1, t_2, \dots, t_N$.

The complete voltage response due to a switching operation is obtained by the addition of the system response before switching (initial condition) to that resulting from applying the current source that performs the switch maneuver.

When analyzing several maneuvers in a single simulation, the number of superposition steps equals the number of events, and all the events must be ordered sequentially with increasing time.

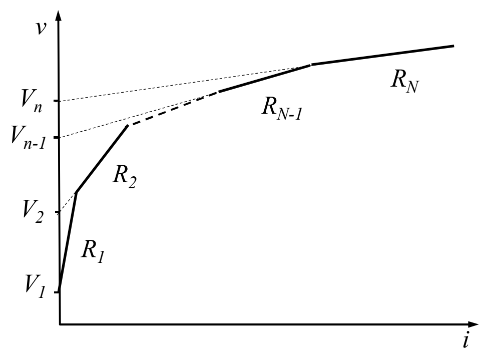
B. Nonlinear Elements Modeling

The inclusion of non-linear elements in frequency domain techniques is performed by approximating their non-linear characteristic using a piece-wise linear form. Once this is made, the simulation procedure is reduced to a sequence of switching operations [7]-[8].

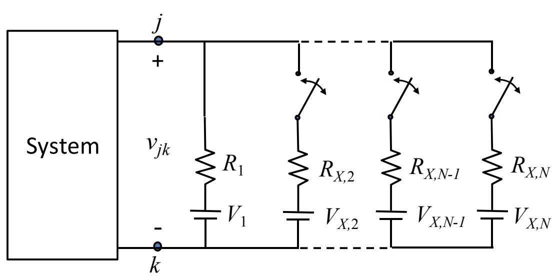
Figure 3(a) shows the *v-i* characteristic of a non-linear element approximated using a piece-wise linear form and Fig. 3(b) shows the circuit model. The function of the switches is to connect or disconnect resistances R_{Xn} and sources V_{Xn} in such a way that the correct Thevenin circuit is connected to the network, this is, at any time instant the network "sees" a resistance R_n in series with a voltage V_n, depending on the value of the voltage v_{jk}.

Consider the change from the n-1 zone (slop) to the n zone. The circuit needed to perform the maneuver is shown in Fig. 4. In order to obtain the required Thevenin equivalent formed by R_n and V_n, the connection of $R_{X,n}$ and $V_{X,n}$ should comply with

$$V_{n-1} / R_{n-1} + V_{X,n} / R_{X,n} = V_n / R_n \tag{6a}$$



(a)



(b)

Fig. 3. Piecewise linear approximation of a nonlinear element by means of N linear segments: (a) v-i characteristic, (b) corresponding circuit.

and

$$1 / R_{n-1} + 1 / R_{X,n} = 1 / R_n \,. \tag{6b}$$

Therefore

$$V_{x,n} = \frac{R_{n-1} V_n - V_{n-1} R_n}{R_{n-1} - R_n} \tag{6c}$$

$$R_{x,n} = \frac{R_{n-1} R_n}{R_{n-1} - R_n} \tag{6d}$$

Closure and opening of the switches in Fig. 3(b) is performed according to the method described in Subsection II.*A*. In both cases, the artificial current source needed for the superposition is given by the direct numerical Laplace transformation:

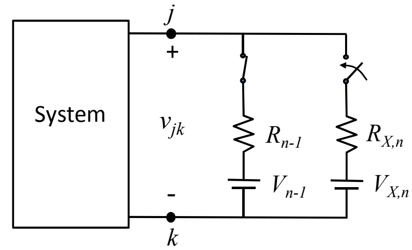


Fig. 4. Circuit for the change from slop n-1 to slop n.

$$I_{sw,n} = NLT\left\{ -\left(\frac{V_{jk} - V_{X,n}}{R_{X,n}} \right) u(t - t_{c,n}) \right\}, \tag{7}$$

where $t_{c,n}$ defines the time in which the element goes from zone n-1 to zone n for the closure case, or from zone n to zone n-1 for the opening case.

It is important to note that in the simulation procedure switch n cannot close if switch n-1 is still opened, and it cannot open if switch n+1 is still closed. To accomplish this, the time step Δt must be small enough to prevent jumps between non-contiguous segments.

C. Network Solution.

Consider the three-phase transmission line represented in Fig. 5. Using a two port admittance model for the transmission line, the switch model and superposition, results in the equivalent network shown in Fig. 6 for computing the effects of switch maneuvers and the non-linear elements connected at the line receiving end. The network nodal equation can be expressed as follows:

$$\begin{bmatrix} \mathbf{J}_{sw} \\ -\mathbf{J}_{sw} \\ \mathbf{I}_{sw,n} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_S + \mathbf{G}_{sw} & -\mathbf{G}_{sw} & \mathbf{0} \\ -\mathbf{G}_{sw} & \mathbf{Y}_{22} + \mathbf{G}_{sw} & \mathbf{Y}_{23} \\ \mathbf{0} & \mathbf{Y}_{32} & \mathbf{Y}_{33} + \mathbf{G}_{X,n} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \mathbf{V}_3 \end{bmatrix} \tag{8}$$

where $\mathbf{G}_{sw}$ is a diagonal conductance matrix that represents the three phase switch condition, $\mathbf{Y}_s$ is the source admittance; $\mathbf{G}_{X,n} = \mathbf{R}_{X,n}^{-1}$, $\mathbf{Y}_{22}$, $\mathbf{Y}_{23}$, $\mathbf{Y}_{32}$ and $\mathbf{Y}_{33}$ represent the admittance model of the transmission line.

The values of the elements of $\mathbf{J}_{sw}$, $\mathbf{G}_{sw}$, $\mathbf{I}_{sw,n}$ and $\mathbf{G}_{X,n}$ change depending on the maneuver and for each switching a complete frequency scan is performed.

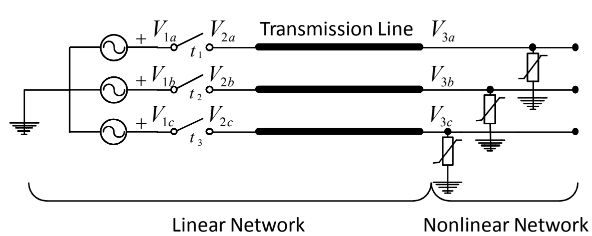


Fig. 5. Representation of the complete network.

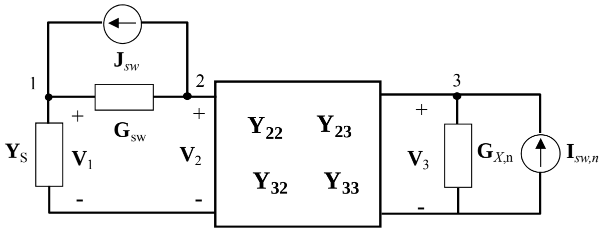


Fig. 6. Network Model.

III. POLYNOMIAL MODELING OF NON-LINEAR ELEMENTS

The basic idea is to split a complete network in a linear and a nonlinear part. For example consider again the transmission system shown in Fig. 5 where the voltage sources and the transmission line form the linear part and the surge arresters the nonlinear part.

To solve for the voltages of the complete network the Newton's method is invoked. Under current balance condition, the current $\mathbf{I}_{3L}$ entering the linear part should be equal to the current $\mathbf{I}_{3NL}$ entering the nonlinear part, this is

$$\mathbf{I}_{3L}(\mathbf{V}_3) + \mathbf{I}_{3NL}(\mathbf{V}_3) = 0 \tag{9}$$

however since the network is split a mismatch will result:

$$\Delta\mathbf{I}_3(\mathbf{V}_3) = \mathbf{I}_{3L}(\mathbf{V}_3) + \mathbf{I}_{3NL}(\mathbf{V}_3) \tag{10}$$

Linearizing (10) around a voltage $\mathbf{V}_3^k$

$$\Delta\mathbf{I}_3(\mathbf{V}_3) = \Delta\mathbf{I}_3(\mathbf{V}_3^k) + \left(\mathbf{J}_{3L} + \mathbf{J}_{3NL}\right)\left(\mathbf{V}_3 - \mathbf{V}_3^k\right) \tag{11}$$

where $\mathbf{J}_L = \mathbf{I}'_{3L}(\mathbf{V}_3^k)$ and $\mathbf{J}_{NL} = \mathbf{I}'_{3NL}(\mathbf{V}_3^k)$ correspond to the Jacobians for the linear and nonlinear parts, respectively.

By computing the root of (11) a Newton recursive scheme to calculate new voltages can be written

$$\mathbf{V}_3^{k+1} = \mathbf{V}_3^k - \left(\mathbf{J}_L + \mathbf{J}_{NL}\right)^{-1} \Delta\mathbf{I}_3(\mathbf{V}_3^k) \tag{12}$$

A. Linear Network

According to Fig. 5, the linear network can be represented as

$$\begin{bmatrix} \mathbf{I}_{in} \\ \mathbf{I}_{3L} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{in,in} & \mathbf{Y}_{in,3} \\ \mathbf{Y}_{3,in} & \mathbf{Y}_{3,3} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{in} \\ \mathbf{V}_3 \end{bmatrix} \tag{13}$$

where $\mathbf{I}_{3L}$ corresponds to the current entering the linear network and $\mathbf{I}_{in}$ to the nodal currents within it, respectively; their corresponding voltages are $\mathbf{V}_L$ and $\mathbf{V}_{in}$; $\mathbf{Y}_{in,in}$, $\mathbf{Y}_{in,3}$, $\mathbf{Y}_{3,in}$ and $\mathbf{Y}_{3,3}$ are formed by linear elements.

Solving for $\mathbf{I}_{3L}$ from (13) yields

$$\mathbf{I}_{3L} = \left(\mathbf{Y}_{3,3} - \mathbf{Y}_{3,in}\mathbf{Y}_{in,in}^{-1}\mathbf{Y}_{in,3}\right)\mathbf{V}_3 + \mathbf{Y}_{3,in}\mathbf{Y}_{in,in}^{-1}\mathbf{I}_{in} \tag{14}$$

From (14) it can be written

$$\Delta\mathbf{I}_{3L} = \mathbf{J}_L\,\Delta\mathbf{V}_3 \tag{15a}$$

where the linear Jacobian at the complex frequency $s_n = c + j\omega_n$ is given by

$$\mathbf{J}_L = \mathbf{Y}_{3,3} - \mathbf{Y}_{3,in}\mathbf{Y}_{in,in}^{-1}\mathbf{Y}_{in,3} \tag{15b}$$

Also from (14) and (15b) it can be written

$$\mathbf{I}_{3L}(\mathbf{V}_3^k) = \mathbf{J}_L\mathbf{V}_3^k + \mathbf{Y}_{3,in}\mathbf{Y}_{in,in}^{-1}\mathbf{I}_{in} \tag{16}$$

B. Nonlinear Network

For the calculation of $\mathbf{J}_{NL}$, the nonlinear load is taken as an input/output relation where the input is the voltage at its terminals and the output corresponds to its nonlinear current (see Fig. 7). For the sake of clarity consider a single nonlinear load whose instantaneous current/voltage relation is represented through a polynomial of order p [9].

$$i_{NL}(t) = \alpha\, v(t) + \beta v^p(t) \tag{17}$$

Given a set of N voltage points in the TD

$$\mathbf{v}_3^k = \left[v_3(t_1), v_3(t_2), \ldots, v_3(t_N)\right]^T \tag{18a}$$

Through (17) a set of current points can be determined

$$\mathbf{i}_{3NL}^k = \left[i_{3NL}(t_1), i_{3NL}(t_2), \ldots, i_{3NL}(t_N)\right]^T \tag{18b}$$

Then $\mathbf{i}_{3NL}^k$ can be converted to the FD through a NLT operation [8]

$$\mathbf{I}_{3NL}(\mathbf{V}_3^k) = \mathrm{NLT}\left\{\mathbf{i}_{3NL}\right\} \tag{18c}$$

For the nonlinear subnetwork the calculation of $\mathbf{J}_{NL}$ should be performed via perturbations [9]. To this end first consider a vector of voltages $\mathbf{V}_0$ in the frequency domain, to which a small perturbation ε is added to yield vectors $\mathbf{V}_1,\ldots,\mathbf{V}_N$ as follows

$$\mathbf{V}_0 = \begin{bmatrix} V_{0,1} \\ V_{0,2} \\ \vdots \\ V_{0,N} \end{bmatrix},\ \mathbf{V}_1 = \begin{bmatrix} V_{0,1}+\varepsilon \\ V_{0,2} \\ \vdots \\ V_{0,N} \end{bmatrix},\ \ldots\mathbf{V}_N = \begin{bmatrix} V_{0,1} \\ V_{0,2} \\ \vdots \\ V_{0,N}+\varepsilon \end{bmatrix} \tag{19}$$

From (19) it can be obtained the following matrix of vector differences

$$\Delta\mathbf{V} = \left[\Delta\mathbf{V}_1 \quad \Delta\mathbf{V}_2 \quad \ldots \quad \Delta\mathbf{V}_N\right] = \varepsilon\mathbf{U} \tag{20}$$

where $\Delta\mathbf{V}_n = \Delta\mathbf{V}_n - \Delta\mathbf{V}_0$ for $n = 1,2,\ldots,N$ and $\mathbf{U}$ is the identity matrix of dimension N.

Each vector $\mathbf{V}_n$ can be transformed to the TD through the inverse numerical Laplace transform

$$\mathbf{v}_n = \mathrm{INLT}\left\{\mathbf{V}_n\right\} \tag{21}$$

Using $\mathbf{v}_n$ with (17) and (18) an output $\mathbf{I}_n$ is determined. Calculating the N+1 current vectors the following vector differences are obtained

$$\Delta\mathbf{I} = \left[\Delta\mathbf{I}_1 \quad \Delta\mathbf{I}_2 \quad \ldots \quad \Delta\mathbf{I}_N\right] \tag{22}$$

where $\Delta\mathbf{I}_n = \Delta\mathbf{I}_n - \Delta\mathbf{I}_0$ for $n = 1,2,\ldots,N$.

Finally with (20) and (22) the nonlinear Jacobian is written as

$$\mathbf{J}_{NL} = \Delta\mathbf{I}/\varepsilon \tag{22}$$

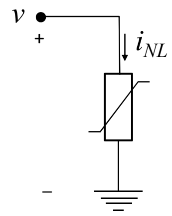


Fig. 7. Nonlinear load.

TABLE I
V-I CHARACTERISTIC OF ARRESTER

Voltage (kV)	Current(kA)
480	0.1760
520	0.3226
560	0.7626
600	1.6426
620	12.6426

IV. Application Example

In this example, consider the sequential energization of a 400 kV 3-phase transmission cable shown in Fig. 8. A surge arrester is connected on each phase at the receiving end of the cable. The parameters of the cables are shown in Fig. 9, the data for the source and switch are $R_S = 0.1\Omega$, L_S = 0.1 H and $R_{SW} = 0.01\Omega$. Closing times for phases A, B and C are 3, 5 and 9 ms, respectively.

For the piece-wise linear method the arresters are represented as nonlinear resistances with *v-i* curves approximated with five linear segments whose values are presented in Table I.

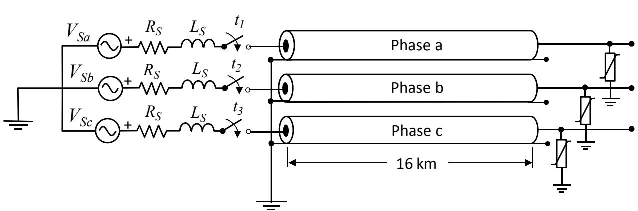


Fig. 8. Underground transmission system.

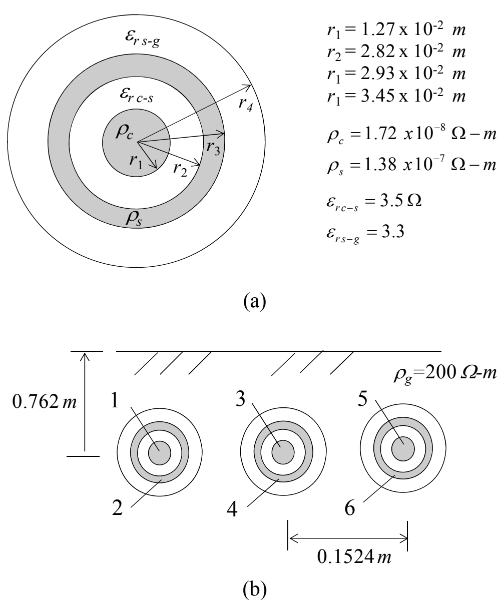


Fig. 9. Transversal configuration of cable system (a) single cable, (b) three phase system..

For the polynomial model, the characteristic *v-i* curve is approximated by the instantaneous voltage/current equation (17) with α =3 .6667×10^{-3}and β =2.253×10^{-7}. The power of the polynomial is set equal to 27.

Figures 10 and 11 show the results obtained with the polynomial and the piece-wise model using 256 samples and a simulation time of 20ms, these results are compared against those from the commercial software PSCAD / EMTDC.

To achieve an error less than 1×10^{-9} the computation time required for the polynomial model was 25.419039s while for the piece-wise model was of only 0.546713s. This time difference is because the piece-wise model required solving *N* matrices of 9×9, while the polynomial model required the calculation of $\mathbf{J}_{NL}$ via perturbations and solving a 6N×6N equations system at each iteration of the Newton's recursive procedure. The number of iterations for the first, second and third switching in the polynomial method was 3, 11 and 11 respectively.

Figures 12 and 13 show the voltage difference error with respect to commercial software PSCAD /EMTDC, as shown both methods possess an acceptable error.

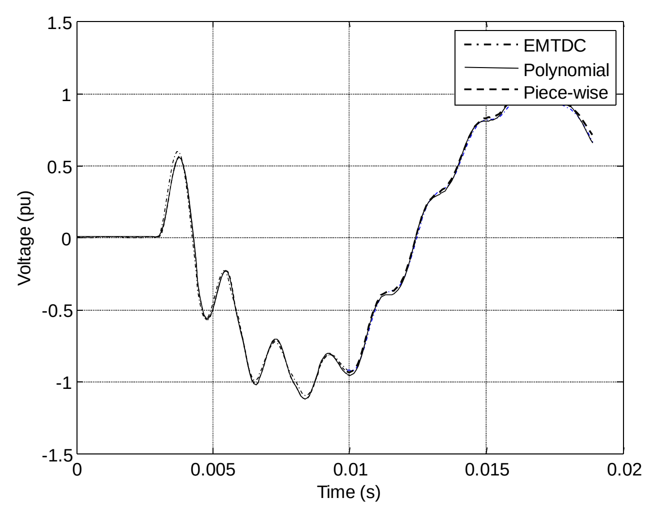


Fig. 10. Voltage at the receiving end of the core of phase a.

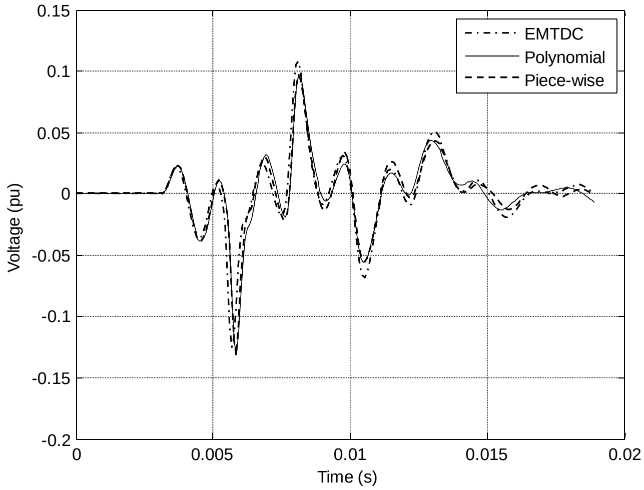


Fig. 11. Voltage at the receiving end of the sheath of phase a.

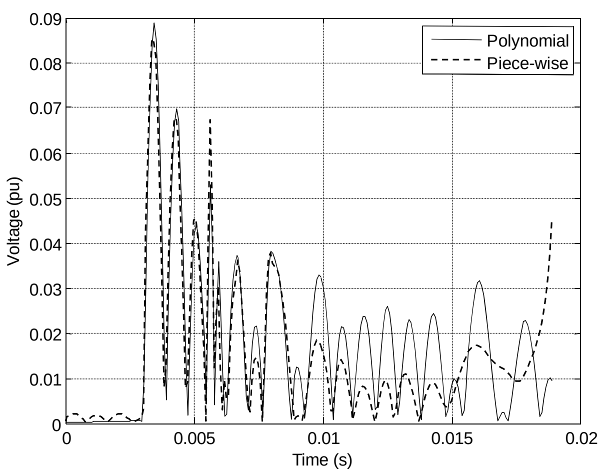


Fig. 12. Error voltage at the receiving end of the core of phase a.

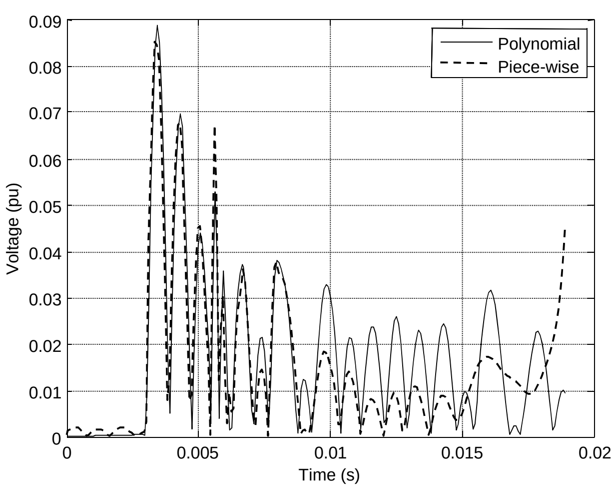


Fig. 13. Error voltage at the receiving end of the sheath of phase a.

V. Conclusions

In this paper two frequency domain methodologies for calculating the transient state of a network including nonlinear loads have been revised. Both methods provide acceptable results and possess advantages and disadvantages. Both methods can handle any number on nonlinear elements at the expense of computing time. Due to the size of the nonlinear Jacobian the method of polynomial representation is slower than the piece-wise method. However this cannot be conclusive since the computation time of this latter depends not only on the number of samples but also on the speed with which the transient waveforms change. Although no convergence problems were found in the numerical examples performed in this work it is believed that Newton's method could present problems with some types of polynomials, especially when the polynomial order is very large.

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