Implementation of Induction Machine VBR Model with Optional Zero-Sequence in SimPowerSystems, ASMG, and PLECS Toolboxes

M. Chapariha, F. Therrien, J. Jatskevich, H. W. Dommel

Abstract**-- In distributed generation systems, zero-sequence may be considered for accurate modeling of induction machines where for monitoring and protection purposes the neutral point is either connected to ground directly or with impedance. This paper describes the so-called voltage-behind-reactance model for induction machines, wherein a zero-sequence branch is introduced in order to obtain an explicit formulation with a constant and decoupled interfacing circuit. The proposed model is demonstrated on an induction generator system implemented in three commercial electrical systems simulation toolboxes. These toolboxes are SimPowerSystems, ASMG, and PLECS which are commonly-used with MATLAB/Simulink. In addition to straightforward implementation in all three toolboxes, the proposed model gives identical results with significant computational speed-up as compared to the previous implicit VBR model. The new model is also compared with the classical *qd*0 model which required snubber interfacing circuit. The studies demonstrate that the proposed VBR model is more accurate than the classical approaches, and has higher numerically stability and efficiency especially when the zero-sequence is considered.**

Keywords**: ac machines, induction generators, power system simulation, voltage-behind-reactance formulation.**

NOMENCLATURE

Throughout this paper, matrix and vector quantities are boldfaced, and scalar quantities are italic non-boldfaced. All the variables are referred to the stator side using the appropriate turns ratio.

$\mathbf{v}_{abcs}$, $\mathbf{i}_{abcs}$	stator actual voltage and current vectors
$\mathbf{v}_{abcn}$	voltage vector from stator terminals to the point *n* in VBR interfacing circuit
$\mathbf{e}''_{abc}$	subtransient voltage vector
i_{0s}	stator's zero-sequence current
v_{ng} , i_{ng}	voltage and current of the VBR interfacing circuit zero-sequence branch
r_s , r_r	stator and rotor winding resistances
L_{ls} , L_{lr}	stator and rotor winding leakage inductances
v_{qr} , v_{dr}	rotor voltage transformed to *q* and *d*-axes
ω_r	angular speed of rotor
ω	angular speed of rotational reference frame
P	number of poles
r_D , L_D	resistance and inductance of the three-phase branch of VBR interfacing circuit
r_0 , L_0	resistance and inductance of the zero-sequence branch of VBR interfacing circuit
r_g	external grounding resistance
R_S , X_S	external source equivalent impedance

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I. INTRODUCTION

ELECTRICAL ac machines are important parts of many small and large power systems. Examples of such systems include electric vehicle drive systems, distributed generators, airplane and vessel electrical systems, microgrids, etc. Synchronous machines are often used as generators in thermal and hydro power plants, while induction machines are commonly used as motors in industrial, commercial, and household applications. More recently, however, induction machines have also frequently been employed as smaller-scale generators in distributed energy systems.

Depending on the required degree of detail, different programs and models are used to simulate electric machines. In this paper, transient simulation programs are considered. Specifically, the focus is on induction machine modeling in state-variable-based programs such as MATLAB/Simulink [1], [2], for which several power systems and electrical circuits toolboxes are commercially available: SimPowerSystems [3], ASMG [4], PLECS [5], etc. These toolboxes provide graphical user interfaces and automated state-model formulation algorithms to simplify the modeling of electrical systems in Simulink environment. Additionally, some of these toolboxes, such as PLECS, have also standalone versions. The formulation considered in this paper can also be discretized for implementation in the nodal-analysis-based EMTP-type simulation programs [6].

For some transient simulations, machine models with magnetically coupled rotor and stator equivalent circuits are considered [7]. Such phase-domain representations are well-established but rarely used directly for numerical simulations due to the presence of variable and coupled inductances: time-varying inductances require re-factorization/regeneration of the system matrix at each time-step and may drastically increase the computational burden of simulation.

The reformulation of phase-domain models using the rotating reference frame theory originally presented in [8] significantly reduces their computation time. These models, herein referred to as classical *qd*0 models, are most frequently found in commercial simulation packages today. Despite being commonly available as built-in components, the *qd*0 models have interfacing problems with external inductive networks [9].

The *qd*0 models are typically interfaced with external networks as voltage-controlled current sources in *abc* phase coordinates. If these current sources are connected in series with inductors, simulation programs encounter problems when formulating a proper explicit state-variable model of the overall system. In such cases, fictitious resistive or capacitive snubber circuits should be added to enable the formulation of input-output coupling variables and the overall state-variable model. However, the snubber circuits add error and can make the system numerically stiff, decreasing the numerical efficiency and stability of the whole simulation.

The so-called voltage-behind-reactance (VBR) formulation offers a solution to the interfacing problem. The VBR formulation takes advantage of the reference frame transformation but leaves enough elements in *abc* phase coordinates to obtain direct interfacing [9]–[13]. A simple VBR formulation was presented in [10] which has a constant and decoupled interfacing circuit; however, therein an algebraic loop is created when zero-sequence is simulated. It was recently enhanced by including zero-sequence in the interfacing circuit instead of in the subtransient voltage equations, making the model explicit. The improved formulation is presented in [11] where it is also extended to synchronous machines. This formulation requires less number of parameters and is claimed to be easy to implement in different simulation programs as will be verified in this paper. The proposed model is suitable for accurate modeling of induction machines with or without grounding. Compared to the *qd*0 model with snubbers, the VBR formulation is more accurate and is numerically less stiff. These numerical advantages stem mostly from the direct interfacing circuit.

No simulation studies are available in the literature to validate the VBR formulation for induction machines given in [11]. In this paper, this formulation (here referred to as the explicit VBR) is compared to the one in [10] (referred to as the implicit VBR-III) and its numerical advantages are shown for a sample machine-network system. The implementation method of the explicit VBR formulation is demonstrated in the three toolboxes mentioned earlier for the first time. A distributed energy system with induction generator is considered for the studies where the machine is connected to an external inductive network and the neutral point is directly grounded for protection purposes [14]; a single-phase-to-ground fault happens in the system close to the generator to verify the model in a more general case including zero-sequence.

Lately, the VBR formulation is being used in the simulation software programs. For example, PLECS has added some variation of VBR machine models into its library of components [5]. This paper demonstrates the simplicity of implementation of the new constant-parameter VBR formulation and confirms its accuracy and shows that it is easily implementable in different state-variable based simulation programs. Comparison with classical *qd*0 model requiring snubbers shows that this model is superior for most of the general applications.

II. VBR Formulation for Induction Machine Model

In this paper, a three-phase wye-connected induction machine with sinusoidally distributed rotor and stator windings is considered. To be consistent with previous publications, specifically [10] and [11], the motor convention with the same variable names are used here. For convenience, the important parameters and variables are defined in the Nomenclature section; the others are explained in the text.

The machine is originally modeled as coupled-circuits in phase-domain as shown in [7]. The classical *qd*0 model, whose advantages and disadvantages were listed in the Introduction, is then obtained by transforming the *abc* variables into the arbitrary reference frame. Finally, algebraic manipulation of the *qd*0 equations yields the VBR formulation [10]. This formulation can be directly interfaced to any external circuit while maintaining an interfacing circuit with constant and decoupled parameters. The branch voltage equation of the interfacing circuit is [10]

$$\mathbf{v}_{abcs} = r_D \mathbf{i}_{abcs} + L_D p \mathbf{i}_{abcs} + \mathbf{e}''_{abc} + \left(3 r_0 \mathbf{i}_{0s} + 3 L_0 p \mathbf{i}_{0s}\right) \tag{1}$$

where

$$r_D = r_s + \frac{L''^2_m}{L^2_{lr}} r_r \tag{2}$$

$$L_D = L_{ls} + L''_m \tag{3}$$

$$r_0 = -\frac{1}{3}\frac{L''^2_m}{L^2_{lr}} r_r \tag{4}$$

$$L_0 = -\frac{1}{3} L''_m . \tag{5}$$

The subtransient magnetizing inductance is given by

$$L''_m = \left(\frac{1}{L_m} + \frac{1}{L_{lr}}\right)^{-1} . \tag{6}$$

The terms inside the parenthesis in (1) are equal to zero if there is no zero-sequence current in the circuit (i.e. the neutral is floating). The vector $\mathbf{e}''_{abc}$ is the subtransient back EMF voltages in *abc* phase coordinates which is defined as

$$\mathbf{e}''_{abc} = [\mathbf{K}_s]^{-1}\cdot[e''_q \quad e''_d \quad 0]^T \tag{7}$$

where $[\mathbf{K}_s]$ is the transformation matrix from the stationary reference frame *abc* to the rotational reference frame *qd*0, and

$$e''_q = \omega_r\lambda''_d + \frac{L''_m r_r}{L_{lr}^2}\left(\lambda''_q - \lambda_{qr}\right) + \frac{L''_m}{L_{lr}} v_{qr} \tag{8}$$

$$e''_d = -\omega_r\lambda''_q + \frac{L''_m r_r}{L_{lr}^2}\left(\lambda''_d - \lambda_{dr}\right) + \frac{L''_m}{L_{lr}} v_{dr}\,. \tag{9}$$

The *q*- and *d*-axes subtransient flux linkages are defined as

$$\lambda''_q = L''_m \frac{\lambda_{qr}}{L_{lr}} \tag{10}$$

$$\lambda''_d = L''_m \frac{\lambda_{dr}}{L_{lr}}\,. \tag{11}$$

Using flux linkages as state variables, the rotor dynamics are represented by the following state equations:

$$p\lambda_{qr} = -\frac{r_r}{L_{lr}}(\lambda_{qr} - \lambda_{mq}) - (\omega - \omega_r)\lambda_{dr} + v_{qr} \tag{12}$$

$$p\lambda_{dr} = -\frac{r_r}{L_{lr}}(\lambda_{dr} - \lambda_{md}) + (\omega - \omega_r)\lambda_{qr} + v_{dr} \tag{13}$$

where the magnetizing fluxes are calculated by

$$\lambda_{mq} = L''_m i_{qs} + \lambda''_q \tag{14}$$

$$\lambda_{md} = L''_m i_{ds} + \lambda''_d\,. \tag{15}$$

The electromechanical torque is

$$T_e = \frac{3P}{4}(\lambda_{md} i_{qs} - \lambda_{mq} i_{ds})\,. \tag{16}$$

On the one hand, induction motors are very rarely grounded. On the other hand, for protection or monitoring purposes, induction generators may be grounded directly or via impedance [14] which in turn requires the terms inside parentheses in (1). Manipulating (1) yields [10]

$$\mathbf{v}_{abcs} = r_D\mathbf{i}_{abcs} + L_D p\mathbf{i}_{abcs} + \mathbf{e}''_{abc} + \left(3r_0 - \frac{3L_0 r_s}{L_{ls}}\right)\mathbf{i}_{0s} + \frac{3L_0}{L_{ls}}\mathbf{v}_{0s} \tag{17}$$

where

$$\mathbf{i}_{0s} = [i_{0s} \quad i_{0s} \quad i_{0s}]^T \tag{18}$$

$$\mathbf{v}_{0s} = [v_{0s} \quad v_{0s} \quad v_{0s}]^T\,. \tag{19}$$

The zero-sequence current i_{0s} and voltage v_{0s} are the algebraic average of their corresponding three phase values as

$$i_{0s} = (i_{as} + i_{bs} + i_{cs})/3 \tag{20}$$

$$v_{0s} = (v_{as} + v_{bs} + v_{cs})/3\,. \tag{21}$$

Eq. (17) is the algebraically exact implicit VBR-III implementation; however, it introduces an algebraic loop when the terminal voltage $\mathbf{v}_{abcs}$ is unknown, e.g. when the machine is in series with inductive elements. To avoid an implicit formulation, the stator voltage equation (1) can be separated into two sections [11]

$$\mathbf{v}_{abcs} = \mathbf{v}_{abcn} + \mathbf{v}_{ng} \tag{22}$$

where

$$\mathbf{v}_{abcn} = r_D\mathbf{i}_{abcs} + L_D p\mathbf{i}_{abcs} + \mathbf{e}''_{abc} \tag{23}$$

$$\mathbf{v}_{ng} = [v_{ng} \quad v_{ng} \quad v_{ng}]^T \tag{24}$$

and the elements of $\mathbf{v}_{ng}$ are

$$v_{ng} = r_0(3i_{0s}) + L_0 p(3i_{0s}) = r_0 i_{ng} + L_0 p i_{ng}\,. \tag{25}$$

The above equation, (25), represents a zero-sequence branch *ng* in series with the three-phase decoupled *RL* branch which is used to represent (23). The resulting interfacing circuit is shown in Fig. 1. This formulation, named explicit VBR, is algebraically exact and explicit and has a constant-parameter decoupled circuit interface.

To summarize, the circuit shown in Fig. 1 with the elements given by (2)–(5), in addition to the rotor state equations given by (12) and (13), the definitions of flux linkages given by (10), (11), (14), and (15), and the reference frame transformation (7), compose the explicit VBR formulation.

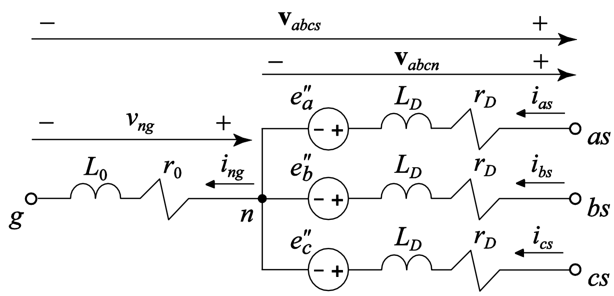

Fig. 1. Interfacing circuit for the explicit VBR formulation given at [11].

III. Model Implementation in Simulation Toolboxes

To demonstrate its ease-of-use, the explicit VBR formulation is implemented in three commonly used toolboxes in MATLAB/Simulink. Fig. 2 shows the SimPowerSystems (SPS) [3] implementation, wherein the interfacing circuit is

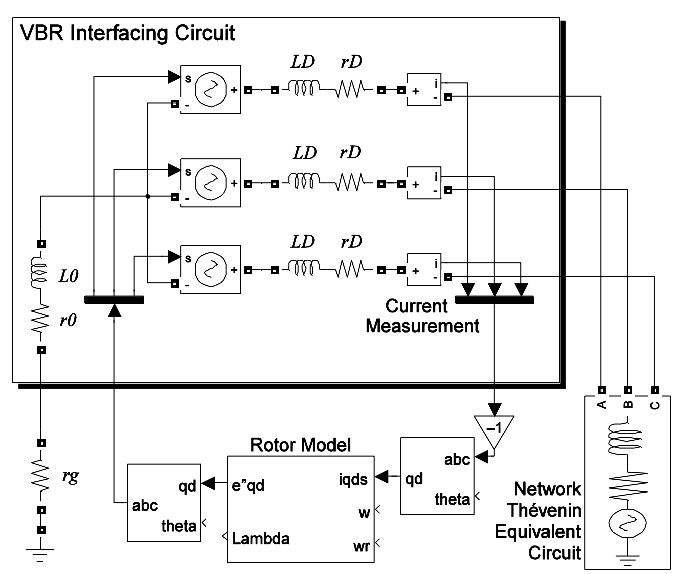

Fig. 2. Implementation of VBR model with zero-sequence in SPS.

shown inside the box. For compactness, the machine is connected to the Thévenin equivalent circuit of a network; however, the network could also be represented in detail. In this figure, the machine is grounded through the resistance r_g. This resistance is not a part of the VBR model and is added externally to limit the short circuit current. If the machine is not grounded, the neutral point is left unconnected without any modification to the model. To focus on the electrical model, the mechanical system and the additional input and output ports are omitted in Fig. 2. For the other two toolboxes, ASMG [4] and PLECS [5], the electrical network is simply replaced with one instance of ASMG-System or PLECS Circuit, respectively, which for conciseness, are not shown here.

IV. Computer Studies

To assess the numerical efficiency of the proposed explicit VBR model, a simple distributed generation system is considered here. It is assumed that an induction generator is connected to a prime mover (e.g. a wind turbine) that in the course of the study maintains a constant speed of 1.027 per-unit. The system is illustrated in Fig. 3, wherein the generator is connected to a Thévenin equivalent circuit. The machine parameters are given in the Appendix. To emulate a severe unbalanced transient situation, a single-phase-to-ground fault is assumed in the system close to the generator feeder and the grounding resistance is zero ($r_g = 0$). Initially the machine is in steady state and the fault happens after one cycle. The fault is modeled by decreasing the phase *a* voltage of the equivalent source to zero ($v_a = 0$).

The VBR model is directly connected to the *RL* network similar to Fig. 2. However, the classical *qd*0 model requires a snubber for interfacing as shown in Fig. 3. To minimize the error, a 10 per-unit resistive snubber is used here. The snubber adds error and makes the system numerically stiff, therefore less numerically stable.

As a reference, the system (including the machine) is represented in the synchronous reference frame and the whole model is solved using the ODE45 solver with the time-step limited to 1 μs. This model is implemented using basic Simulink blocks. Having a very simple and linear system, the conversion to rotational reference frame is straightforward in this example. However, for general cases e.g. when the system

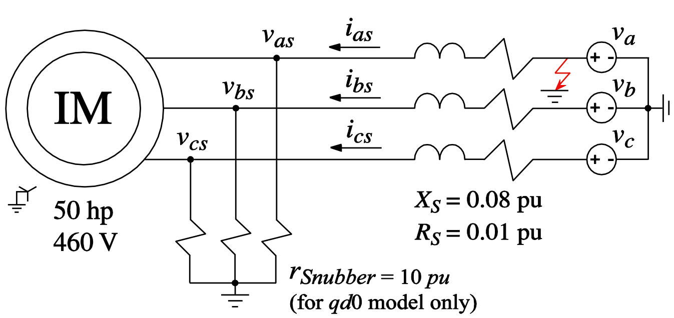

Fig. 3. Induction generator connected to the Thévenin equivalent circuit of a network..

is large or includes non-linear elements such as rectifiers, such a conversion is not trivial, if possible at all.

The implicit VBR-III [10] is implemented in the SimPowerSystems (SPS) toolbox, as SPS conveniently allows solving algebraic loops in Simulink. The explicit VBR [11] is implemented in the SPS, ASMG, and PLECS toolboxes; the three implementations give identical results.

Since the built-in *qd* models in the library of SPS and PLECS do not include zero-sequence, the *qd*0 machine model is implemented using basic Simulink blocks. The model is interfaced to an instance of PLECS circuit by means of controlled current sources.

To show the consistency of the explicit VBR [11] and the implicit VBR-III [10], both models are run with the ODE45 solver using the same settings. The maximum and minimum time-steps are set to 1 *m*s and 0.1 μs, respectively. The models are represented in per-unit and the relative and absolute tolerances of the solver are 10^{-4}. For the *qd*0 model with snubbers, the stiffly-stable Simulink solver ODE15s is used. Its settings are identical to those used for the VBR models.

The simulation results shown in Fig. 4 demonstrate the consistency between the VBR models and the reference solution. Three windows highlighted in Fig. 4 are enlarged and shown in Fig. 5 to Fig. 7.

Fig. 5 shows phase *c* stator current in steady-state; it shows that both VBR models yield exactly the same results. Moreover, they have chosen identical time-steps. For the *qd*0 model, the snubbers sink part of the machine output current and therefore produce some error. Comparatively, the *qd*0 model with snubbers has chosen several times more time-steps and has a visible steady-state error. Fig. 6 and Fig. 7, which show phase *c* stator current and electromagnetic torque during fault, confirm that the VBR models have no visible error in transient as well. For the *qd*0 model with snubbers, the transient response has some error, although it is less visible due to the comparatively large fault current. The stiffly-stable solver ODE15s has chosen even smaller time-steps during the transient period (after the fault) than in steady state.

A quantitative evaluation of the simulation study is summarized in Table I. The time-steps and calculation data are obtained from Simulink Profiler [2]. The relative error is calculated by comparing the predicted trajectory with the reference solution using the 2-norm of the error and normalizing the difference [15]. For example, the error for i_{as} is given by

$$\varepsilon(i_{as}) = \frac{\left\| \tilde{i}_{as} - i_{as} \right\|_2}{\left\| \tilde{i}_{as} \right\|_2} \times 100\% \tag{26}$$

where $\tilde{i}_{as}$ is the reference solution trajectory. The average three-phase current error $\varepsilon(\mathbf{i}_{abcs})$, which is shown in Table I, is evaluated using

$$\varepsilon(\mathbf{i}_{abcs}) = \left[\varepsilon(i_{as}) + \varepsilon(i_{bs}) + \varepsilon(i_{cs})\right]/3 \tag{27}$$

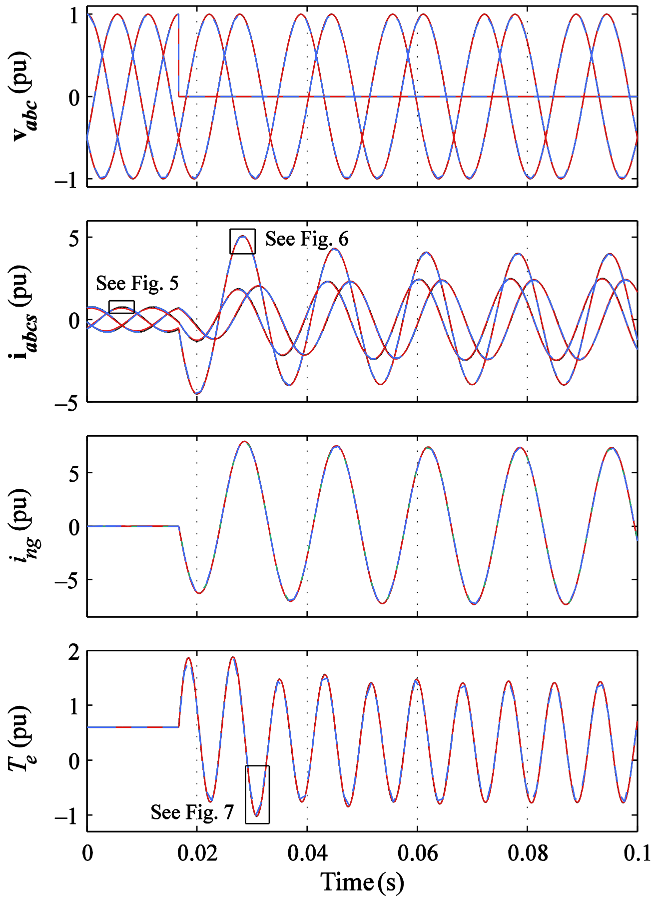

Fig. 4. Simulation results for the single-phase-to-ground fault study: source voltage; source current; machine neutral current; and electromechanical torque.

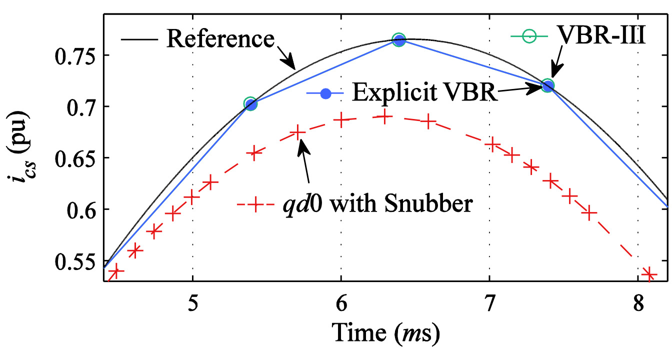

Fig. 5. Magnified view of the source current in steady state from Fig. 4.

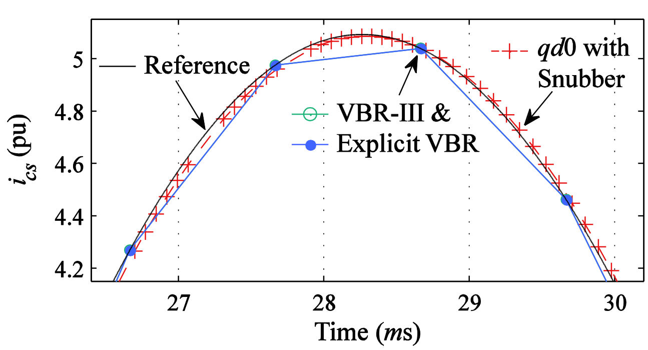

Fig. 6. Magnified view of the source current during the fault from Fig. 4.

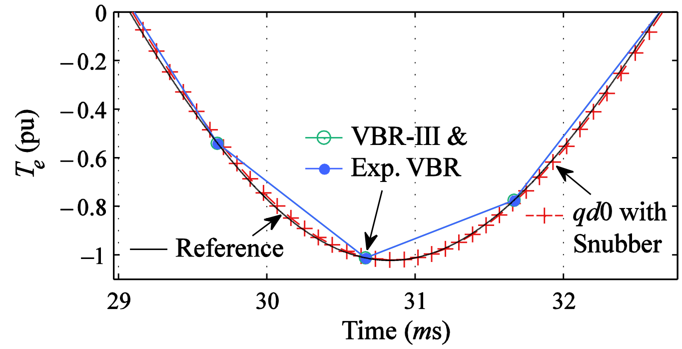

Fig. 7. Magnified view of the electromechanical torque during the fault from Fig. 4.

Table I verifies that both implicit and explicit VBR formulations are algebraically identical to the reference. It also reveals the difference between the computational costs of the two VBR formulations by comparing the number of subtransient voltage calculations. Practically, the implicit VBR-III is significantly slower since it requires iterations in each time-step for the algebraic loop solution (3865 calculations compared to 764 for the explicit VBR).

The *qd*0 model is explicit but numerically stiff, thus it used several times more time-steps (989 compared to 110 times for the VBR models) and even more internal current calculations (7070 times) than the implicit VBR subtransient voltage calculations (3865 times). As shown in Table I, the largest eigenvalue of the *qd*0 model with snubbers is several orders of magnitude bigger than the largest eigenvalue of the VBR models.

TABLE I
COMPARISON OF NUMERICAL EFFICIENCY OF VBR FORMULATIONS FOR SINGLE-PHASE FAULT STUDY

Simulation Index	Formulations		
	Implicit VBR-III [10]	*Explicit VBR* [11]	*qd0 with Snubber*
Major Time Steps	110	110	989
Minor Loop Calculations*	3865	764	7070
Current $\mathbf{i}_{abcs}$ Prediction Error	0.000 %	0.000 %	2.861 %
Largest eigenvalue	–199 ± j118	–199 ± j118	-1.18×10^5

* This row shows the number of subtransient voltage calculations for the VBR models and the number of injected current calculations for the *qd*0 model.

V. CONCLUSIONS

The proposed explicit VBR model for induction machine was verified, the implementation in commercially available simulation toolboxes was demonstrated, and the model was compared to the previous implicit version VBR-III and the classical *qd*0 models. As shown for the three commonly used simulation toolboxes, SPS, ASMG, and PLECS, the interfacing circuit and internal subtransient voltage equations for the explicit VBR formulation are simple and easy to implement. A single-phase-to-ground fault study demonstrates that the VBR models are more accurate and numerically more efficient than the $qd0$ model. The studies presented in this paper show that when zero-sequence is considered among the two VBR formulations the numerical efficiency of the explicit VBR model is noticeably higher.

Based on the studies presented in this paper, as well as the ones in [10] and [11], the explicit VBR model is suggested as the general purpose model for squirrel-cage induction machines in state-variable-based simulation programs. It yields identical results to the reference and does not require snubbers when connected to an inductive network/system, while offering high accuracy, numerical stability, and simulation efficiency.

VI. Appendix

Induction Generator Parameters [16]: 4 poles, 60 Hz, 3-phase, 460 V, 50 hp, 1705 rpm, $r_s = 0.087\,\Omega$, $X_{ls} = 0.302\,\Omega$, $X_m = 13.08\,\Omega$, $r_r = 0.228\,\Omega$, $X_{lr} = 0.302\,\Omega$.

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