Sympathetic Inrush Current Phenomenon with Loaded Transformers

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Abstract-- **The magnetization of a transformer is usually associated with the well-known phenomenon of inrush current. However, inrush current does not only affect the transformer being switched on. Instead, it has a significant impact also on all parallel connected transformers, which most certainly includes measurement transformers. This is known as a sympathetic inrush phenomenon. While the transformer being switched on might be a subject to a *sudden* high saturation level, the parallel transformers are *gradually* drawn to saturation as well, but of opposite polarity. The sympathetic inrush is expected in situations where the ohmic part represents a significant portion of the total system impedance. In contrast to other available literature on this subject, in this paper a modal approach to solving equivalent circuit by expressing differential equations in the state-equation form is selected. From the derivation of eigenvalues and eigenvectors, the phenomenon can be systematically investigated. A special attention will be given to circumstances, when the already operational transformer is fully loaded as within substations two or more transformers often operate in parallel, among which at least one of them is usually loaded. Finally, simulation results are compared to captured Wide Area Monitoring System (WAMS) measurements of the phenomenon and reasons for discrepancies are discussed.**

Keywords: **inrush current, magnetization curve, sympathetic interaction, power system dynamics simulation, WAMS measurements.**

I. Introduction

THE transformer inrush phenomenon is very known and widely described for a long time. If the prospective magnetic flux (imaginative steady-state conditions prior to switching, describing the situation as if the switch was already on) in the transformer iron core at the moment of transformer energization differs from the value of zero, the flux DC component appears in the transformer core, which decays exponentially with time. Due to non-linear magnetizing characteristic of the transformer iron core (magnetic flux versus magnetizing current), a DC component of the magnetic flux can drive the transformer into saturation, in which the current drawn from the power system no longer changes linearly with the magnetic flux. Instead, each change in magnetic flux generates large currents that contain higher harmonic content. This usually happens during a transformer energization, especially in case of three-phase transformers, as the majority of circuit breakers perform a simultaneous switching of all three phase terminals. The intensity of the phenomenon can be significantly reduced or even omitted by controlling the moment of individual phase switching. However, not all transformers are equipped with mechanisms allowing such switching.

According to [1] transformer inrush currents might occur in different forms and can be divided into the following sub-categories:

– energization inrush (caused by re-application of voltage source to the transformer which has previously been de-energized. However, remanence could be still present),
– sympathetic inrush (caused by re-application of voltage source to the transformer, which operates in parallel to two or more other transformers),
– recovery inrush (caused by restoration of a voltage after clearance of a fault).

The subject of this paper is the second sub-category from the above list. Even though the first explanations of the phenomenon can be found in the quite early literature [2], the topic is also very relevant these days. Authors think it is reasonable to speculate that the need for additional explanation of such important phenomena always exist. This paper is based on findings and presentations, published in [1], [3], [4], [5], [6], [7], [8] and [9]. Certain additional explanations are added to papers already published with the strong support from clear graphical representation. Besides, in contrast to other available literature on this subject, in this paper a modal approach to solving equivalent circuit by expressing differential equations in the state-equation form is selected. From the derivation of eigenvalues and eigenvectors, the phenomenon can be systematically investigated.

Authors are of opinion that this paper would be very appreciated among TSOs as well as researches that are beginning their research on this topic.

II. Energization Inrush Current

At first, it is reasonable to briefly explain the basic transformer energization inrush current. The most explanatory way to do so is by writing a voltage equation based on the Kirchhoff's second law for the transformer equivalent circuit, connected via system impedance $Z_s = R_s + j\omega L_s$ to a voltage source $u(t)$ (Fig. 1). Transformer equivalent circuit is usually

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thought of as a “T” circuit, where the denotations represent the following:

- R_1......... resistance of the transformer’s primary winding,
- $L_{\sigma 1}$........ leakage inductance of the transformer’s primary winding,
- R_2’........ resistance of the transformer’s secondary winding (recalculated on the number of primary winding turns N_1),
- $L_{\sigma 2}$’ ...... leakage inductance of the transformer’s secondary winding (recalculated on the number of primary winding turns N_1),
- R_m .... magnetizing resistance of the transformer, representing iron losses,
- L_m......... magnetizing inductance of the transformer.

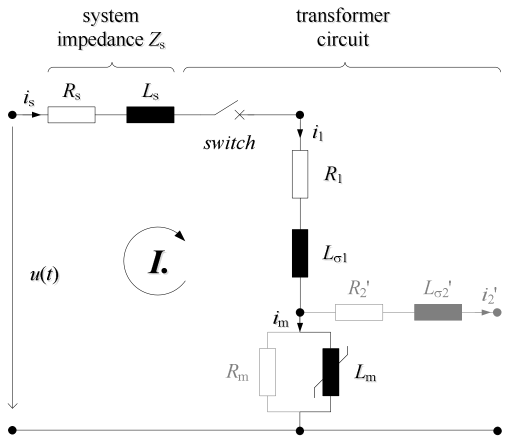

Fig. 1. Equivalent circuit of a power transformer, connected to a voltage source via system impedance

As a transformer inrush current occurs due to the non-linearity of L_m in the magnetizing branch, the simplest way to analyze the phenomenon is to assume that the transformer is un-loaded (i_2’ = 0, consequently the secondary branch is inactive and depicted in grey). This enables writing the following loop voltage equation, where the voltage source $u(t) = U_m \cdot \sin(\omega \cdot t + \alpha)$ is assumed ideally sinusoidal and the iron losses are neglected ($R_m \to \infty$, also depicted in grey):

$$-u(t)+\left[R_s+R_1\right]\cdot i(t)+\frac{\mathrm{d}}{\mathrm{d}t}\left(\left[L_s+L_{\sigma 1}+L_m\right]\cdot i(t)\right)=0 \tag{1}$$

where $i(t) = i_s(t) = i_1(t) = i_m(t)$. For the purpose of simpler mathematical derivation, let us assume that L_m is linear (even though it is in fact the non-linearity of L_m the ground reason for the inrush phenomenon to be so important). Further, let us merge the transformer quantities and denote $R = R_1$ and $L = L_{\sigma 1} + L_m$. If we assume the relation between current and magnetic flux $L = \Phi/i$, the following expression can be obtained by rewriting (1):

$$\dot{\Phi}(t)=-\frac{R+R_s}{L+L_s}\cdot\Phi(t)+\frac{L}{L+L_s}\cdot u(t) \tag{2}$$

What we are interested in is the passive circuit response, i.e. (2) without voltage source $u(t) = 0$. The so-called homogeneous solution of differential equation (2) with a constant C_1 is therefore more or less trivial and equals:

$$\Phi_{DC}(t)=C_1\cdot e^{-\frac{R+R_s}{L+L_s}\cdot t} \tag{3}$$

By taking into account that the switch from Fig. 1 is turned on at time $t = 0$ when flux equals $\Phi_{DC}(0) = \Phi_0$ (initial conditions), (3) becomes:

$$\Phi_{DC}(t)=\Phi_0\cdot e^{-\frac{R+R_s}{L+L_s}\cdot t} \tag{4}$$

(4) represents the passive circuit response, which is obviously a DC component with a decaying rate determined by ratio between circuit serial ohmic resistance and serial inductance. Particular part of the differential equation solution on the other hand (AC flux component), which is not of main concern within this paper, can be obtained by using method of undetermined coefficient and some trigonometric laws. Assuming the solution in the form of:

$$\Phi_{AC}(t)=C_2\cdot\sin(\omega t+a)+C_3\cdot\cos(\omega t+a) \tag{5}$$

its insertion into (2) gives the unknown constants:

$$C_2=\frac{L\cdot U_m\cdot(R+R_s)}{(R+R_s)^2+\omega^2\cdot(L+L_s)^2}$$
$$C_3=-\frac{\omega L\cdot U_m\cdot(L+L_s)}{(R+R_s)^2+\omega^2\cdot(L+L_s)^2} \tag{6}$$

Considering trigonometric equalities:

$$\frac{1}{\sqrt{1+\beta^2}}=\cos(\arctan(\beta))$$
$$\frac{\beta}{\sqrt{1+\beta^2}}=\sin(\arctan(\beta)) \tag{7}$$

the particular solution equals:

$$\Phi_{AC}(t)=\frac{L\cdot U_m}{Z}\cdot\sin(\omega t+a-\Theta) \tag{8}$$

where

$$\Theta=\arctan\left(\frac{\omega\cdot(L+L_s)}{(R+R_s)}\right) \tag{9}$$

$$Z=\sqrt{(R+R_s)^2+\omega^2(L+L_s)^2} \tag{10}$$

It is clear that the voltage across transformer inductance $U_L(t)$ and its magnetic flux $\Phi(t) = \Phi_{DC}(t) + \Phi_{AC}(t)$ are π/4 shifted, as the definition dictates:

$$U_L(t)=N_1\cdot\frac{\mathrm{d}\Phi(t)}{\mathrm{d}t} \tag{11}$$

This is shown also in Fig. 2. By applying $u(t)$ at the moment when $U_L(t)$ is at its peak, $\Phi(t)$ would experience no DC component ($\Phi_0 = 0$) as the prospective flux would at that time be equal to zero – dashed grey curve. However, by applying it at $U_L(t)$ zero-crossing when the prospective flux would be at its peak value, it would experience the worst-case DC component ($\Phi_0 = \Phi_{0,max} = L \cdot U_m/Z$) – solid black curve. Of course, in above derivations no residual flux in the transformer core was assumed and consequently, Φ_0 is in such case always between $-L \cdot U_m/Z \leq \Phi_0 \leq L \cdot U_m/Z$. Also, due to impedance angle Θ being very close to $\pi/4$ it is clear that $U_L(t)$ is almost in phase with $u(t)$.

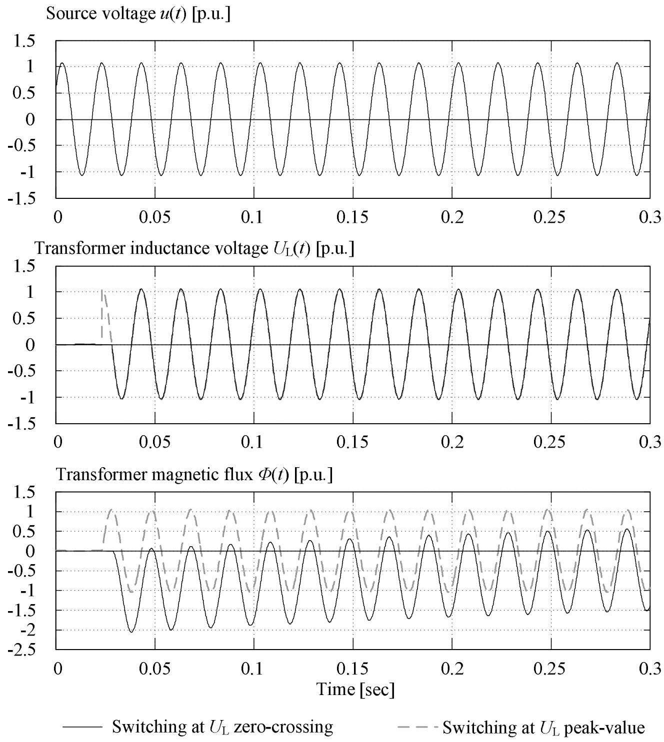

Fig. 2. System variables for circuit of Fig. 1 with respect to time for two typical moments of transformer energization

As long as our goal is merely understanding the reason behind the flux DC component $\Phi_{DC}(t)$ occurrence, the simplification of considering L_m (or in above derivation L, as $L = L_{\sigma 1} + L_m$) linear is more or less irrelevant. However, as soon as inrush currents are concerned, one has to keep in mind and suitably consider that L_m is in fact nonlinear. Magnetizing current $i_m(t)$ is directly dependent on the non-linearity of L_m. This is why in order to simulate the actual phenomenon, a non-linear saturation curve has to be incorporated into the model. In such case, high value of magnetic flux in the transformer core causes extremely high currents flowing through transformer winding (see lower graph on Fig. 3), which in turn cause significant voltage drops on system impedance during high current period – see upper graph on Fig. 3. Following (11) which describe the flux as being the integral of voltage, one can easily come to a conclusion that due to high currents the transformer core does not reach as much into saturation zone that would reach in case of e.g. Fig. 2 where L_m was considered linear. The comparison can be seen on the second graph on Fig. 3. So to a certain extent, this can be thought of as some kind of a self-regulating mechanism similar to self-regulating effect of power system load, which due to voltage and frequency drop usually decrease its power withdrawal from the grid [10]. Not only that, the decay of $\Phi_{DC}(t)$ is clearly faster in the presence of non-linear magnetizing characteristic.

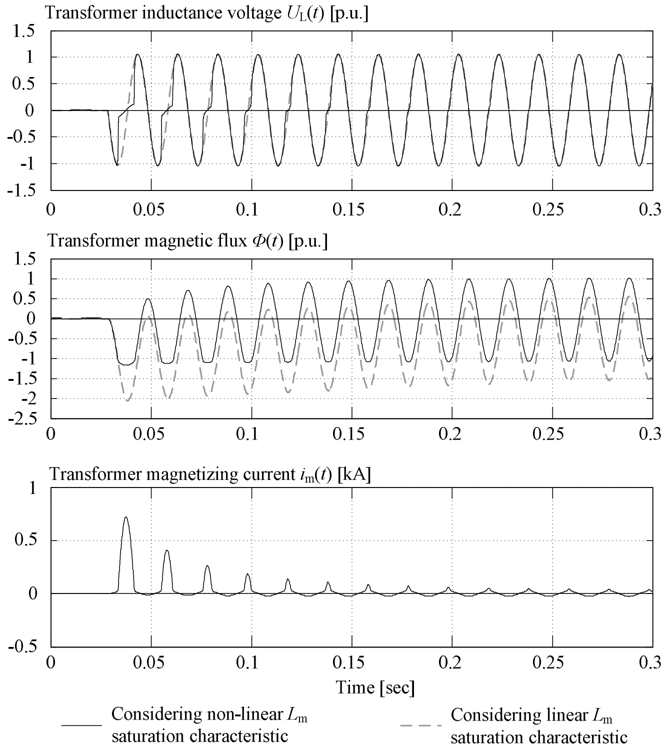

Fig. 3. Transformer inductance voltage, magnetic flux and magnetizing current in case of non-linear saturation characteristic

At this point, additional remark has to be discussed concerning transformer saturation curve. Transformer no-load tests usually provide a non-linear characteristic between transformer voltage and winding current. This data should not be used for simulation purposes carelessly, as no-load test results usually provide voltages and currents in RMS values. However, according to Fig. 3 (lower graph) the shape of the current while transformer is in saturation is highly non-sinusoidal. While the voltage used in tests is said to be purely sinusoidal and therefore peak values can simply be obtained by multiplying with $\sqrt{2}$, this is not the case with the current [11]. Measuring equipment is not specially calibrated to expect such extreme non-sinusoidal current conditions so measured RMS current values do not directly reflect the quantity of momentary currents values. This is why the no-load test curve has to be converted into simulation-suitable form by e.g. using an algorithm in [11].

III. Sympathetic Inrush Current

A. Derivations

Transformer sympathetic inrush current is a consequence of a sympathetic interaction between two or more transformers operating in parallel, after applying a voltage source $u(t)$ to one of them. A similar approach as for a single transformer energization can therefore be performed for the derivation of magnetic flux of two parallel transformers. As only the DC flux component is responsible for transformer saturation, AC flux will be ignored in this section. Similar equivalent circuit is used for derivation of equations with the only difference having two unloaded transformers instead of just one (Fig. 4). The switching is performed on transformer T2, whereas transformer T1 is already energized. Again, currents on the secondary winding of both transformers are assumed to be zero and the iron losses are also neglected. The situation is shown in Fig. 4, where the elements corresponding to secondary winding and iron losses are again deliberately depicted in grey color, as they can be treated as inactive. Two loop voltage equations (corresponding to two voltage loops, identified by denotations *I.* and *II.*) can be written in a similar manner than (1):

$$\begin{aligned}&-u(t)+\left[R_{\mathrm{s}}+R_{1,\mathrm{T1}}\right]\cdot i_{1,\mathrm{T1}}+R_{\mathrm{s}}\cdot i_{1,\mathrm{T2}}+\\&+\frac{\mathrm{d}}{\mathrm{d}t}\left(\left[L_{\mathrm{s}}+L_{\sigma1,\mathrm{T1}}+L_{\mathrm{m,T1}}\right]\cdot i_{1,\mathrm{T1}}+L_{\mathrm{s}}\cdot i_{1,\mathrm{T2}}\right)=0\end{aligned} \qquad (12)$$

$$\begin{aligned}&-R_{1,\mathrm{T1}}\cdot i_{1,\mathrm{T1}}+R_{1,\mathrm{T2}}\cdot i_{1,\mathrm{T2}}-\\&-\frac{\mathrm{d}}{\mathrm{d}t}\left(\left[L_{\sigma1,\mathrm{T1}}+L_{\mathrm{m,T1}}\right]\cdot i_{1,\mathrm{T1}}-\left[L_{\sigma1,\mathrm{T2}}+L_{\mathrm{m,T2}}\right]\cdot i_{1,\mathrm{T2}}\right)=0\end{aligned} \qquad (13)$$

Renaming and merging individual transformer quantities ($R_{1,\mathrm{T1}} = R_{\mathrm{T1}}$, $R_{1,\mathrm{T2}} = R_{\mathrm{T2}}$, $L_{\sigma1,\mathrm{T1}} + L_{\mathrm{m,T1}} = L_{\mathrm{T1}}$, $L_{\sigma1,\mathrm{T2}} + L_{\mathrm{m,T2}} = L_{\mathrm{T2}}$) (12) and (13), considering the relation between the magnetic flux and the current $L = \Phi/i$, can be written in the following matrix form:

$$\underbrace{\begin{bmatrix}\dot{\Phi}_{\mathrm{T1}}(t)\\ \dot{\Phi}_{\mathrm{T2}}(t)\end{bmatrix}}_{\dot{\mathbf{\Phi}}}=\underbrace{\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}}_{\mathbf{A}}\cdot\underbrace{\begin{bmatrix}\Phi_{\mathrm{T1}}(t)\\ \Phi_{\mathrm{T2}}(t)\end{bmatrix}}_{\mathbf{\Phi}}+\mathbf{B}\cdot u(t) \qquad (14)$$

where **A** is the state matrix, **B** is the control matrix [12] and individual matrix elements of **A** being (as already written, elements of **B** are not of interest within this section and are consequently not provided):

$$\begin{aligned}A_{11}&=-\frac{L_{\mathrm{s}}R_{\mathrm{T1}}+L_{\mathrm{T2}}R_{\mathrm{T1}}+L_{\mathrm{T2}}R_{\mathrm{s}}}{L_{\mathrm{T2}}L_{\mathrm{s}}+L_{\mathrm{T1}}L_{\mathrm{T2}}+L_{\mathrm{T1}}L_{\mathrm{s}}}\\A_{12}&=\frac{L_{\mathrm{T1}}\cdot\left(L_{\mathrm{s}}R_{\mathrm{T2}}-L_{\mathrm{T2}}R_{\mathrm{s}}\right)}{L_{\mathrm{T2}}\cdot\left(L_{\mathrm{T2}}L_{\mathrm{s}}+L_{\mathrm{T1}}L_{\mathrm{T2}}+L_{\mathrm{T1}}L_{\mathrm{s}}\right)}\\A_{21}&=\frac{L_{\mathrm{T2}}\cdot\left(L_{\mathrm{s}}R_{\mathrm{T1}}-L_{\mathrm{T1}}R_{\mathrm{s}}\right)}{L_{\mathrm{T1}}\cdot\left(L_{\mathrm{T2}}L_{\mathrm{s}}+L_{\mathrm{T1}}L_{\mathrm{T2}}+L_{\mathrm{T1}}L_{\mathrm{s}}\right)}\\A_{22}&=-\frac{L_{\mathrm{s}}R_{\mathrm{T2}}+L_{\mathrm{T1}}R_{\mathrm{T2}}+L_{\mathrm{T1}}R_{\mathrm{s}}}{L_{\mathrm{T2}}L_{\mathrm{s}}+L_{\mathrm{T1}}L_{\mathrm{T2}}+L_{\mathrm{T1}}L_{\mathrm{s}}}\end{aligned} \qquad (15)$$

We are dealing with the second-order system so two eigenvalues are expected. In case of two equal transformers ($R_{\mathrm{T1}} = R_{\mathrm{T2}} = R$, $L_{\mathrm{T1}} = L_{\mathrm{T2}} = L$) the eigenvalues are the following (also available from e.g. [5]):

$$\begin{aligned}\lambda_1&=-\frac{R}{L}\\\lambda_2&=-\frac{R+2R_{\mathrm{s}}}{L+2L_{\mathrm{s}}}\end{aligned} \qquad (16)$$

whereas in general case when transformers T1 and T2 are different they become:

$$\begin{aligned}\lambda_1&=-\frac{L_{\mathrm{T1}}L_{\mathrm{T2}}\begin{pmatrix}L_{\mathrm{s}}\left(R_{\mathrm{T1}}+R_{\mathrm{T2}}\right)+L_{\mathrm{T2}}\left(R_{\mathrm{T1}}+R_{\mathrm{s}}\right)+\\+L_{\mathrm{T1}}\left(R_{\mathrm{T2}}+R_{\mathrm{s}}\right)\end{pmatrix}+\sqrt{D}}{2L_{\mathrm{T1}}L_{\mathrm{T2}}\left(L_{\mathrm{T2}}L_{\mathrm{s}}+L_{\mathrm{T1}}L_{\mathrm{T2}}+L_{\mathrm{T1}}L_{\mathrm{s}}\right)}\\\lambda_2&=\frac{-L_{\mathrm{T1}}L_{\mathrm{T2}}\begin{pmatrix}L_{\mathrm{s}}\left(R_{\mathrm{T1}}+R_{\mathrm{T2}}\right)+L_{\mathrm{T2}}\left(R_{\mathrm{T1}}+R_{\mathrm{s}}\right)+\\+L_{\mathrm{T1}}\left(R_{\mathrm{T2}}+R_{\mathrm{s}}\right)\end{pmatrix}+\sqrt{D}}{2L_{\mathrm{T1}}L_{\mathrm{T2}}\left(L_{\mathrm{T2}}L_{\mathrm{s}}+L_{\mathrm{T1}}L_{\mathrm{T2}}+L_{\mathrm{T1}}L_{\mathrm{s}}\right)}\end{aligned} \qquad (17)$$

where

$$D=L_{\mathrm{T1}}^2L_{\mathrm{T2}}^2\begin{pmatrix}L_{\mathrm{s}}^2\left(R_{\mathrm{T1}}+R_{\mathrm{T2}}\right)^2+L_{\mathrm{T2}}^2\left(R_{\mathrm{T1}}+R_{\mathrm{s}}\right)^2+\\+L_{\mathrm{T1}}^2\left(R_{\mathrm{T2}}+R_{\mathrm{s}}\right)^2-2L_{\mathrm{T1}}L_{\mathrm{s}}\begin{pmatrix}\left(R_{\mathrm{T1}}-R_{\mathrm{T2}}\right)R_{\mathrm{T2}}+\\+\left(R_{\mathrm{T1}}+R_{\mathrm{T2}}\right)R_{\mathrm{s}}\end{pmatrix}+\\+2L_{\mathrm{T2}}\begin{pmatrix}R_{\mathrm{T1}}\left(L_{\mathrm{s}}R_{\mathrm{T1}}-\left(L_{\mathrm{T1}}+L_{\mathrm{s}}\right)R_{\mathrm{T2}}\right)-\\-\left(L_{\mathrm{T1}}+L_{\mathrm{s}}\right)\left(R_{\mathrm{T1}}+R_{\mathrm{T2}}\right)R_{\mathrm{s}}+L_{\mathrm{T1}}R_{\mathrm{s}}^2\end{pmatrix}\end{pmatrix} \qquad (18)$$

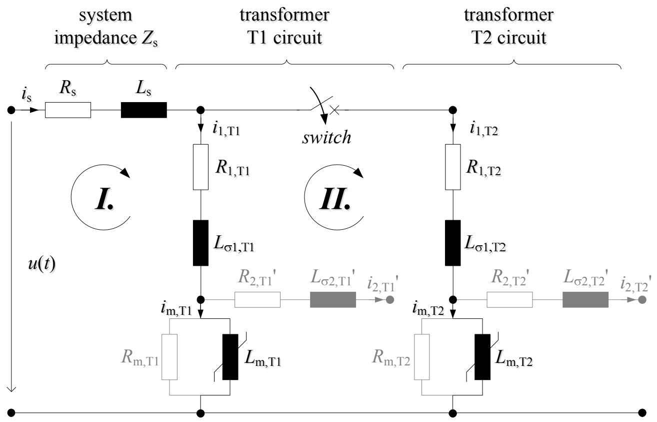

Fig. 4. Common equivalent circuit of two un-loaded power transformers operating in parallel, connected to a voltage source via system impedance

Provided equations (17) and (18) offer investigation of sympathetic interaction between transformers of different sizes, whereas in the available literature usually equal transformers are considered. For physically sensible parameters, both eigenvalues (λ_1 and λ_2) are real and the corresponding right eigenvectors of matrix **A** ($\boldsymbol{\eta}_1$ and $\boldsymbol{\eta}_2$) determine the DC flux components of both transformers, as shown:

$$\begin{bmatrix}\Phi_{\mathrm{T1}}(t)\\ \Phi_{\mathrm{T2}}(t)\end{bmatrix}=\begin{bmatrix}\boldsymbol{\eta}_1\end{bmatrix}\cdot K_1\cdot e^{\lambda_1 t}+\begin{bmatrix}\boldsymbol{\eta}_2\end{bmatrix}\cdot K_2\cdot e^{\lambda_2 t} \qquad (19)$$

Expressions for right eigenvectors $\boldsymbol{\eta}_1$ and $\boldsymbol{\eta}_2$ are a bit more complex and are due to the space limitation omitted from this paper. In order to obtain exact solutions for both fluxes, first the amplitudes of exponent terms (product of right eigenvector and unknown constant, i.e. $\boldsymbol{\eta}_1 \cdot K_1$ and $\boldsymbol{\eta}_2 \cdot K_2$) in (19) have to be determined. This is done by considering initial conditions at $t = 0$, when (19) becomes:

$$\begin{bmatrix} \Phi_{\mathrm{T1}}(0) \\ \Phi_{\mathrm{T2}}(0) \end{bmatrix} = \begin{bmatrix} \Phi_{\mathrm{T10}} \\ \Phi_{\mathrm{T20}} \end{bmatrix} = \begin{bmatrix} [\boldsymbol{\eta}_1] & [\boldsymbol{\eta}_2] \end{bmatrix} \cdot \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} \tag{20}$$

Solving (20) gives us expressions for constants K_1 and K_2. By multiplying those with eigenvectors $\boldsymbol{\eta}_1$ and $\boldsymbol{\eta}_2$ as determined by (19) and at the same time considering the fact that transformer T1 is already operational ($\Phi_{\mathrm{T10}} = 0$) and hence T2 is the sole subject to initial flux DC component at the moment of switching ($\Phi_{\mathrm{T20}} \neq 0$), amplitudes of exponent terms are obtained:

$$\begin{bmatrix} \Phi_{\mathrm{T1}}(t) \\ \Phi_{\mathrm{T2}}(t) \end{bmatrix} = \begin{bmatrix} E_1 / \sqrt{D} \\ 1/2 + E_2 / \sqrt{D} \end{bmatrix} \cdot \Phi_{\mathrm{T20}} \cdot e^{\lambda_1 t} + \begin{bmatrix} -E_1 / \sqrt{D} \\ 1/2 - E_2 / \sqrt{D} \end{bmatrix} \cdot \Phi_{\mathrm{T20}} \cdot e^{\lambda_2 t} \tag{21}$$

where

$$E_1 = L_{\mathrm{T1}}^2 \left(L_{\mathrm{T2}} R_{\mathrm{s}} - L_{\mathrm{s}} R_{\mathrm{T2}} \right)$$
$$E_2 = \frac{L_{\mathrm{T1}} L_{\mathrm{T2}} \begin{pmatrix} L_{\mathrm{s}} \left(R_{\mathrm{T2}} - R_{\mathrm{T1}} \right) - L_{\mathrm{T2}} \left(R_{\mathrm{T1}} + R_{\mathrm{s}} \right) + \\ + L_{\mathrm{T1}} \left(R_{\mathrm{T2}} + R_{\mathrm{s}} \right) \end{pmatrix}}{2} \tag{22}$$

By assuming the equality of both transformers, which is suitable from flux versus time visualization’s point of view, (21) becomes:

$$\begin{bmatrix} \Phi_{\mathrm{T1}}(t) \\ \Phi_{\mathrm{T2}}(t) \end{bmatrix} = \begin{bmatrix} -1/2 \\ 1/2 \end{bmatrix} \cdot \Phi_{\mathrm{T20}} \cdot e^{\lambda_1 t} + \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} \cdot \Phi_{\mathrm{T20}} \cdot e^{\lambda_2 t} \tag{23}$$

It is obvious that magnetic flux DC components of both transformers is determined by the same two exponent terms ($1/2 \cdot \Phi_{\mathrm{T20}} \cdot e^{\lambda 1 \cdot t}$ and $1/2 \cdot \Phi_{\mathrm{T20}} \cdot e^{\lambda 2 \cdot t}$), the only difference being that in case of T1 they are mutually subtracted whereas in case of T2 they are added. The situation is graphically depicted in Fig. 5. So basically each of the two exponent terms has its own decay rate (λ_1 and λ_2), among which the one containing system impedance (λ_2 in case of (16)) is clearly faster. This term is also the main factor determining the speed of the gradual transition into saturation of already operational transformer T1. Namely, transformer T2 is being energized and therefore its DC flux component appearance is sudden (as in case of Fig. 2 and Fig. 3). Transformer T1 on the other hand experiences gradual transition towards the saturation zone as seen in Fig. 5. It is of vital importance for one to notice that DC flux components of both transformers are of opposite sign and therefore, their saturations of opposite polarity as well. Physical background of the reason for such phenomenon is given in the following subchapter.

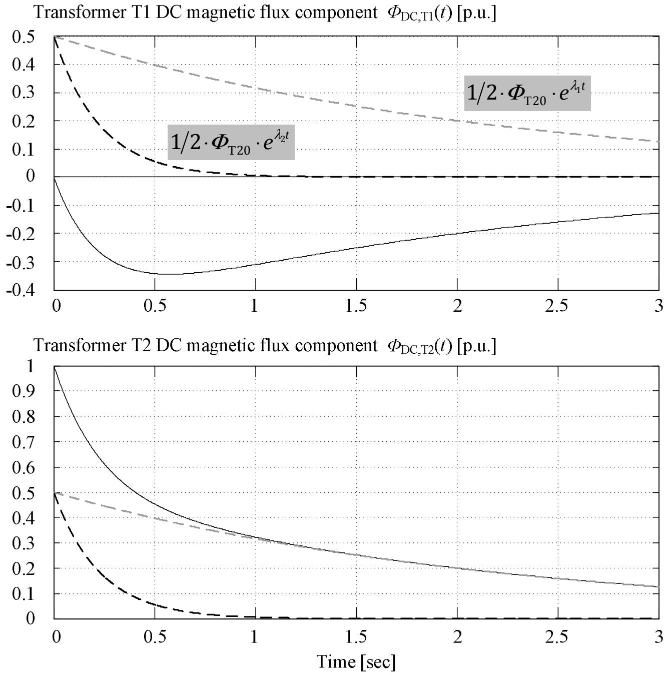

Fig. 5. Magnetic flux DC components of both (equal) transformers in case of transformer T2 energization

B. Physical background

For the purpose of understandable explanation of the physical situation, we will use (23) derived for two equal unloaded transformers. In the steady-state operation, currents can be thought of as purely sinusoidal, as transformer cores are not saturated. On the other hand, during the described phenomenon this is clearly not the case (lower graph on Fig. 3) and consequently, currents (i) have to be in general thought of as a sum of alternating AC component ($\tilde{i}$) and the direct DC component (I):

$$\begin{aligned} i_{1,\mathrm{T1}} &= \tilde{i}_{1,\mathrm{T1}} + I_{1,\mathrm{T1}} \\ i_{1,\mathrm{T2}} &= \tilde{i}_{1,\mathrm{T2}} + I_{1,\mathrm{T2}} \\ i_{\mathrm{s}} &= i_{1,\mathrm{T1}} + i_{1,\mathrm{T2}} = \tilde{i}_{\mathrm{s}} + I_{\mathrm{s}} \end{aligned} \tag{24}$$

According to (11) magnetic flux can be written with an integral of the voltage applied to the nonlinear inductance, so:

$$\begin{aligned} \Phi_{\mathrm{T1}}(t) &= \int \left(u(t) - \Delta u_1(t) \right) \mathrm{d}t \\ \Phi_{\mathrm{T2}}(t) &= \int \left(u(t) - \Delta u_2(t) \right) \mathrm{d}t \end{aligned} \tag{25}$$

where the voltage drops $\Delta u_1(t)$ and $\Delta u_2(t)$ encompass all elements between the source and the substitute inductances $L_{\mathrm{T1}} = L_{\sigma 1,\mathrm{T1}} + L_{\mathrm{m,T1}}$ and $L_{\mathrm{T2}} = L_{\sigma 1,\mathrm{T2}} + L_{\mathrm{m,T2}}$ and are therefore:

$$\begin{aligned} \Delta u_1(t) &= R_{\mathrm{s}} \cdot i_{\mathrm{s}} + L_{\mathrm{s}} \frac{\mathrm{d}i_{\mathrm{s}}}{\mathrm{d}t} + R_{1,\mathrm{T1}} \cdot i_{1,\mathrm{T1}} + \frac{\mathrm{d}}{\mathrm{d}t} \left(L_{\mathrm{T1}} \cdot i_{1,\mathrm{T1}} \right) \\ \Delta u_2(t) &= R_{\mathrm{s}} \cdot i_{\mathrm{s}} + L_{\mathrm{s}} \frac{\mathrm{d}i_{\mathrm{s}}}{\mathrm{d}t} + R_{1,\mathrm{T2}} \cdot i_{1,\mathrm{T2}} + \frac{\mathrm{d}}{\mathrm{d}t} \left(L_{\mathrm{T2}} \cdot i_{1,\mathrm{T2}} \right) \end{aligned} \tag{26}$$

For clearer understanding, let us observe the magnetic flux change $\Delta\Phi(t)$ in only a short time interval, e.g. single voltage period $T = 1 / f_n$ (where f_n is the nominal system frequency) so:

$$\Delta\Phi_{T1}(t) = \int_t^{t+T} \left(u(t) - \Delta u_1(t)\right)\mathrm{d}t = \int_t^{t+T} u(t)\,\mathrm{d}t - \int_t^{t+T} \Delta u_1(t)\,\mathrm{d}t$$
$$\Delta\Phi_{T2}(t) = \int_t^{t+T} \left(u(t) - \Delta u_2(t)\right)\mathrm{d}t = \int_t^{t+T} u(t)\,\mathrm{d}t - \int_t^{t+T} \Delta u_2(t)\,\mathrm{d}t \qquad (27)$$

The first summand represents the integral of a pure sinusoidal signal over one period and is therefore by the definition equal to zero in both cases. On the other hand, the second summand equals zero only when the currents constitute only an alternating component. Namely, the integral of a purely sinusoidal voltage drop over one period again equals zero. This means that the magnetic flux changes only when there is a direct current component present. By considering this fact and that a DC current does not cause any voltage drop on reactive elements, (27) can be rewritten as:

$$\Delta\Phi_{T1} = \int_t^{t+T} \left(-\left(R_s + R_{1,T1}\right)\cdot I_{1,T1} - R_s\cdot I_{1,T2}\right)\mathrm{d}t$$
$$\Delta\Phi_{T2} = \int_t^{t+T} \left(-R_s\cdot I_{1,T1} - \left(R_s + R_{1,T2}\right)\cdot I_{1,T2}\right)\mathrm{d}t \qquad (28)$$

One should keep in mind that currents in (28) represent DC currents. At the moment of switching $I_{1,T1} = 0$, whereas $I_{1,T1} > 0$ due to the assumption that the switching takes place in the worst possible moment and so DC flux component is suddenly present at transformer T2. According to (28) it can be seen that under such circumstances both flux changes are negative. This negative change builds up with each voltage period T, which gradually drags positive DC flux component of transformer T2 out of *positive* polarity saturation and on the other hand pushes DC flux component of transformer T1 in the same direction. However, as it is initially zero this means it is being driven towards *negative* polarity saturation.

The optimal way to support the above explanation is to add some graphic representation – see Fig. 6. The two graphs depict phase diagrams of magnetic flux versus magnetizing current of both transformers T1 and T2. Black dots represent DC magnetic flux components at the moment slightly after the application of $u(t)$ to transformer T2. In addition, black arrows indicate the direction towards which the dots are *initially* travelling as time t progresses. As can be seen from Fig. 6 DC flux component corresponding to T1 is at the origin of the diagram (no DC component at the switching moment), whereas DC flux component corresponding to T2 is at value $\Phi_{T20}>0$ (determined by the moment of connecting T2 to the grid – see Fig. 2 and Fig. 3). The sinusoidal curves on the graphs indicate the AC component of both fluxes, which summed together with a DC component represent the value of the total flux at each moment in time t. This makes it clear that transformer T1 at the moment of $u(t)$ application still operates in the linear part of the saturation characteristic (depictured by the thick gray line), whereas transformer T2 is already in positive polarity saturation. After several voltage periods T, both DC components are gradually shifted in the direction of black arrows. Since transformer T2 is building a negative polarity DC flux component, high magnetizing currents appear on its terminals as soon as the saturation curve knee is reached. In this way, $I_{1,T1}$ is gaining in amplitude of negative sign and eventually takes a substantial part of overall DC flux change – see (28). However, according to (28), $I_{1,T1}$ has a stronger influence on the flux change of transformer T1 (multiplied by the sum of two ohmic resistances) and $I_{1,T2}$ of transformer T2. This is why the direction of DC flux component of T1 eventually reverses, whereas that of T2 does not. After the reversal, DC flux components of both transformers travel towards zero with an important addition compared to single transformer energization, i.e. DC current of a neighboring transformer slows down the DC flux decaying rate due to opposite saturation polarity. This is why this transition is slower compared to the energization inrush phenomenon of a single transformer, as the currents of *both* transformers (of different signs) influence the speed of flux changes in *both* transformers.

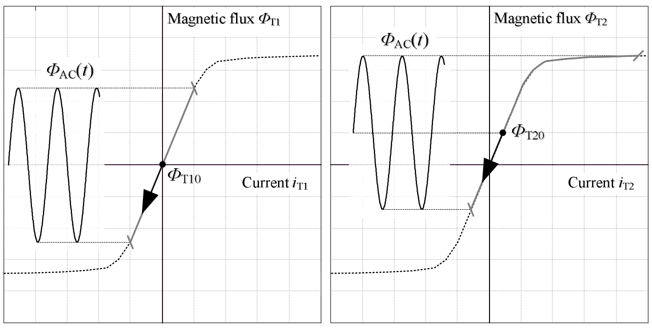

Fig. 6. Phase diagrams of transformers T1 and T2 at the moment slightly after application of a voltage source $u(t)$ to transformer T2

As both transformers have the same connection point (connected to the same busbar within a power plant or a substation), the voltages on both inductances L_{T1} and L_{T2} are in phase and consequently so are both fluxes. This means that the opposite polarity saturation zones of both transformers are not reached simultaneously, but 180° apart. As current and magnetic flux are in phase, this also goes for high current intervals, so called "current spikes" – see Fig. 7.

IV. Sympathetic Inrush Within a Substation

The phenomenon, extensively described in previous sections, does not differ in different topological situations, e.g. in a power plant or in a substation. However, in a power plant, transformer is used to increase the generator voltage and therefore the power flow is always in the same direction (towards the grid). Not only that, situation when one of several transformers would already be loaded and the other one being energized is rare. Within a substation on the other hand, such operation is common. The reason why this is important from the transformer inrush's point of view is provided in the following subsections.

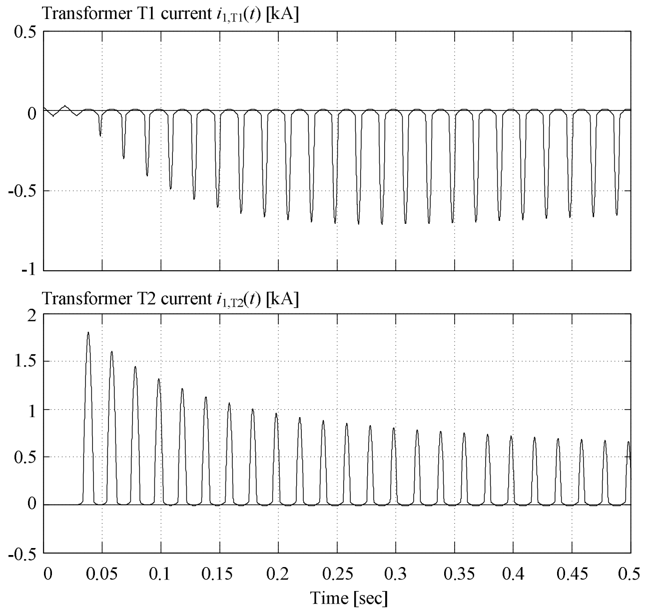

Fig. 7. Magnetizing currents "spikes" of both transformers during sympathetic inrush are 180° apart

A. *Power plant situation*

Within a power plant it is reasonable to perform transformer energization in such a way that generator is not affected. This means that energization of both parallel transformers is done while the generator is off-line, which means on the grid-side of the transformer. The first switching magnetizes first transformer (energization inrush from Fig. 1) while the second switching causes sympathetic inrush phenomenon – the situation is identical to that of Fig. 4. The currents, measured on both transformer windings, are equal to magnetizing current of each transformer.

B. *Substation situation*

Within a substation, situation usually occurs when a certain transformer is being energized while other parallel transformers are not only previously energized but are also fully loaded. In such case the currents, measured on the loaded transformer windings, encompass not only magnetizing current of the corresponding transformer but also the load current and especially a certain portion of the magnetizing current of neighboring transformer(s). The situation is graphically represented in Fig. 8. It is evident that highlighted primary $i_{1,\mathrm{T1}}$ and secondary $i_{2,\mathrm{T1}}$ currents are equal to:

$$\begin{aligned} i_{1,\mathrm{T1}} &= i_{\mathrm{load}} - f_2 \cdot i_{\mathrm{m,T2}} + f_1 \cdot i_{\mathrm{m,T1}} \\ i_{2,\mathrm{T1}} &= i_{\mathrm{load}} - f_2 \cdot i_{\mathrm{m,T2}} - (1 - f_1) \cdot i_{\mathrm{m,T1}} \end{aligned} \tag{29}$$

In Fig. 9, simulation results confirm (29). Within each graph in Fig. 9, the highlighted area is depicted on shorted time interval of 200 ms. It can be seen that a portion of magnetizing current $i_{\mathrm{m,T2}}$, flowing from LV grid, is present in both currents in the same extent. Gradually increasing $i_{\mathrm{m,T1}}$ is however present on depicted currents differently: first, with an opposite sign and second with a different share, depending on transformer and system impedances.

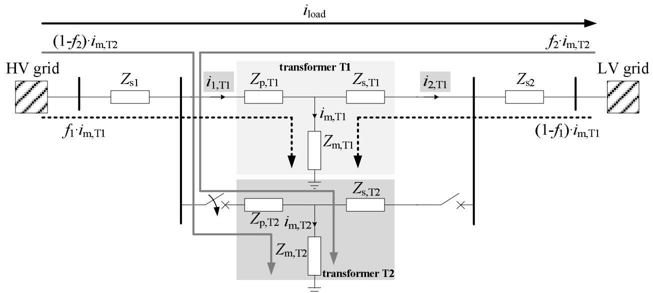

Fig. 8. Currents in the parallel connection of two transformers (operational one being loaded) during sympathetic inrush phenomenon

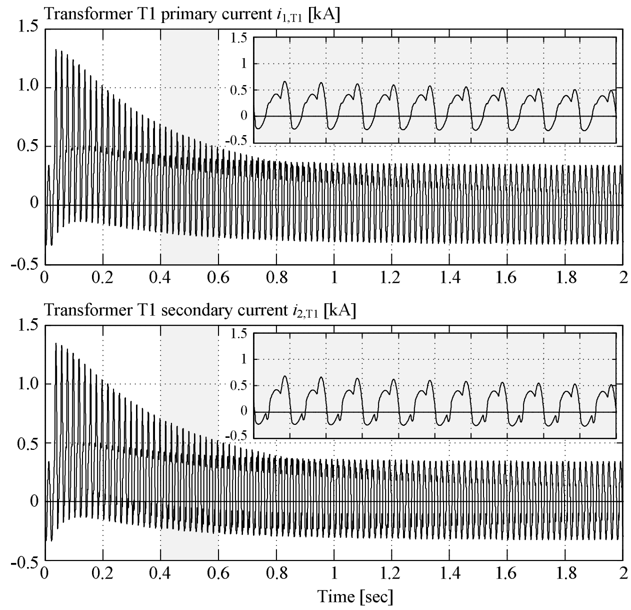

Fig. 9. Primary and secondary winding currents of loaded transformer T1 during energization of neighboring transformer T2

In the continuation, simulation results from Fig. 9 are compared to captured Wide Area Monitoring System (WAMS) measurements of the presented phenomenon (Fig. 10). However, before looking into WAMS measurements, few facts should be shortly discussed. WAMS system is based on Phasor Measurement Unit (PMU) operation, which provide measurements of phasors (not momentary values of AC variables) by following the IEEE standard [13]. According to [13], there are two performance classes of PMU requirements, the "M class" (intended for monitoring, which should provide more accurate data with no special need for fast reporting) and "P class" (intended for protection, which should provide less accurate data but in shorter period of time). WAMS results from Fig. 10 are in accordance with [13], but not yet intended to be a part of the protection scheme and therefore, the exact sliding window length of RMS calculation is relatively long. Besides, current transformers used for obtaining those measurements did most likely also suffer from saturation. Consequently, authors were unable to reconstruct the curves from Fig. 10. Nevertheless, the initial fast increase of current (see lower graph in Fig. 10) is in the order of 200 ms, which implies the cause for that might be a portion $f_2 \cdot i_{\mathrm{m,T2}}$. On the other hand, slower increase of current within the following 3 seconds (see upper graph in Fig. 10) might be the consequence of $(1-f_1)\cdot i_{m,T1}$. One should be aware that during the calculation of RMS values, all measurements are first squared and consequently the sign of the current is not of any significance.

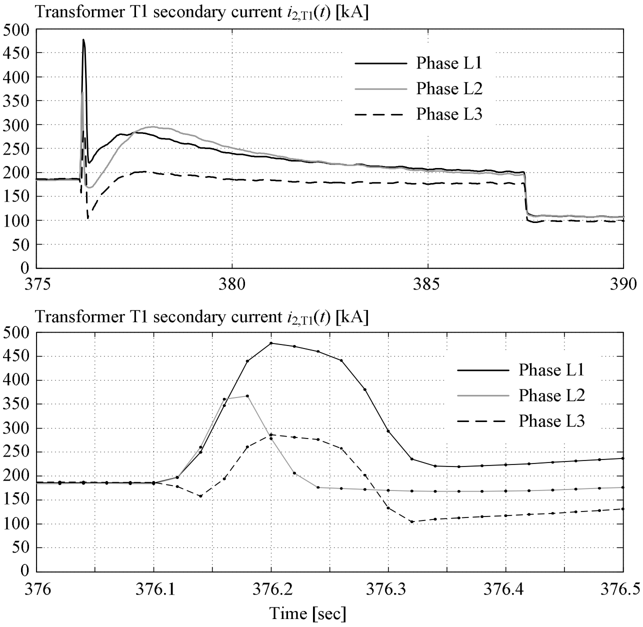

Fig. 10. Captured WAMS measurements of a sympathetic inrush current phenomenon (different time scales)

V. Conclusions

In this paper the sympathetic inrush current of two parallel transformers is presented by the use of a modal approach to solving equivalent circuit. Equivalent circuit's differential equations were expressed in the state-equation form which is usually not practice in similar papers. By doing so, authors made the analysis of DC flux component possible for several kinds of situations, e.g. with more than two parallel transformers, with transformers of different sizes, etc.

Special attention was given to explaining the phenomenon in such a way that the paper content would be useful to both scientists and engineers in the process of studying the phenomenon. In addition, the situation with already operational transformer being fully loaded was addressed, the reason being availability of currents that can be measured in reality. Namely, in case of un-loaded transformer e.g. the winding current equals the magnetizing current, whereas in case of loaded transformer, the winding current encompasses several components. The simulation results were compared to captured WAMS measurements of the phenomenon in a real power system. A few problematic issues that make the comparison difficult were stressed. In order to reconstruct the captured WAMS measurements, further work will include modelling PMU's phasor length calculation from momentary signal values. Also, the model will be expanded to include not only the saturation characteristic but hysteresis as well. In such a way, it will be possible to approach the real situation, as at the moment of transformer switching there is most certainly some remanent flux present in the transformer iron core. Its modelling is possible only through hysteresis.

VI. References

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