Supplementary Techniques for 2S-DIRK-Based EMT Simulations

Taku Noda, Toshiaki Kikuma and Rikido Yonezawa

***Abstract*— Electromagnetic transient (EMT) simulations are now often used for system studies including power electronics converters, which control voltages and currents using switching devices. The trapezoidal method, used in EMTP (Electro-Magnetic Transients Program) for numerical integration, produces "fictitious" sustained numerical oscillation, when an inductor current or a capacitor voltage is suddenly changed especially by switching. To cope with this problem, the authors have proposed applying the two-stage diagonally implicit Runge-Kutta (2S-DIRK) method which is an inherently "oscillation-free" numerical integration method. In order to bring out full accuracy of the 2S-DIRK method, supplementary numerical techniques are required for the representation of voltage sources, current sources and switches. This paper illustrates these techniques with numerical examples.**

***Index Terms*— Numerical integration, Numerical oscillation, Switching, Trapezoidal method, and 2S-DIRK method.**

I. Introduction

ELECTROMAGNETIC TRANSIENT (EMT) simulations are becoming more and more important. Traditional applications of EMT simulations include studies of overvoltages, inrush currents, abnormal oscillations, harmonics and relay settings. In addition to these traditional applications, EMT simulations are now often used for system studies including power electronics converters. Since power-electronics converters control voltages and currents using switching devices, waveform-level simulations, in other words, EMT simulations are often essential.

For EMT simulations, Electro-Magnetic Transients Program (EMTP) has been used, and now other programs are also used. The legacy versions of EMTP use the genuine trapezoidal method for the numerical integration of dynamic circuit elements such as inductors and capacitors [1]. The trapezoidal method is a simple single-step integration method which calculates the solution at the present time step only from values at the previous time step. In spite of its simple formula, the trapezoidal method has a second-order accuracy and is A-stable. When an integration method is said to be A-stable, it does not diverge for any time step size. However, the trapezoidal method produces "fictitious" sustained numerical oscillation, which does not exist in real EMT phenomena, for a sudden change of a variable. This numerical oscillation appears when an inductor current or a capacitor voltage is suddenly changed, and this can be a serious drawback for EMT simulations including switches and highly-nonlinear elements. To deal with this problem, critical damping adjustment (CDA) has been proposed and implemented in newer versions of EMTP [2], [3]. In CDA, the trapezoidal method is basically used, and only at a moment a sudden change of a variable is detected the integration method is replaced by the backward Euler method. In most cases, an acceptable result is obtained, since the backward Euler method does not produce sustained numerical oscillation. In practical implementations [3], switching events, sudden changes of voltage and current source values, and changes of the operating points of nonlinear components represented by piece-wise linear curves are detected to switch the integration method. However, the detection of a sudden change of a variable is not always feasible. For instance, a voltage or current source controlled by a control system can produce a sudden change due to a limiter operation, but detecting this is quite difficult. Detecting all types of sudden changes by programming can be a serious overhead of execution time. Consequently, an inherently "oscillation-free" integration method is essential to solve this problem. In [4], the authors have proposed applying the two-stage diagonally implicit Runge-Kutta (2S-DIRK) method for the numerical integration of power system EMT simulations. The 2S-DIRK method has a second-order accuracy and is A-stable like the trapezoidal method. In addition, it is mathematically guaranteed that the 2S-DIRK method does not produce sustained numerical oscillation, that is, it is an oscillation-free numerical integration method.

In order to bring out full accuracy of the 2S-DIRK method, supplementary numerical techniques are required for the representation of voltage/current sources and switches. In [4], the 2S-DIRK formulas of inductors and capacitors for both linear and nonlinear cases are derived, and analytical and numerical comparisons of the 2S-DIRK method with other integration methods are presented. However, detailed numerical techniques for the representation of voltage/current sources and switches are not shown except those for ideal switches (see Appendix B of [4]). Considering this, supplementary numerical techniques for the representation of voltage/current sources and switches with on and off resistances are presented in this paper. The voltage value of a voltage source or the current value of a current source used at the first stage of the 2S-DIRK scheme should be appropriately interpolated, when it is activated. A switch with on and off resistances should operate, turn on or turn off, only in the calculation

T. Noda and R. Yonezawa are with Electric Power Engineering Research Laboratory, CRIEPI (Central Research Institute of Electric Power Industry), 2-6-1 Nagasaka, Yokosuka, Kanagawa 240-0196, Japan (e-mail: {takunoda, yonezawa}@criepi.denken.or.jp).

T. Kikuma is with System Engineering Research Laboratory, CRIEPI, 2-11-1 Iwado-kita, Komae, Tokyo 201-8511, Japan (e-mail: kikuma@criepi.denken.or.jp).

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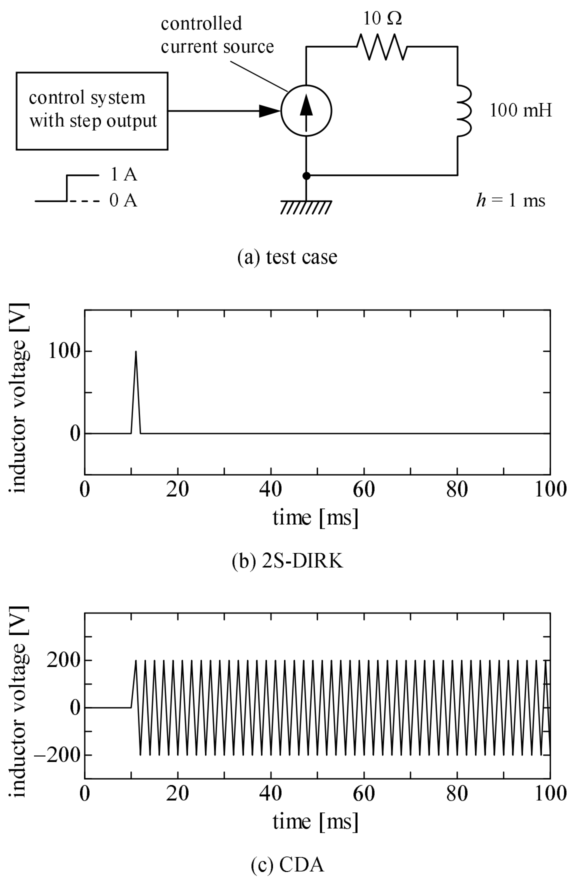
process of the second stage not in that of the first stage. This paper illustrates these supplementary techniques in detail with numerical examples.

II. Sustained Numerical Oscillation

In this section, a simple example of sustained numerical oscillation is presented. Fig. 1 (a) shows a simple circuit in which a series RL circuit is excited by a current source [5]. The current through the current source is controlled by a control system. The voltage across the inductor is calculated for the case where the output from the control system is changed stepwise from 0 to 1 at $t = 10$ ms. The time step used is 1 ms. The calculated result obtained by the 2S-DIRK method is shown in Fig. 1 (b). A spike voltage is generated when the current is changed stepwise. This is consistent with the circuit theory. On the other hand, Fig. 1 (c) shows the calculated result obtained by CDA. An implementation of CDA failed to detect the stepwise change of the current, and thus sustained numerical oscillation is produced (a version of EMTP was used for this simulation). Of course, this sustained numerical oscillation does not exist in reality. For other examples, such as sustained numerical oscillation of a capacitor current caused by a step voltage change transmitted by a distribution line, see [4].



(a) test case

(b) 2S-DIRK

(c) CDA

Fig. 1. Calculated results of a sudden change of an inductor current. (a) shows the test circuit where the current through an inductor is suddenly changed from 0 A to 1 A by a current source controlled by a control system. (b) shows the result obtained by the 2S-DIRK method. (c) shows the result obtained by CDA.

III. 2S-DIRK Method

A. Review of the 2S-DIRK Method

This section briefly reviews the algorithm of the 2S-DIRK (two-stage diagonally implicit Runge-Kutta) method [4]. The dynamics of inductors and capacitors, including both linear and nonlinear cases, can be described by the differential equation

$$\frac{dy}{dt} = f(t, y), \tag{1}$$

where t is time and f is function of time and the variable y. In order to carry out an EMT simulation, we have to integrate this equation from the previous time step $t = t_{n-1}$ to the present time step $t = t_n$, for each dynamic component at each time step. Since the 2S-DIRK method is a two-stage integration method, it uses the intermediate time step at $t = \tilde{t}_n$ which is in between $t = t_{n-1}$ and $t = t_n$. The intermediate time step is given by

$$\tilde{t}_n = t_{n-1} + ah, \quad a = 1 - \frac{1}{\sqrt{2}}, \tag{2}$$

where h is the time step size used. Here, $\tilde{h} = ah$ is defined and used below for simple notation.

From the solution $y = y_{n-1}$ at $t = t_{n-1}$, the first-stage formula calculates the intermediate solution $y = \tilde{y}_n$ at $t = \tilde{t}_n$.

$$\tilde{y}_n = y_{n-1} + \tilde{h} f(\tilde{t}_n, \tilde{y}_n) \tag{3}$$

Before moving on to the second stage, $\tilde{y}_n$ is converted to

$$\tilde{y}_{n-1} = \alpha y_{n-1} + \beta \tilde{y}_n, \tag{4}$$

where $\alpha = -\sqrt{2}$ and $\beta = 1 + \sqrt{2}$. Then, the second-stage formula calculates the solution $y = y_n$ at $t = t_n$.

$$y_n = \tilde{y}_{n-1} + \tilde{h} f(t_n, y_n) \tag{5}$$

It should be noted that both the first-stage and the second-stage formula, respectively shown in (3) and (5), are in the same form as the backward Euler formula. This simplifies the coding of the 2S-DIRK method, since the code can be written in a way that the backward Euler method is applied twice. Another point to note is that solving a set of n circuit equations using a general two-stage implicit method in which two stages are coupled each other results in solving a set of $2n$ simultaneous equations. The 2S-DIRK method, however, is designed so as to decouple the two stages, and thus we can sequentially solve two sets of n simultaneous equations. This leads to a faster calculation.

In comparison with the trapezoidal method, the 2S-DIRK method has the following points in common. First of all, both methods are classified into the single-step integration scheme, which calculates the solution at the present time step only from the solution at the previous time step. In terms of accuracy and stability, both methods have second-order accuracy and both are A-stable. The most important different point between the two methods is on sustained numerical oscillation. It is mathematically guaranteed that the 2S-DIRK method does not produce sustained numerical oscillation, while the trapezoidal method produces it when an inductor current or a capacitor voltage is suddenly changed. Another different point is that the 2S-DIRK method uses two stages while the trapezoidal method uses a single stage. Thus, the 2S-DIRK method has to calculate the intermediate solution at $t = \tilde{t}_n$ in addition. As mentioned above, the 2S-DIRK method is faster than other two-stage implicit methods but slower than the trapezoidal method due to the additional intermediate solution.

In [4], 2S-DIRK formulas for linear inductor, linear capacitor, nonlinear inductor and nonlinear capacitor are derived, and the 2S-DIRK method is compared with the backward Euler, the trapezoidal, and the Gear-Shichman method in terms of accuracy and sustained numerical oscillation.

For general comparison between explicit and implicit numerical integration methods, see Appendix A.

B. Roles of the First and the Second Stage

In terms of coding, the 2S-DIRK method can be viewed as applying the backward Euler method twice, as mentioned above. In terms of numerical integration, however, the first and the second stage have different roles. The calculated result obtained at the first stage is just an intermediate solution, and thus it should not be considered as a proper output from the 2S-DIRK method. On the other hand, the calculated result obtained at the second stage is the final solution, and it is the proper output from the 2S-DIRK method. When the waveform of a variable is plotted as a result of an EMT simulation, a sequence of second-stage solutions must be used and first-stage solutions should be ignored.

Here, note that the 2S-DIRK method is a single-step integration method consisting of two stages. When a multi-stage integration method is used, a sudden change should not be applied to an integration variable within the process of an integration step consisting of successive stages, since a smooth change of the variable is assumed within the integration process of a single step. This suggests that a sudden change of the variable should be applied before the process of successive integration stages, and thus, before the integration process of the first stage, in order to get good accuracy. This principle, illustrated in Fig. 2, of course applies to the 2S-DIRK method. In the following sections, the principle is taken into account in the representations of voltage/current sources and switches with on and off resistances, as supplementary techniques.

IV. Supplementary Techniques

A. Representation of Voltage and Current Sources

Assume that $t = t_0$ is the time when a voltage source is activated. When $t < t_0$, the voltage source is not activated, and thus its voltage v remains zero. When $t \geq t_0$, the voltage v is defined by the function $f(t)$.

If t_0 is an exact multiple of the time step h used, the following procedure may be appropriate in terms of the principle mentioned in the preceding section. When $t < t_0$, v is set to zero. At $t = t_0$, v is also set to zero. In order to realize a smooth transition from $t = t_0$ to the next time step at $t = t_0 + h$, the value of v used by the first stage at $t = t_0 + \tilde{h}$ is obtained by the linear interpolation.

$$v|_{t=t_0+\tilde{h}} = af(t_0 + h), \tag{6}$$

where a is defined in (2). This is illustrated in Fig. 3 for the case where the waveform of the voltage source is a step function. Apparently, the procedure mentioned above is applicable to arbitrary waveforms. If t_0 is not exactly a multiple of the time step h, it is shifted to the nearest multiple of h. If h is set to a sufficiently small value, shifting the activation timing of a voltage source by a fraction of h should not cause a problem. Note that the voltage waveform $f(t)$ itself is not shifted.

EMT simulations often start from their steady-state solutions using a steady-state initialization method [6]. In this case, the voltage source is activated from the beginning, and thereby the procedure above is not necessary. For current sources, exactly the same interpolation procedure can be applied.

B. Representation of Switches with On and Off Resistances

The technique described below is applied to switches with finite on and off resistances. For ideal switches (with zero on resistance and infinite off resistance), the technique described in Appendix B of [4] should be used.

An operation of a switch may cause sudden changes of variables in the circuit. The principle mentioned in Section III-B suggests that a sudden change should be applied before the integration process of the first stage. An operation of a

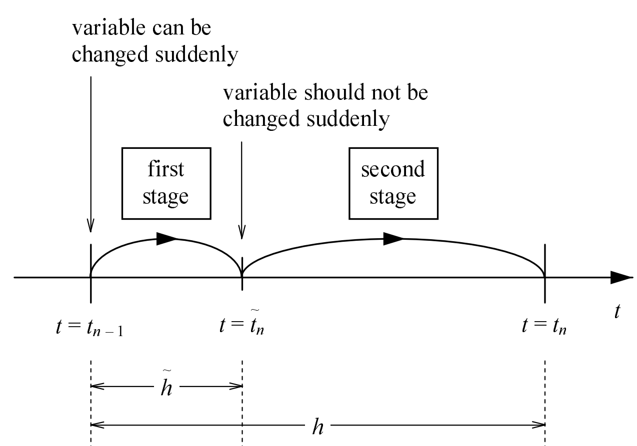


Fig. 2. Illustration of the first and the second stage of the 2S-DIRK method.

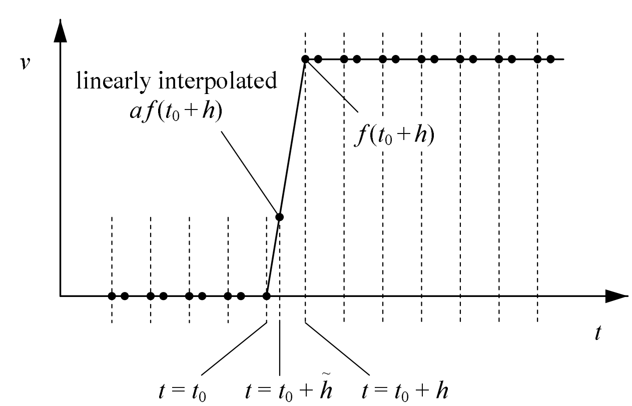


Fig. 3. Linear interpolation of the voltage of a voltage source used in the calculation of the first stage at the moment when it is activated. A step function case is shown.

switch thereby should take place in the calculation process of the second stage. If the state of the switch is determined during the calculation process of the second stage, the resultant switch state is first reflected in the solution of the second stage, thus before the first stage. This is in accordance with Fig. 2.

The procedure for time-controlled switches is simple. Their states are determined just before entering the calculation process of the second stage so that their state changes are first reflected in the solution of the second stage, thus before the first stage.

A switch whose state is dependent on its voltage or current is often used for the modeling of diodes and thyristors. The procedure for this kind of switches is slightly different from that for time-controlled switches. The states of those switches cannot be determined before entering the calculation process of the second stage. Actually, their states are determined in the iterative process in the second stage to solve the nonlinear circuit equations in which their switch equations are embedded [7]. Once the circuit equations have been solved, their switch states are determined and reflected in the solution of the second stage, thus before the first stage.

V. Numerical Examples

A. Voltage Sources

Fig. 4 shows a simple test circuit consisting of a step voltage source and a capacitor. It is used for the validation of the proposed voltage source representation described in Section IV-A. The step voltage source is activated at $t = 5$ μs, and its voltage rises from 0 V to 100 V at that time. The value of the capacitor is 1 μF. If the trapezoidal method is used for the EMT simulation of this circuit, sustained numerical oscillation is produced in the capacitor current i_C due to the sudden change of the capacitor voltage at $t = 5$ μs. The 2S-DIRK method, on the other hand, does not produce sustained numerical oscillation in i_C but produces a spike which instantaneously charges the capacitor. The spike current is also observed in reality.

Since the 2S-DIRK method is a 2-stage integration method, there exist three different ways to represent a step voltage change as shown in Fig. 5. Representation A is the proposed one with a linearly-interpolated point at $t = t_0 + \tilde{h}$. In Representation B, the voltage rises within the first integration stage. In Representation C, the voltage rises within the second integration stage. The calculated results of the capacitor current i_C obtained using the 2S-DIRK method for these three cases of the voltage source representation are shown in Fig. 6.

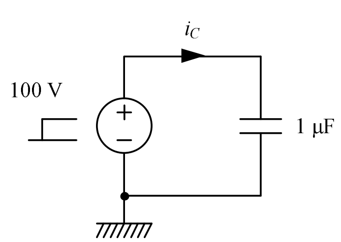


Fig. 4. Test circuit used for the validation of the proposed voltage source representation.

A time step of 1 μs has been used in the calculations. The peak value of the spike observed in i_C can be calculated by the following simple equation.

$$I_{\text{peak}} \cong C\frac{\Delta v}{\Delta t} = C\frac{\Delta v}{h} = 100 \text{ A}$$

Out of the three cases of the voltage source representation, only Representation A closely agrees with the value above. We can confirm that Representation A, which is the proposed method, is the correct choice for the voltage source representation.

B. Current Sources

Fig. 7 shows another simple test circuit consisting of a step current source and an inductor. This one is used for the validation of the proposed current source representation described in Section IV-A. In the same way as the previous section, the step current source is activated at $t = 5$ μs, and its current rises from 0 V to 100 V at that time. The value of the inductor is 0.1 mH. It is also the same that sustained numerical oscillation is produced in the inductor voltage v_L if the trapezoidal method is used for its EMT simulation. The simulation result obtained by the 2S-DIRK method produces a spike in v_L, instead of the sustained numerical oscillation, due to the sudden change of the inductor current applied at $t = 5$ μs. The spike voltage instantaneously creates the flux of the inductor, as observed in reality.

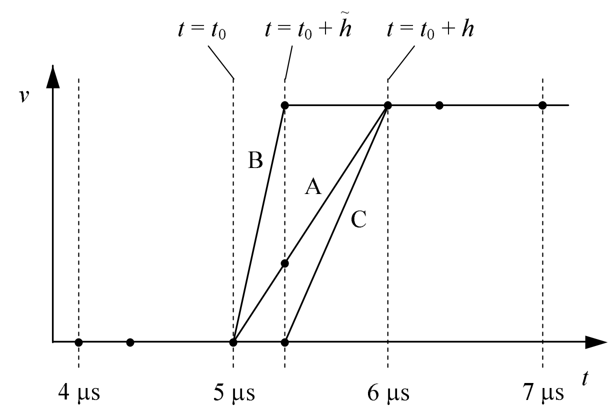


Fig. 5. Three different ways to represent a step voltage change in the 2S-DIRK method.

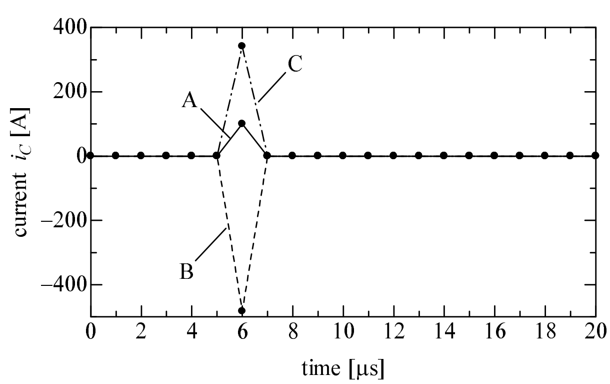


Fig. 6. Calculated results of the capacitor current i_C obtained using the 2S-DIRK method for the three cases of the voltage source representation.

The same three representations as those shown in Fig. 5 can be applied to the current source representation. Representation A is the proposed one with a linearly-interpolated point. The calculated results of the inductor voltage v_L obtained using the 2S-DIRK method for the three cases of the current source representation are shown in Fig. 8. A time step of 1 μs has been used. Just in the same way as the previous section, the peak value of the spike observed in v_L can be calculated by the following simple equation.

$$V_{\text{peak}} \cong L\frac{\Delta i}{\Delta t} = L\frac{\Delta i}{h} = 100 \text{ V}$$

Out of the three cases of the current source representation, again, only Representation A closely agrees with this value. We can confirm that Representation A, which is the proposed method, is the correct choice also for the current source representation.

C. Switches with On and Off Resistances

Fig. 9 shows an RL series circuit whose current is supplied by a dc voltage source. This circuit is used for the validation of the proposed switch representation described in Section IV-B. The values of the resistance and the inductance are respectively 100 Ω and 0.1 mH, and the voltage of the dc voltage source is 100 V. The circuit is closed by a switch in the initial steady state, and thus the steady-state current is 1 A. The switch is opened at $t = 5$ μs. The on resistance of the switch is 1 mΩ, and the off resistance is 1 MΩ. If the trapezoidal method is used for the EMT simulation of this circuit, it produces sustained numerical oscillation in the inductor voltage v_L starting with the switching operation. If the 2S-DIRK method is used, on the other hand, it produces a spike in v_L, which also exists in reality.

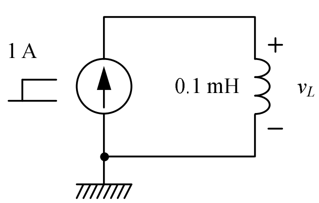


Fig. 7. Test circuit used for the validation of the proposed current source representation.

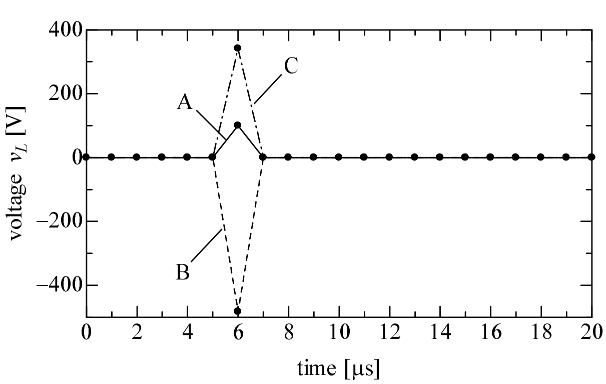


Fig. 8. Calculated results of the inductor voltage v_L obtained using the 2S-DIRK method for the three cases of the current source representation.

According to the proposed switch representation described in Section IV-B, the state of a time-controlled switch is determined just before entering the calculation process of the second stage. The other possible option is to determine the switch state before entering the calculation process of the first stage. Here, these two options are compared. Fig. 10 shows the calculated results obtained using the 2S-DIRK method with the two options mentioned above. The current through the inductor is shown in Fig. 10 (a), and the voltage across the inductor in Fig. 10 (b). The waveforms of the inductor current obtained by both options are identical, and their difference cannot be seen. The waveform of the inductor voltage obtained by the proposed representation, which determines the state of the switch before the second stage, shows a negative spike. The negative spike is in accordance with $L(di_L/dt) < 0$, where L is the value of the inductor and i_L is the current through it. On the other hand, the waveform obtained by the other

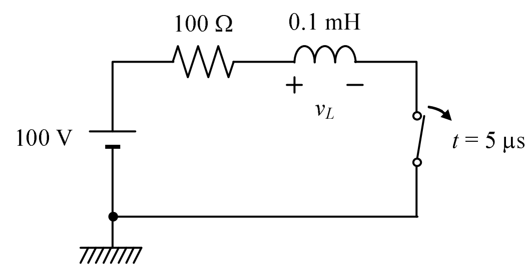


Fig. 9. Test circuit used for the validation of the proposed switch representation.

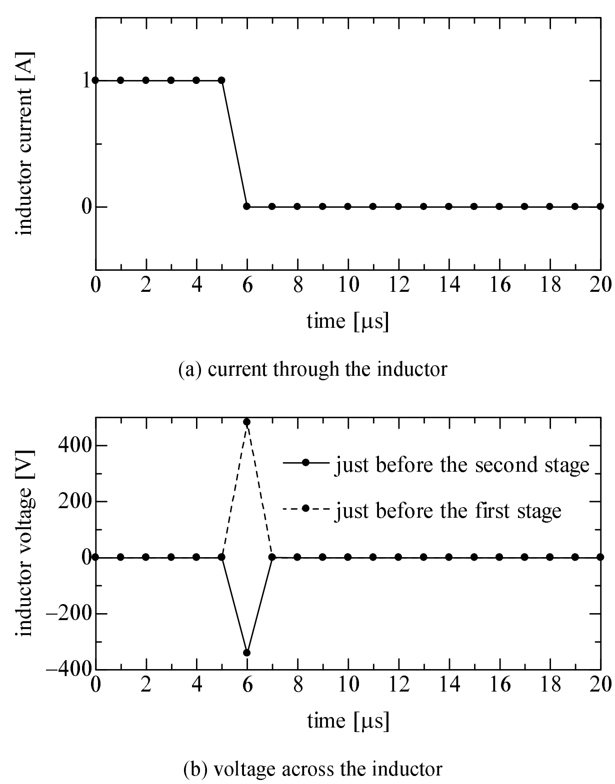


Fig. 10. Calculated results of the current through and the voltage across the inductor obtained using the 2S-DIRK method for the two options of the switch representation.

option, which determines the state of the switch before the first stage, shows a positive spike. This is inconsistent with $L(di_L/dt) < 0$. From this result, we can conclude that the proposed switch representation for the 2S-DIRK method is appropriate.

VI. Conclusion

In order to fundamentally solve the problem of sustained numerical oscillation, the authors have proposed applying the two-stage diagonally implicit Runge-Kutta (2S-DIRK) method for the numerical integration of power system EMT simulations. It is mathematically guaranteed that the 2S-DIRK method does not produce sustained numerical oscillation, that is, it is an oscillation-free numerical integration method.

In order to bring out full accuracy of the 2S-DIRK method, supplementary numerical techniques are required for the representation of voltage sources, current sources and switches. In this paper, these techniques have been illustrated in detail. The proposed techniques have been validated by numerical examples.

The 2S-DIRK integration method for EMT simulations [4], the robust and efficient iterative scheme for solving nonlinear circuit equations [7], the practical steady-state initialization method [6], and the supplementary techniques related to the 2S-DIRK method proposed in this paper have been implemented in the EMT analysis program XTAP (eXpandable Transient Analysis Program). The authors hope that XTAP enables accurate and robust EMT simulations especially related to power-electronics converters.

Appendix

A. Explicit or Implicit?

An explicit numerical integration formula explicitly expresses the output at the present time step using past output values. Thereby it does not require solving the circuit equations for obtaining the output at the present time step, and this results in an efficient simulation. However, its numerical stability is limited, and its output diverges unless a sufficiently small time step size which is determined by the stability criterion is used. The critical time step size is dependent on the shortest time constant that the circuit to be solved has.

On the other hand, an implicit numerical integration formula requires solving the circuit equations to obtain the output at the present time step. This is computationally demanding, when compared with an explicit formula. An implicit formula, however, gives a stable solution for any time step size used.

The circuit to be solved in an EMT simulation is usually a stiff system, i.e. it has both long and short time constants. For such simulations, implicit numerical formulas are suitable. This is because estimating the shortest time constant in a circuit is not an easy task, and an implicit formula gives at least stable solution for any time step specified.

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