Evaluation of the Effect of Parameters of Three-Phase Transformer Core Models on the Harmonic Content of Inrush Currents: Implications on the Setting of Inrush Detection Functions

João Ricardo M. S. Souza, Clever S. Pereira Filho, Alberto De Conti

Abstract-- **Nowadays, several functions related to transient phenomena are implemented in transformer differential relays, especially inrush detection algorithms based on harmonic content. Analysis of the behavior of this kind of function is more properly carried out in time domain. However, to perform a time domain study it is necessary to pay attention to the modeling of the system components. To perform an analysis of inrush detection algorithms, for instance, the accuracy of the transformer model is significant, especially of its core. The aim of this paper is to analyze the effect of the parameters of the transformer core model on the harmonic content of inrush currents and discuss how these parameters could affect the setting of the inrush detection functions based on the harmonic content. To do this, hundreds of simulations of the energization of a three-phase power transformer modeled by the Hybrid Model available in ATP are performed using statistic switches for different parameters related to the core model. The obtained results, which are compared to a theoretical curve obtained for a single-phase transformer, indicate that core topology and dimensions could affect significantly the harmonic content of the inrush current and, consequently, the setting of inrush detection functions.**

Keywords: **Transformers, differential protection, time domain analysis, hybrid model, ATP.**

I. Introduction

Nowadays, there are several functions implemented in transformer differential relays related to transient phenomena, especially inrush detection algorithms based on harmonic content. Analysis of the behavior of this kind of function is more properly carried out in time domain [1, 2]. However, to perform a time domain study it is necessary to pay attention to the modeling of the system components. To evaluate inrush detection algorithms, for instance, the accuracy of the transformer model is significant, especially of its core.

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The study of transformer inrush currents is of great importance for electrical systems, and has been the subject of several investigations in the last century (see, e.g., [3,4], for a review). Some efforts have been made to investigate the harmonic content of inrush currents on single-phase transformers (e.g., [5, 6]). For three-phase transformers, some studies have attempted to correlate the core topology and waveform shape with the peak value of inrush currents [7]. However, the investigation of the relationship between the design of the transformer core and the harmonic content of inrush currents is subject of a limited number of references [2, 8].

The Alternative Transient Program (ATP) is one of the most used tools for performing electromagnetic transient studies [9]. The most advanced transformer model available in ATP is the so-called hybrid model [10]. In the hybrid model, the core equivalent circuit obtained by duality principle is connected to the leakage network of the windings by a fictitious winding located on the core. The use of the duality principle increases the model accuracy, but produces different models for each core topology of three-phase transformers. Moreover, information about the relative dimensions of each core component is necessary. However, final users rarely have access to this type of information and are forced to make inferences about it.

The aim of this paper is to analyze the effect of the parameters of the transformer core model on the harmonic content of inrush currents and discuss how these parameters could affect the setting of the inrush detection functions based on the harmonic content. To do this, hundreds of simulations of the energization of a three-phase power transformer modeled by the hybrid model are performed using the statistic switches available at ATP. The inrush currents are treated by an algorithm implemented in MODELS that calculates the fundamental component and the rate of second and fourth harmonics using cosine filters. The analysis is performed varying the core topology and the relative dimensions of the core components. The obtained results are compared with each other and with a theoretical curve obtained for a single-phase transformer. It is important to point that the residual flux was not considered in this study.

This paper is organized as follows. Section II presents a theoretical analysis of the fundamental, second harmonic and fourth harmonic content of inrush currents for a single-phase transformer. Section III presents simulation results obtained varying the core topology and its relative dimensions. Section IV presents a correlation analysis of the obtainer results. Finally, Section V presents the conclusions.

II. Theoretical Behavior of the Harmonic Content of the Inrush Currents for a Single-phase Transformer

A. Analytical Calculation

When a single-phase transformer is energized, a magnetic flux λ will appear in its core. This flux is basically composed of a sinusoidal oscillating at the fundamental frequency superimposed to an exponentially decaying function. By using a simplification, the current-flux curve of the magnetization branch of a single-phase transformer could be represented by three points. These three points define two lines: one that represents the linear region and another that represents the saturation region. So, in this case, the current *i* that flows through the primary winding is composed by two different regions as shown in Fig. 1 [11].

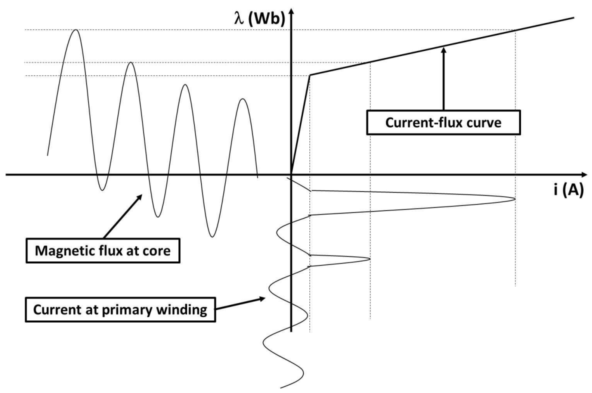

Fig. 1. Approximation of the behavior of the magnetic flux at core and the current at the primary winding in a single-phase transformer [11].

Another possible approximation is to make the slope of the linear region of the current-flux curve of the magnetization branch approach π radians [11]. This is reasonable because this slope is higher than the slope of the saturation region. By doing this, the inrush current becomes a sequence of pulses such as shown in Figure 2.

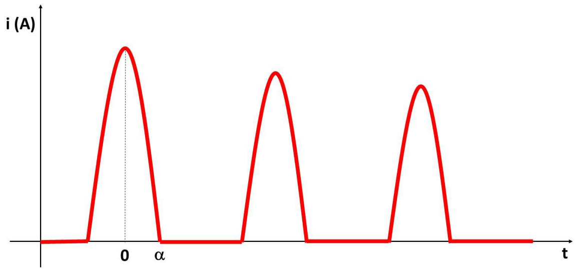

Fig. 2. Inrush current approximation for a single-phase transformer.

The current pulses shown in Figure 2 may be characterized by its opening angle α. By neglecting the exponential decay, the inrush current can be expressed as [11]:

$$i(\theta)=\begin{cases} I_m\cdot(\cos\theta-\cos\alpha), & -\alpha\le\theta\le\alpha \\ 0, & -\pi\le\theta<-\alpha,\quad \alpha<\theta\le\pi \end{cases} \tag{1}$$

Applying the Fourier transform, the n^{th} harmonic content of the inrush current for a given opening angle α can be obtained as follows

$$i_n(\alpha)=\frac{I_m}{\pi}\cdot\left(\begin{array}{l}\frac{1}{n+1}\cdot\sin[(n+1)\cdot\alpha]+\frac{1}{n-1}\cdot\sin[(n-1)\cdot\alpha]+ \\ +2\cdot\cos(\alpha)\cdot\frac{1}{n}\cdot\sin(n\cdot\alpha)\end{array}\right) \tag{2}$$

where *n* is the harmonic order, and I_m is the maximum possible value of the inrush current, which is reached if α=π radians. Equation (2) is valid for $n > 1$. The fundamental component is given by [11]:

$$i_1(\alpha)=\frac{I_m}{\pi}\cdot\left[\alpha-\frac{1}{2}\cdot\sin(2\cdot\alpha)\right] \tag{3}$$

B. Validation of the Analytical Formulas

Equations (2) and (3) show that, for single-phase transformers, the current harmonic content could be used to detect transformer energization. However, to obtain (2) and (3) some simplifications are made. To evaluate the effect of these simplifications and validate these equations, a set of 200 simulations dealing with the energization of a three-phase transformer was performed using ATPDraw according to the circuit shown in Fig. 3. The energization was performed only at the primary winding and the others windings (secondary and tertiary) remained opened.

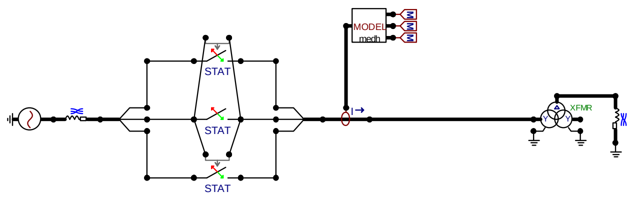

Fig. 3. Circuit simulated in ATPDraw.

In the simulations, the three-phase transformer was represented with the XFMR model, which allows the user to consider different core topologies. A triplex core was used because it is equivalent to three single-phase transformers. The transformer parameters, which are listed in Tables I-III, were obtained from the test report of an actual transformer.

Table I – Transformer Parameters

	Primary	Secondary	Tertiary
V (kV)	138	34.5	13.8
S (MVA)	18	18	18
Connection	Yg	Yg	D
Phase Shift (°)	-	0	30

TABLE II – SHORT-CIRCUIT TEST RESULTS

	Impedance (%)	Power Base (MVA)	Loss (kW)
P – S	4.28	18	34.47
P – T	7.46	18	39.95
S – T	2.50	18	31.03

TABLE III – OPEN CIRCUIT TEST RESULTS (PERFORMED AT TERTIARY)

Voltage (%)	80	85	90	95	100	105	110
Loss (kW)	15.825	17.945	21.292	24.752	28.906	32.126	46.788
Current (%)	0.117	0.128	0.151	0.189	0.295	0.638	2.378

The statistical switches available in ATP were used. The closing time of the reference switch was defined by a uniform distribution with mean 0.025 s and standard deviation 0.00481 s, which results in one 60 Hz cycle interval. The closing times of the other two switches were defined by a normal distribution whose mean is equal to the closing time of the first switch and the standard deviation is 0.0015 s. This procedure aims to take into account the pole discrepancy of a 138-kV circuit breaker.

The system equivalent seen from the busbar at which the transformer protection is analyzed was modeled with a balanced three-phase voltage source behind an RL coupled element represented in symmetrical components. The values of the positive and zero sequence resistance and reactance used in this study are shown in Table IV.

TABLE IV – SYSTEM EQUIVALENT

	R(Ω)	X(Ω)
Positive Sequence	0.826	5.728
Zero Sequence	0.703	5.245

A MODELS code was written to evaluate the value of the fundamental, second and fourth harmonic components of the inrush currents. This code uses a cosine filter and a rectangular window with a single 60 Hz cycle length and 64 points per cycle. The peak values obtained for phase A after each simulation are illustrated in Fig. 4 in the form of sets of points relating the second and fourth harmonics to the fundamental component. These plots were generated by Scilab routines.

From the second harmonic content curve shown in Fig. 4, a regression to a fourth order polynomial curve was performed using a least squares technique following (4)–(6). The obtained curve is shown in the first plot of Fig. 4. A similar procedure was adopted for the fourth harmonic content curve. However, in this case, the square of the fundamental current and a different order were used, according the polynomial in (7). The obtained curve is shown in the second plot of Fig. 4.

$$I_1 = \theta_1 + \theta_2 \cdot I_2 + \theta_3 \cdot I_2^2 + \theta_4 \cdot I_2^3 + \theta_5 \cdot I_2^4 \quad (4)$$

$$\begin{bmatrix} I_{1-1} \\ I_{1-2} \\ \vdots \\ I_{1-n} \end{bmatrix} = \begin{bmatrix} 1 & I_{2-1} & I_{2-1}^2 & I_{2-1}^3 & I_{2-1}^4 \\ 1 & I_{2-2} & I_{2-2}^2 & I_{2-2}^3 & I_{2-1}^4 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & I_{2-n} & I_{2-n}^2 & I_{2-n}^3 & I_{2-1}^4 \end{bmatrix} \cdot \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix} \Rightarrow \bar{y} = \tilde{X} \cdot \bar{\theta} \quad (5)$$

$$\bar{\theta} = [(\tilde{X}^T \cdot \tilde{X})^{-1} \cdot \tilde{X}^T] \cdot \bar{y} = \tilde{P} \cdot \bar{y} \quad (6)$$

$$I_1^2 = \sum_{n=0}^{9} \theta_n \cdot I_4^n \quad (7)$$

The dashed lines in Fig. 4 were obtained with (2) and (3) by varying the opening angle from 0 to 0.6 π radians and choosing an adequate value for I_m (in this case, 100 A) so that a comparison of the curves obtained by the analytical equations with those obtained by the regression curves could be done. This comparison shows that (2) and (3) are sufficiently accurate in the investigated case and that the simplifications performed in their derivation do not affect the results significantly.

A linear blocking characteristic was also plotted in both viewgraphs in Fig. 4. Several manufactures use the second and fourth harmonic content ratio to block their differential relays during a transformer energization. Thus, the points located above the blocking characteristic represent cases in which the differential relay will operate incorrectly.

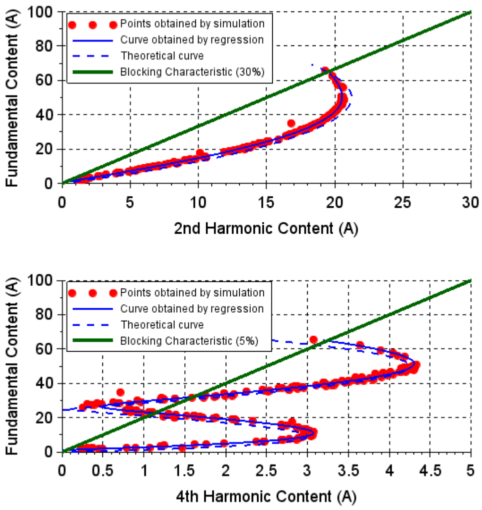

Fig. 4. Second and fourth harmonic characteristic of inrush currents.

III. EFFECTS OF CORE TOPOLOGY AND RELATIVE CORE DIMENSIONS ON THE HARMONIC CONTENT OF INRUSH CURRENTS IN THREE-PHASE TRANSFORMERS

A. Effect of Core Topology

As mentioned previously, the hybrid model available in ATP is capable of modeling three-phase transformers taking into account their core topology. To obtain a theoretical estimate of the effect of this parameter on the setting of inrush detection functions based on harmonic content, the simulation performed in section II was repeated varying the type of core and the relative dimensions of its parts. To assure repeatability, the seed of the random number generator used to define the closing time of the switches was set to the same value in the beginning of each set of simulations.

The plots in Figs. 5 and 6 illustrate simulation results for all phases of the three-phase transformer evaluated in Section II. However, this time it was assumed that this transformer consisted either of a 3-leg or a 5-leg stacked core transformer. For simplicity, the same parameters presented in Tables I, II and III were considered. Likewise, Figs. 7 and 8 show simulation results corresponding to a form A and a form B shell core transformer, respectively.

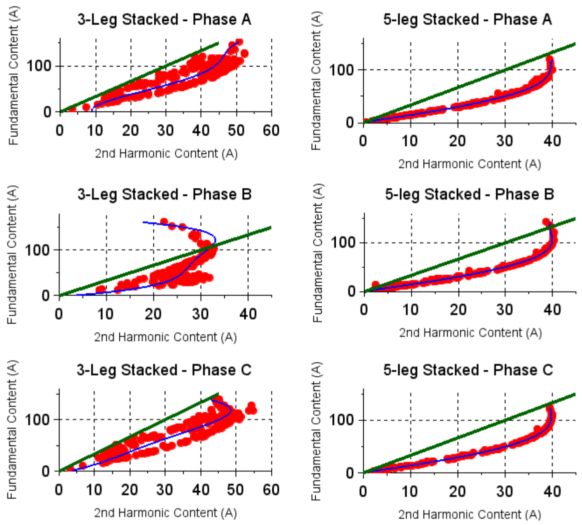

Fig. 5. Second harmonic characteristic of inrush currents obtained for 3-leg and 5-leg stacked core transformers. Red dots: points obtained by simulation; blue line: curve obtained by regression; green line: blocking characteristic (5%).

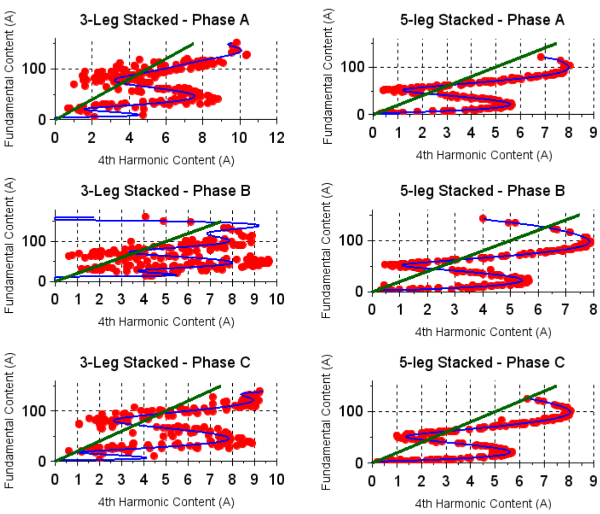

Fig. 6. Fourth harmonic characteristic of inrush currents obtained for 3-leg and 5-leg stacked core transformers. Red dots: points obtained by simulation; blue line: curve obtained by regression; green line: blocking characteristic (5%).

It is seen in Figs. 5 and 6 that during the energization of a 5-leg stacked core transformer the inrush current harmonic content presents the same characteristic in all three phases, such as during the energization of a triplex core transformer. However, this is not observed during the energization of a 3-leg stacked core transformer, which could be explained by the lack of a ferromagnetic path for the zero sequence flux component in the latter. It is also seen in Figs. 5 and 6 that during the energization of a 3-leg stacked core transformer the inrush harmonic content reaches higher values than in the case of a triplex core transformer. This could be explained by the coupling of the there-phases in the 3-leg stacked core transformer, which increases the saturation. It is to be noted, however, that the increase of the fundamental component is greater than the increase of the second harmonic component. This suggests that the percentage of the second harmonic content used for the inrush detection function associated to a 3-leg stacked core transformer should be possibly set to a smaller value than in the case of a triplex core transformer.

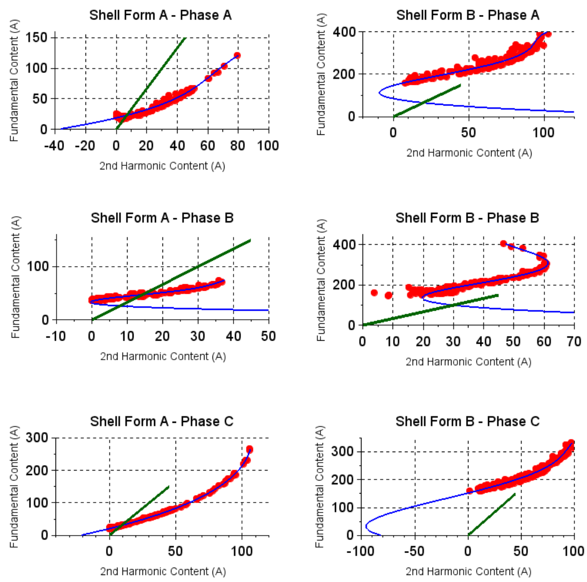

Fig. 7. Second harmonic characteristic of inrush currents obtained for a form A and a form B shell core transformer. Red dots: points obtained by simulation; blue line: curve obtained by regression; green line: blocking characteristic (5%).

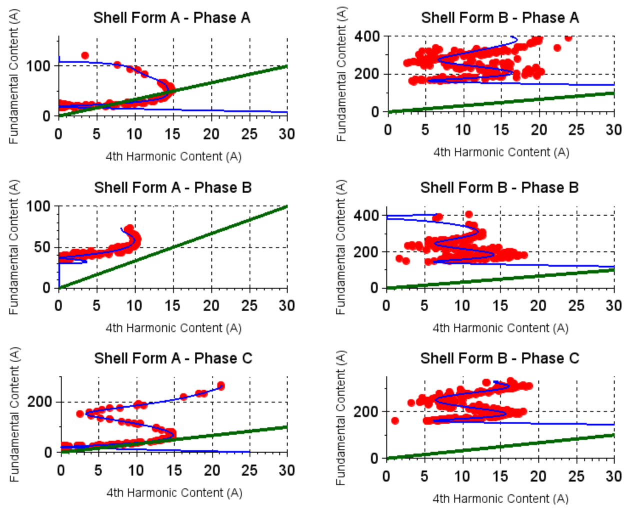

Fig. 8. Fourth harmonic characteristic of inrush currents obtained for a form A and a form B shell core transformer. Red dots: points obtained by simulation; blue line: curve obtained by regression; green line: blocking characteristic (5%).

The plots in Figs. 7 and 8 for a shell core transformer suggest the existence of an offset in the relationship between the fundamental and the second and fourth harmonic characteristics of the inrush currents. This is true especially for transformers of form B, in which the winding of the center leg is installed in the opposite direction of the other two legs. The existence of this offset suggests that the percentage of the second and fourth harmonic content used for the inrush detection function should be set to very small values. This could pose serious difficulties in the use of the harmonic blocking technique for the protection of this specific transformer.

B. Effect of Relative Core Dimensions

In the simulation results shown in Figs. 5-8 the relative dimensions of all transformer core parts (yoke, outer leg and middle limb) were set to 1 for areas and 2 for lengths in the hybrid model. However, this parameter could vary for each manufacturer and transformer. Furthermore, this information is usually unknown by the final users. In order to verify, from a theoretical point of view, the significance of varying the relative dimensions of the core on the harmonic content associated with transformer inrush currents, Figs. 9 and 10 show, respectively, the second and the fourth harmonic characteristics obtained by varying the relative area of a 3-leg stacked core transformer. Likewise, Figs. 11 and 12 show, respectively, the second and the fourth harmonic characteristics obtained by varying the relative length of the same 3-leg stacked core transformer.

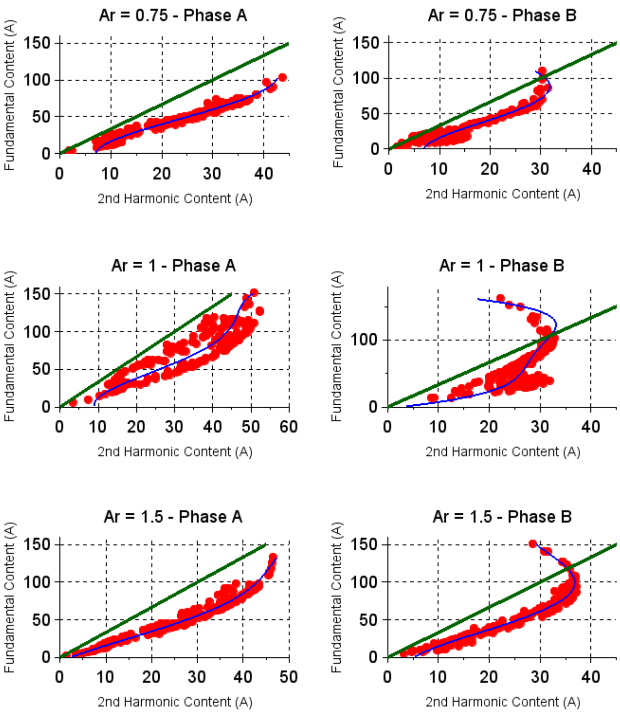

Fig. 9. Second harmonic characteristic of inrush currents obtained for a 3-leg stacked core transformer varying the relative area. Red dots: points obtained by simulation; blue line: curve obtained by regression; green line: blocking characteristic (5%).

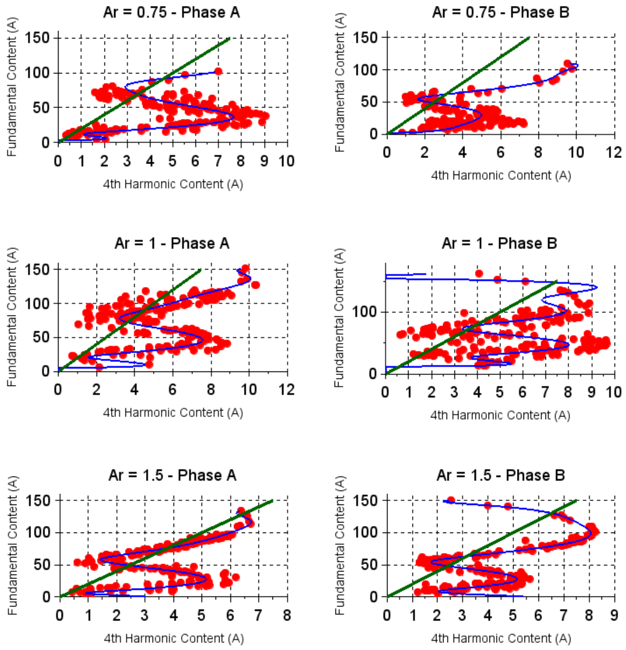

Fig. 10. Fourth harmonic characteristic of inrush currents obtained for a 3-leg stacked core transformer varying the relative area. Red dots: points obtained by simulation; blue line: curve obtained by regression; green line: blocking characteristic (5%).

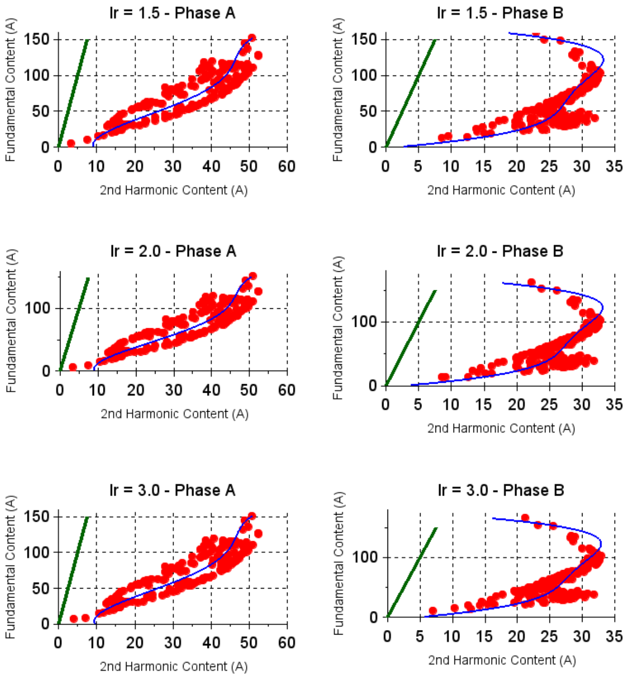

Fig. 11. Second harmonic characteristic of inrush currents obtained for a 3-leg stacked core transformer varying the relative length. Red dots: points obtained by simulation; blue line: curve obtained by regression; green line: blocking characteristic (5%).

The plots shown in Figs. 9 and 10 suggest that the relative area affects the harmonic content of the inrush current significantly. Then, it could also affect the percentage of the second harmonic content used for the inrush detection function. This happens because changing the relative area modifies the current-flux curve of the yoke. This is illustrated in Fig. 13, which shows the current-flux curve of the yoke and of the legs in the cases corresponding to Figs. 9 and 10. It can be seen that for a relative area of 0.75, the saturation of the yoke is more significant and for a relative area of 1.5 the saturation of the leg is more significant. However, for a relative area of 1, the knees of the both current-flux curves are located almost at the same point.

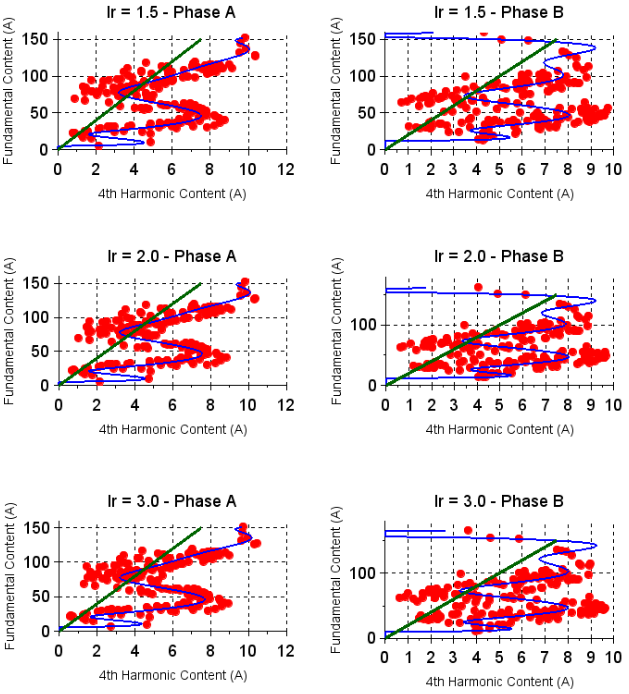

Fig. 12. Fourth harmonic characteristic of inrush currents obtained for a 3-leg stacked core transformer varying the relative length. Red dots: points obtained by simulation; blue line: curve obtained by regression; green line: blocking characteristic (5%).

In the plots shown in Figs. 11 and 12, the relative length does not seem to affect significantly the harmonic content of the inrush current. As a consequence, this parameter is unlikely to affect the percentage of the second and fourth harmonic contents used for the inrush detection function. This happens because the relative length does not change the current-flux curve significantly, as shown in Fig. 14.

IV. CORRELATION ANALYSIS

When analyzing the energization of a 3-leg stacked core transformer, it is seen that the shape of the curves obtained by regression differ from the shape of the theoretical curve discussed in Section II, especially for phase B (the center leg one), as shown in Figs. 5 and 6. This happens because in this case there is a higher dispersion between the points of the fundamental component and the peak values of the second and fourth harmonic components. In other words, a small correlation is observed between the fundamental component and the second and fourth harmonics in this particular case. It also happens for the fourth harmonic component of the shell core transformer, especially for form B, as illustrated in Fig. 8. The reason for this loss of correlation is the lack of a ferromagnetic path for the circulation of zero sequence components and the proximity of the knee points of the current-flux curves of the leg and the yoke, such as shown in Fig. 13(b).

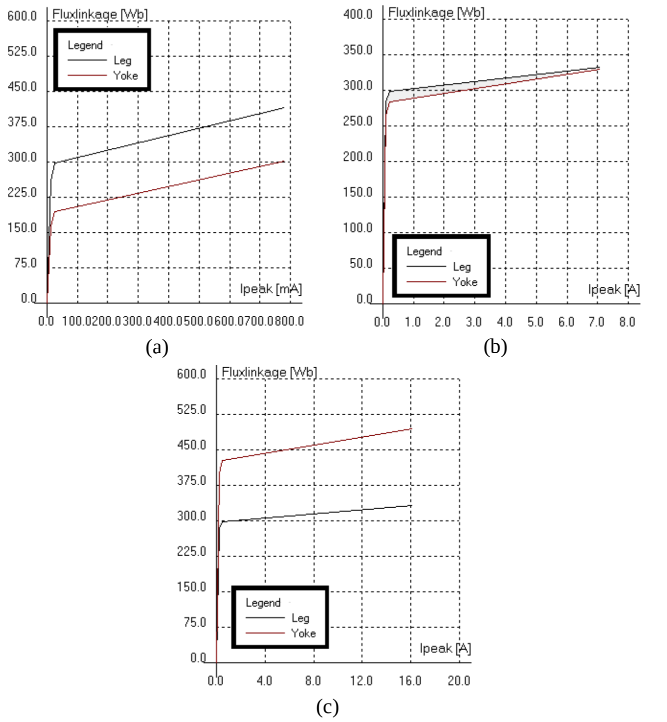

Fig. 13. Current-flux curves for a 3-leg stacked core transformer with relative area set to (a) 0.75, (b) 1.0 and (c) 1.5.

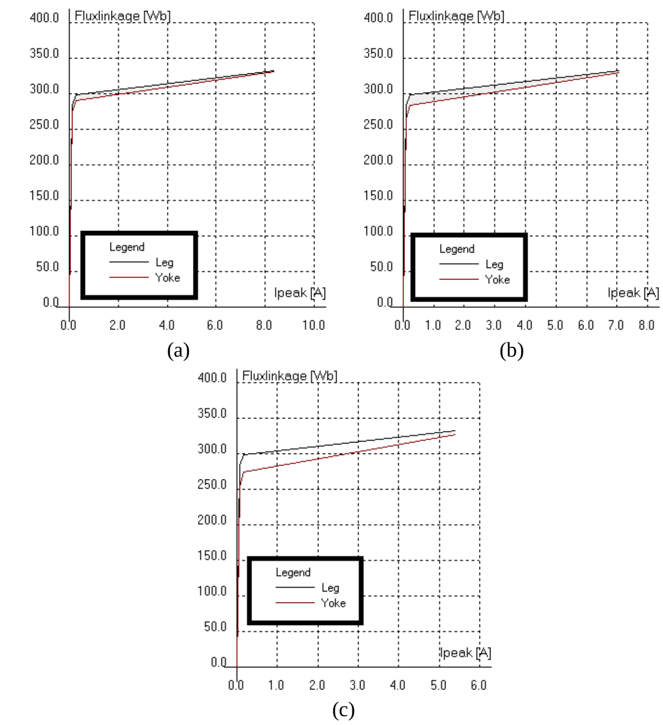

Fig. 14. Current-flux curves for a 3-leg stacked core transformer with relative length set to (a) 1.5, (b) 2.0 and (c) 3.0.

Table V shows the correlation factors between the maximum fundamental and the second harmonic component of the inrush current obtained for different core topologies. Similarly, Table VI shows the correlation factors between the maximum fundamental and the fourth harmonic component. The correlation factor $\rho_{X,Y}$

$$\rho_{X,Y} = \frac{\operatorname{cov}(X,Y)}{\sqrt{\operatorname{var}(X)\cdot\operatorname{var}(Y)}} \tag{8}$$

shows how significant is the relationship between two random variables X and Y, where cov(X, Y) is the covariance between X and Y, and var(X) and var (Y) are their individual variances [10]. The correlation factor is a number between 0 and 1. If its value is close to 1, then the relationship between the two random variables is strong. However, if its value is close to 0 these variables are called uncorrelated.

TABLE V – CORRELATION FACTORS BETWEEN THE MAXIMUM FUNDAMENTAL AND THE MAXIMUM SECOND HARMONIC COMPONENT

Phase	Triplex	3-leg Stacked	5-leg stacked	Shell Form A	Shell Form B
A	0.947	0.898	0.963	0.960	0.945
B	0.920	0.617	0.942	0.981	0.891
C	0.924	0.928	0.955	0.968	0.966

TABLE VI – CORRELATION FACTORS BETWEEN THE MAXIMUM FUNDAMENTAL AND THE MAXIMUM FOURTH HARMONIC COMPONENT

Phase	Triplex	3-leg Stacked	5-leg stacked	Shell Form A	Shell Form B
A	0.629	0.361	0.603	0.691	0.270
B	0.547	0.136	0.536	0.907	0.227
C	0.627	0.449	0.709	0.592	0.310

According to Table V, despite the dispersion observed for the 3-leg stacked core, seen in Figs. 5 and 6, the second harmonic and the fundamental components still have a good correlation. Nevertheless, because of such dispersion the percentage of the second harmonic content used in these cases in the inrush detection function should in theory be set to smaller values.

According to Table VI, the correlation factor between the fourth harmonic and the fundamental components are close to 0 for the 3-leg stacked core and the shell core form B. This result could indicate that, in these cases, the rate of fourth harmonic content is not a good indicator of the occurrence of transformer energization.

V. CONCLUSIONS

This paper presents a theoretical analysis about the prospective effect of parameters of three-phase transformer core models on the harmonic content of inrush currents and discusses its implications on the setting of inrush detection functions. As a result, several possible recommendations about the setting of inrush detection functions of differential relays are presented.

Moreover, this study could help to define which topology and dimensions could be used to reach a more conservative situation in a study of the behavior of an inrush detection scheme if these data were unknown. Thus, the user could take advantage of the better precision of the hybrid model even without sufficient information about the transformer constructive characteristics.

Although promising, the proposed methodology requires proper validation based on experimental data, which is a topic for future research. The proposed methodology could be used to evaluate the effect of other parameters, such as residual flux, on the behavior of inrush detection functions. Furthermore, it could also be used to study and validate novel techniques for detection of transformer energization.

VI. ACKNOWLEDGEMENTS

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