An Adaptive Aggregation Algorithm for Power System Dynamic Equivalencing in Transient Stability Studies for Future Energy Grids

Sajjad Zadkhast, Arash Alimardani, Juri Jatskevich, Ebrahim Vaahedi

Abstract—**Aggregating coherent generators is an important step in dynamic equivalencing to create a reduced order system. The available methods tend to keep dynamics of large generators which is a valid approach as these generators can provide a large amount of power. However, this approach loses the accuracy if a large generator is not able to provide enough power in the post-fault state of the system. The typical methods also fail to create an accurate reduced system when there is an error in coherency identification. This manuscript presents a new adaptive aggregation method wherein the weight of each generator is determined based on its impact on the system dynamics. The proposed method keeps the dynamics of key generators which are more influential in dynamic behavior of the system. Additionally, by using the trapezoidal integration rule, a new criterion is defined to evaluate the accuracy of the reduced system. If the reduced system is not accurate, the non-coherent generators are identified and separated from their groups. The effectiveness of the proposed method is investigated and demonstrated using a IEEE 50-gen test system by comparing the results with Time-Domain simulation and Inertial Aggregation algorithm.**

***Keywords*: Aggregating generators, dynamic equivalencing, transient stability.**

I. Introduction

DYNAMIC equivalencing has been an effective tool to expedite transient stability studies. In this approach, the coherent generators are aggregated to create a reduced order system. There are three steps involved in the dynamic equivalencing: (i) identifying groups of coherent generators; (ii) aggregating buses corresponding to coherent generators; and (iii) aggregating excitation and control system of the coherent generators. The first step has been extensively studied in the literature [1]-[5]. This step generally involves identification of the generators with similar angular swing curves. In contrast, there have been fewer efforts for aggregation of coherent generators. Inertial and slow coherency aggregation method [6] is one of the well-established methods in this area. In this approach, the aggregation is performed at the generator's internal nodes using weighted average of generators. The weight of each generator is determined by its inertia. Accordingly, the generators with large inertia contribute more to the dynamic characteristics of the corresponding equivalent generator. Using terminal bus method for aggregating generators was investigated in [7]. In this approach, the terminals of the coherent generators are connected to an equivalent bus via an ideal transformer with complex ratio. The method is based on the power preservation at the terminal bus of the equivalent generator. This method is less accurate than the inertial and slow coherency method since the terminals of the coherent generators are less coherent comparing to the internal nodes. Using participation factors for aggregating generators was investigated in [8]. In this approach, a reference generator is selected for each group of coherent generators and the weight of each generator is determined based on the participation factor between this generator and the reference generator. The synchrony aggregation approach was presented in [9], where the reference generator is represented in detailed model and the rest of the generators in the group are represented as a current source.

Typically, the equivalent generator is created using a weighted average of the coherent generators, wherein the weight of each generator is determined by the inertia. This approach tends to keep dynamic characteristic of the large generators in the reduced order system. This is a reasonable assumption since large generators are usually more influential on the overall system dynamics. However, this approach is not accurate when a large generator is not able to deliver enough power in the post-fault system. Besides, all available aggregation methods assume that the generators in a group are tightly coherent. Thus, if there is an error in coherency identification, the conventional aggregation methods typically fail to give an accurate reduced order system. The method proposed in this paper addresses these challenges and makes the following overall contribution:

1) The proposed method determines the weight of each generator based on its impact on the overall system dynamics.
2) In order to derive an accurate reduced order system, internal nodes of coherent generators are aggregated. It is shown that the reduced order system is as accurate as full system as long as all generators in a group are tightly

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coherent.

3) Using the trapezoidal integration rule, a new methodology is presented to identify the non-coherent generators. In the proposed approach, the reduced order system is continuously monitored to ensure that all generators are indeed coherent. If a non-coherent generator is detected in a group, this generator will be separated from its group and the aggregation will be updated.
4) The IEEE transient stability benchmark system with 50 generators and 145 buses is used to validate the proposed method. The test system is solved using the commercial-grade TSAT tool from Powertech Labs Inc. and the results are used as a reference to validate and compare the results.
5) The proposed method is compared with the inertial and slow coherency aggregation method [6] and it is demonstrated that the proposed method is more accurate.

II. Relative Weight of Each Generator

As it is commonly assumed, the equivalent generator is a weighted average of the generators belonging to the same group of coherent generators. This approach is very effective since, to some extent, it keeps the dynamic characteristics of all generators. Without loss of generality, assume that generators 1 through n constitute a group of coherent generators. Then, the rotor angle δ_{eq} and speed ω_{eq} of an equivalent generator can be written as

$$\begin{aligned} \delta_{eq} &= \sum_{i=1}^{n} w_i \delta_i , \\ \omega_{eq} &= \sum_{i=1}^{n} w_i \omega_i , \end{aligned} \tag{1}$$

where w_i represents the relative weight of the i th generator in the given group. The weights w_i affect the accuracy of the equivalent generator as well as the reduced system. Therefore, w_i should be chosen carefully to have an accurate reduced system. In inertial aggregation method [6], the weight of each generator is determined by its inertia. In this approach, the contribution of small generators becomes negligible and the equivalent generator mainly represents the dynamic characteristics of the large generators. The inertial aggregation method gives accurate results in most cases since the small generators have negligible impact on the overall system. However, if the effect of small generators is considerable, the inertial aggregation algorithm fails to give an accurate reduced system.

In order to have a more accurate reduced system, effect of generators on the dynamic characteristics of the overall system should be taken into account. Generally, the generators affect the system's dynamic characteristic by injecting active and reactive powers. In this regard, the key generators are those which inject more power and have considerable impact on the system dynamic characteristics. Therefore, the energy transferred to the system can be a good measure of the impact of the generators on the overall system.

Suppose all nonlinear loads (i.e. constant current and constant power loads) are linearized around the initial conditions corresponding to the post-fault system. The linearized loads as well as constant impedance loads are then included into the network admittance matrix (Y-Bus), which makes it possible to eliminate the load buses from the system using Kron method [11] to obtain the reduced admittance matrix ($\mathbf{Y}$). Moreover, assume that the generators are represented by Voltage Behind Reactance (VBR) model [10]. This model can be used to represent both classical (in the simplest form) and the detailed model for the generators. Using the reduced admittance matrix and the VBR model, the power transferred to bus j from bus i is expressed as

$$P_{ij} = E'_{qi} E'_{qj} \left\{ G_{ij} \cos\left(\delta_i - \delta_j\right) + B_{ij} \sin\left(\delta_i - \delta_j\right) \right\}, \tag{2}$$

where E'_{qi} and E'_{qj} represent the magnitude of the voltage behind reactance of the generators i and j, respectively; G_{ij} and B_{ij} are the real and imaginary parts of the Y_{ij}, respectively, which represent the conductance and susceptance between terminal buses of the generators i and j in the reduced admittance matrix. Since G_{ij} and B_{ij} are the elements of i th row in $\mathbf{Y}$, (2) shows that the generator which has larger voltage behind reactance and larger elements in the corresponding row in the reduced admittance matrix, is capable of transferring more power to the system. According to this approach, we define the weight of each generator as

$$w_i = \frac{E'_{qi} \sum_{j=1}^{m} \left|Y_{ij}\right|^2}{\sum_{i=1}^{m} E'_{qi} \sum_{j=1}^{m} \left|Y_{ij}\right|^2}, \tag{3}$$

where m represents total number of the generators. This equation defines the weight of generator i in the corresponding group of coherent generators. The proposed approach ranks the generators based on their capability of injecting power into the system. If there is a poor connection between a generator and the system due to a line outage and the generator is not able to inject considerable amount of power to the system, the proposed approach identifies the poor connection using reduced-admittance matrix and assigns a small weight to the generator.

III. Aggregating Coherent Generators

Suppose the generators G_1 and G_2 in Fig. 1 are coherent. In the VBR model, the coherency condition between generators is expressed in terms of the voltage behind reactance as follows

$$\frac{E'_{q1} e^{j\delta_1}}{E'_{q2} e^{j\delta_2}} = c_0 + \varepsilon\left(t\right), \tag{4}$$

where δ_1 and δ_2 represent rotor angle; c_0 is a constant complex number; and ε represents the error in the coherency between two generators. If the generators are tightly coherent, ε is negligible and the ratio between the VBR of the coherent generators becomes approximately constant in all times. According to (4), the coherency condition between the generators is determined based on machine internal voltage. Therefore, it is more appropriate to aggregate the generators at the internal node rather than terminal bus. Suppose G_{eq} is the equivalent generator representing generators 1 and 2. As Fig. 1 shows, the equivalent generator receives the aggregated mechanical power ($P_{m,eq} = P_{m1} + P_{m2}$) and provides the electrical power $P_{e,eq}$. In order to have systems 1 and 2 equivalent, G_{eq} should transfer P_{e1} and P_{e2} to terminal buses of generators 1 and 2, respectively. If we suppose the two systems are equivalent, the rotor speed dynamics of the equivalent generator can be written as:

$$\frac{d\omega_{eq}}{dt} = P_{m1} + P_{m2} - (P_{e1} + P_{e2}), \tag{5}$$

which can be reformulated in terms of the rotor speed dynamics of generators 1 and 2 as

$$\frac{d\omega_{eq}}{dt} = M_1 \frac{d\omega_1}{dt} + M_2 \frac{d\omega_2}{dt}, \tag{6}$$

where M_1 and M_2 represent constant inertia of generators 1 and 2, respectively. Suppose the weight of each generator is determined by the method presented in section II. The parameters of the equivalent generator can be calculated using weighted average of the generators as follows

$$\begin{aligned} \delta_{eq} &= w_1\delta_1 + w_2\delta_2, \\ E'_{q,eq} &= w_1 E'_{q1} + w_2 E'_{q2}, \\ M_{q,eq} &= w_1 M_1 + w_2 M_2. \end{aligned} \tag{7}$$

If the generators 1 and 2 are tightly coherent, E'_{q1}/E'_{q2} and $\delta_1 - \delta_2$ are almost constant in all times. Using this assumption, the parameters of generators 1 and 2 can be reformulated in terms of the parameters of the equivalent generator as:

$$\begin{aligned} \delta_i(t) &= \delta_{eq}(t) + \Delta\delta_{i0},\ i = 1,2 \\ E'_{qi}(t) &= \beta_{i0} E'_{q,eq}(t),\ i = 1,2 \end{aligned} \tag{8}$$

where $\Delta\delta_{i0}$ represents the difference between δ_i and δ_{eq} at $t = 0$ and β_{i0} represents the ratio between E'_{qi} and $E'_{q,eq}$ at $t = 0$.

In Fig. 1, the total power transferred to buses 1 and 2 is equal to

$$\begin{aligned} S_1 + S_2 = {} & V_1 e^{j\theta_1}\left(\frac{E'_{q1}e^{-j\delta_1} - V_1 e^{-j\theta_1}}{-jX'_{d1}}\right) \\ & + V_2 e^{j\theta_2}\left(\frac{E'_{q2}e^{-j\delta_2} - V_2 e^{-j\theta_2}}{-jX'_{d2}}\right). \end{aligned} \tag{9}$$

Using (8), it is shown in the Appendix that (9) can be reformulated in terms of the parameters of the equivalent machine as

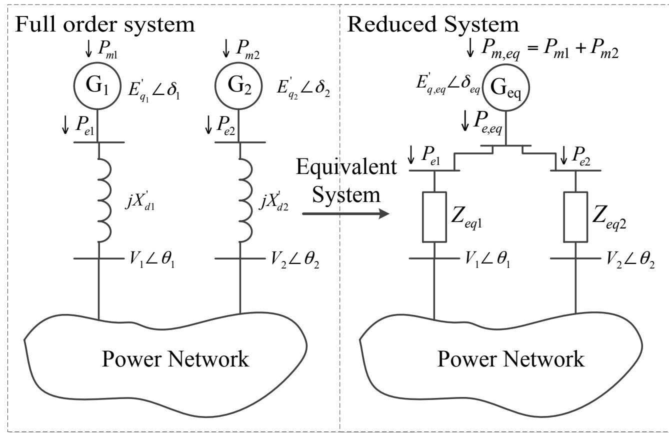

Fig. 1. Creating an equivalent generator from two coherent generators.

$$\begin{aligned} S_1 + S_2 = {} & \widetilde{V}_1 e^{j\widetilde{\theta}_1}\left(\frac{E'_{q,eq}e^{-j\delta_{eq}} - \widetilde{V}_1 e^{-j\widetilde{\theta}_1}}{-j\widetilde{X}'_{d1}}\right) \\ & + \widetilde{V}_2 e^{j\widetilde{\theta}_2}\left(\frac{E'_{q,eq}e^{-j\delta_{eq}} - \widetilde{V}_2 e^{-j\widetilde{\theta}_2}}{-j\widetilde{X}'_{d2}}\right), \end{aligned} \tag{10}$$

where, $\widetilde{V}_i = V_i/\beta_i$, $\widetilde{X}'_{di} = X'_{di}/\beta_i^2$, $i = 1,2$. This equation represents the total power transferred from the equivalent generator to buses 1 and 2. The single-line diagram representing (10) is shown in Fig. 2. In this figure, the equivalent generator is connected to each bus through the modified transient reactance ($\widetilde{X}'_{di}$) and a transformer/phase-shifter. By comparing Fig. 1 and Fig. 2, it can be seen that the connections between the equivalent generator and the terminal buses are different from the full order model, shown in Fig. 1. By adding the transformer and modified transient reactance, the equivalent system tries to keep the same ratio between P_{e1} and P_{e2} as the full order model.

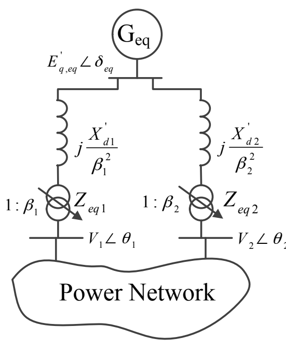

Fig. 2. The diagram representing the connection between equivalent generator and terminal buses of generators.

IV. Identifying Errors in Coherency Identification

Equations (9) and (10) establish a connection between the reduced system and the full order system. These equations are equivalent as long as the assumption (8) holds. This assumption is the underlying assumption in all available aggregation methods and its accuracy highly depends on the coherency between the generators in a group. If the generators are not tightly coherent, (8) is not a valid assumption and consequently, the reduced model will not be valid. Therefore, in order to have an accurate reduced system, the coherency identification algorithm should provide only tightly coherent generators. In this section, the problem of coherency identification error is addressed and an algorithm is presented to identify the errors in coherency identification.

The transient stability problem consists of differential-algebraic equations. The general form of the problem can be expressed as

$$\begin{aligned} \dot{\mathbf{x}} &= f(\mathbf{x},\mathbf{y}), \\ 0 &= g(\mathbf{x},\mathbf{y}), \end{aligned} \tag{11}$$

where, vectors $\mathbf{x}$ and $\mathbf{y}$ represent dynamical and algebraic variables, respectively; vector function f relates dynamical and algebraic variables to the time derivative of the dynamic variables; and vector function g represents power flow constraints in the transmission system. Among different methods, trapezoidal discretization rule [12] is one of the best methods for discretizing (11) at each integration step. In this approach, the solution at each step satisfies following equation:

$$\left\| \mathbf{x}_{n+1} - \mathbf{x}_n + \frac{T}{2}\left(f\left(\mathbf{x}_{n+1},\mathbf{y}_{n+1}\right) + f\left(\mathbf{x}_n,\mathbf{y}_n\right)\right)\right\|_\infty < \varepsilon_T, \tag{12}$$

where T represents integration time-step; subscripts n and $n+1$ represent number of integration step; ε_T represents the tolerance; and $\|\cdot\|_\infty$ represents the infinite norm of a vector, which is the element with the largest magnitude in the vector. Suppose ($\tilde{\mathbf{x}},\tilde{\mathbf{y}}$) represents the state of the full order system, which is calculated from the state of the reduced system using (8). If the reduced order system is accurate, the solution of the reduced system will be close to the solution of the full order system and accordingly, ($\tilde{\mathbf{x}},\tilde{\mathbf{y}}$) satisfies (12). However, if a generator is not tightly coherent with other machines in the group, the reduced system will not be accurate and (12) will not be satisfied.

In order to identify the misplaced generators, we define the trapezoidal error as

$$\mathbf{e}_T = \tilde{\mathbf{x}}_{n+1} - \tilde{\mathbf{x}}_n + \frac{T}{2}\left(f\left(\tilde{\mathbf{x}}_{n+1},\mathbf{y}^r_{n+1}\right) + f\left(\tilde{\mathbf{x}}_n,\mathbf{y}^r_n\right)\right), \tag{13}$$

where, vector $\mathbf{e}_T$ represents the error in trapezoidal integration rule; and $\mathbf{y}^r_n$ represents the algebraic variables calculated using $\tilde{\mathbf{x}}_n$. If the reduced system is not accurate due to an error in coherency identification, some of the elements of $\mathbf{e}_T$ will be large and therefore, by evaluating $\mathbf{e}_T$ at each integration step, we can determine whether or not reduced system is accurate. Besides, the error mainly appears in the variables corresponding to the misplaced generator and accordingly, by checking (13) it is possible to identify the source of inaccuracy in the reduced system. Based on this approach, we present following algorithm to identify errors in the reduced system:

At each integration step, following steps are performed:

1. The state of each individual generator is calculated using (8).
2. The trapezoidal error (13) is checked for all dynamical variables.
3. The elements of $\mathbf{e}_T$ which do not satisfy (12) are identified and the corresponding generators are separated from their groups.
4. The groups which have been modified are updated and a new equivalent generator is calculated for them.

The proposed algorithm checks validity of the reduced system at each integration step. Typically, the error created by a misplaced generator grows very quickly and the proposed algorithm identifies the misplaced generators at first few integration steps. If any misplaced generator is identified at the beginning of the simulation, it would be efficient to restart the simulation using the updated groups.

It should be stressed that if a generator is not coherent with the rest of the group, the error may also appear in the variables corresponding to other generators, which are close to the misplaced generator. Typically, this error is smaller than the error appearing in the variables corresponding to the misplaced generator. However, if there are several non-generators in a group, a large number of variables will be affected. In this situation, it can become hard to identify the misplaced generators and separate them from the rest of the group. Therefore, the proposed algorithm works best when there are few misplacement errors in coherency identification.

V. Simulation Results

The IEEE transient stability test system with 50 generators and 145 buses, presented in [13], has been used to evaluate effectiveness of the proposed aggregation method. Power Tech's TSAT tool is used to solve the transient stability problem using trapezoidal integration method and the results are used as a benchmark. We use *Time-Domain* Simulation to refer to the solution of TSAT tool throughout this section. The proposed method and Inertial Aggregation method [6] are also coded into MATLAB. A fault is applied at bus #7 and cleared by opening transmission line between buses #7 and #6 after 0.08sec. The post-fault simulation time is 8 seconds.

Table I. presents tightly coherent generators identified by the time-domain simulation. In this table, each bracket represents a group of coherent generators and the numbers inside the bracket represent the bus number of each generator. For example, {67, 97, 124} represent a group of coherent generators located at buses #67, #97, and #124. The groups of coherent generators are fed into the proposed method and inertial aggregation method to create a reduced system. In order to compare the results, the rotor angle of generator #60 is plotted in Fig. 3. This figure shows that the results of time-domain simulation, inertial aggregation, and the proposed method. As can be seen, both methods show good performance and the results are close to time-domain simulation throughout the study.

In the second study, the generator #91 is added to the sixth group. This generator is slightly less tightly coherent with the rest of the group. Moreover, a new group consisting of generators #104 and #111 is added to the groups of coherent generators. The rotor angle of generators #104 and #111 in the full order system is plotted in Fig. 4. As can be seen, while the rotor angle plots of the generators are very similar, they are not tightly coherent. For example, at $t = 3.6$ sec, the different between δ_{104} and δ_{111} is almost zero and at $t = 4$ sec, the difference becomes 0.4 rad (22 degrees).

In order to compare aggregation results, the plot of rotor angle of generator #60 is depicted in Fig. 5. As it is expected, the difference between the results of the reduced systems and time-domain simulation is larger than the first study, where only tightly coherent generators were used. However, Fig. 5 shows that the proposed method is able to provide much better reduced system and the results are much closer to the time-domain simulation.

TABLE I
GROUPS OF TIGHTLY COHERENT GENERATORS IDENTIFIED BY TIME-DOMAIN SIMULATION

Group Number	Generators' bus number	Group Number	Generators' bus number	Group Number	Generators' bus number
1	{79, 80}	2	{67,97,124}	3	{82,109}
4	{101,112}	5	{105,106}	6	{108,121}

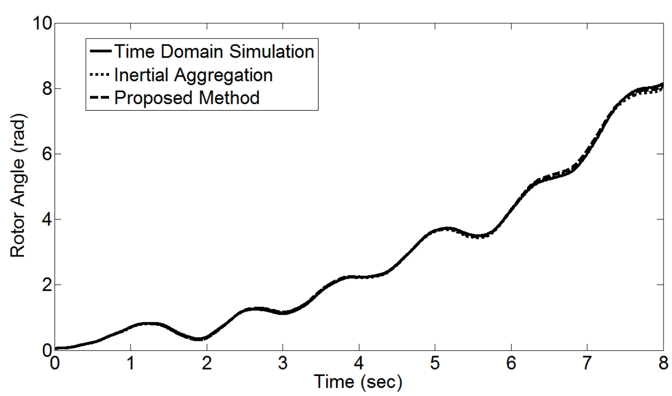

Fig. 3. Rotor angle of generator #60 in the first study.

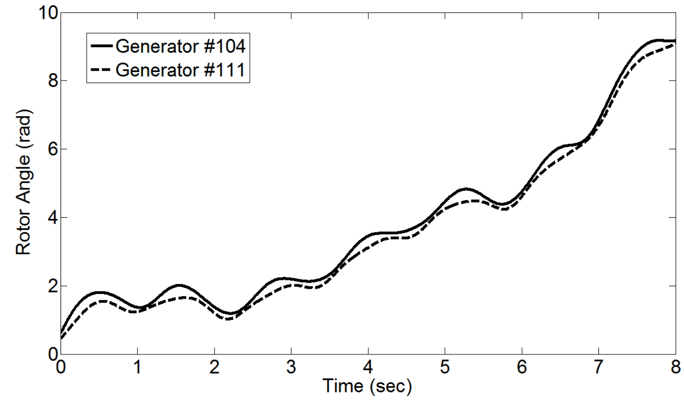

Fig. 4. Rotor angle of generators #104 and #111 in the full order system.

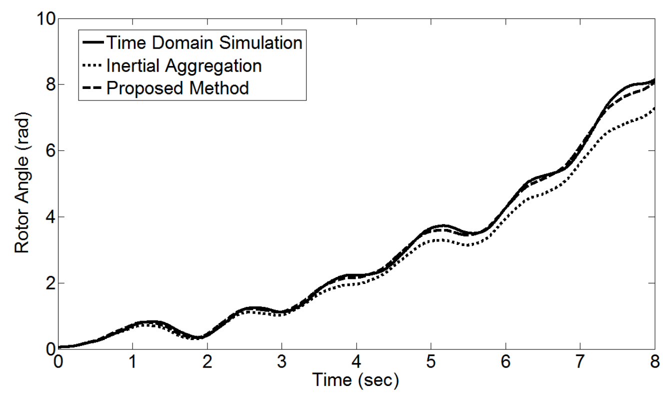

Fig. 5. Plot of rotor angle of generator #60 in the second study.

In order to investigate the effectiveness of the proposed adaptive aggregation method, presented in section IV, the generator #111 is replaced with the generator #115 in the previous study. The new groups of coherent generators are shown in Table II. The rotor angles of the generators #104 and #115 (in full order system) are plotted in Fig. 6. As this figure shows, the generators #104 and #115 are not coherent and accordingly, if these generators are placed in the same group, it is expected that the reduced system will not be accurate. In this case, the error in coherency condition should be detected to avoid making inaccurate reduced system. In order to investigate the accuracy of the reduced systems in the presence of non-coherent generators, rotor angle of generator #60 is plotted in Fig. 7. In this figure, the solid line represents the results of time-domain simulation which is the benchmark. As can be seen, the inertial aggregation method and the proposed aggregation method are not able to provide an accurate reduced order system. The reason is that the assumption (8) does not hold in this case and therefore, the equivalent generator for group 7 cannot represent both generators in this group (i.e. generators #104 and #115). The inaccuracy in this equivalent generator affects other generators in the system and therefore, all the results will be inaccurate. Still, it can be seen, the proposed aggregation method demonstrates a better performance than inertial aggregation method even in the presence of error in coherency identification.

The last plot in Fig. 7 represents the rotor angle of generator #60 calculated by the proposed adaptive aggregation method. Using trapezoidal discretization rule, the proposed adaptive aggregation method is able to separate generators #104 and #115 at the first step of the integration. Additionally, the proposed method separates generator #91 from generators #108 and #121 since generator #91 is not tightly coherent with the other two generators. As can be seen, the resulting reduced order system is highly accurate.

VI. Conclusions

In this paper, a new adaptive aggregation algorithm is proposed. The proposed methodology uses weighting average of coherent generators to create the equivalent machine. The reduced admittance matrix and the voltage behind reactance of generators are used to determine the impact of each generator on dynamic behavior of the system. The weight of each generator is determined using its impact on the dynamic characteristic of the system. Instead of keeping the generators with large inertia, the proposed methodology keeps the dynamics of more influential generators. Besides, based on trapezoidal integration rule, a new criterion is defined to evaluate the results of the reduced order system. In the proposed approach, while integrating the reduced system, the accuracy of the results is continuously monitored. If the results are not accurate enough, the generators contributing in the error are found and separated from their group. Using IEEE 50-gen test system, it is demonstrated that the proposed method shows a better performance comparing to the inertial aggregation algorithm and is able to provide accurate results even if there are errors in the coherency identification.

TABLE II
GROUPS OF TIGHTLY COHERENT GENERATORS IDENTIFIED BY TIME-DOMAIN SIMULATION

Group Number	Generators' bus number	Group Number	Generators' bus number	Group Number	Generators' bus number
1	{79, 80}	2	{67,97,124}	3	{82,109}
4	{101,112}	5	{105,106}	6	{91,108,121}
7	{104, 115}				

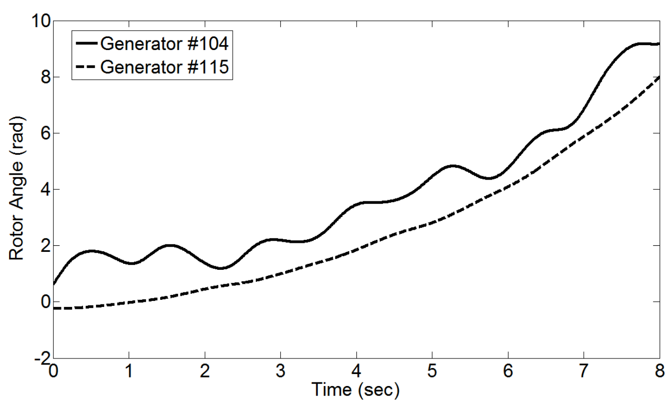

Fig. 6. Rotor angles of generators #104 and #115 in the full order system.

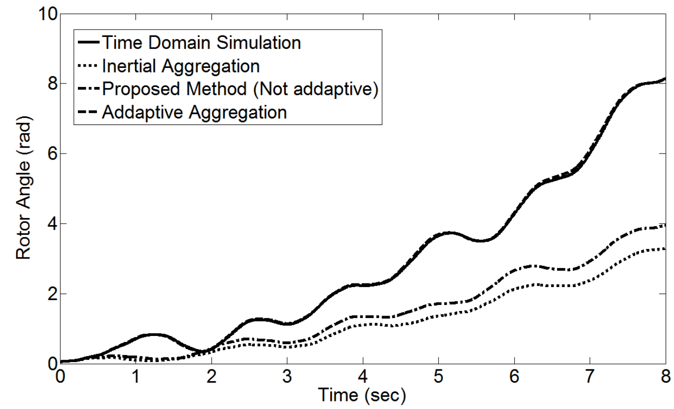

Fig. 7. Rotor angle of generator #60 in the presence of an error in coherency identification.

VII. Appendix

In this appendix it is shown that (9) and (10) are equivalent. Using (8), we can write (9) in terms of parameters of the equivalent generator as

$$S_1 + S_2 = V_1 e^{j\theta_1}\left(\frac{\beta_1 E'_{q1} e^{-j\left(\delta_{eq}+\Delta\delta_1\right)} - V_1 e^{-j\theta_1}}{-jX'_{d1}}\right) + V_2 e^{j\theta_2}\left(\frac{\beta_2 E'_{q2} e^{-j\left(\delta_{eq}+\Delta\delta_2\right)} - V_2 e^{-j\theta_2}}{-jX'_{d2}}\right). \tag{14}$$

Since β_i and $\Delta\delta_i$ are both constant, they can be taken out of the parenthesis.

$$S_1 + S_2 = \beta_1 V_1 e^{-j(\theta_1-\Delta\delta_1)}\left(\frac{E'_{q1} e^{-j\delta_{eq}} - \left(V_1/\beta_1\right) e^{-j(\theta_1-\Delta\delta_1)}}{-j\,X'_{d1}/\beta_1}\right) \beta_2 V_2 e^{-j(\theta_2-\Delta\delta_2)}\left(\frac{E'_{q2} e^{-j\delta_{eq}} - \left(V_2/\beta_2\right) e^{-j(\theta_2-\Delta\delta_2)}}{-j\,X'_{d2}/\beta_2}\right). \tag{15}$$

In (15), the β_1 and β_2 outside the parenthesis are multiplied by the denominator as

$$S_1 + S_2 = V_1 e^{-j(\theta_1-\Delta\delta_1)}\left(\frac{E'_{q1} e^{-j\delta_{eq}} - \left(V_1/\beta_1\right) e^{-j(\theta_1-\Delta\delta_1)}}{-j\,X'_{d1}/\beta_1^2}\right) V_2 e^{-j(\theta_2-\Delta\delta_2)}\left(\frac{E'_{q2} e^{-j\delta_{eq}} - \left(V_2/\beta_2\right) e^{-j(\theta_2-\Delta\delta_2)}}{-j\,X'_{d2}/\beta_2^2}\right), \tag{16}$$

which is equivalent to (10).

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