

Accuracy and Realization Issues in Frequency Dependent Sequence Networks

João P. L. Salvador, Felipe Camara Neto, Antonio C. S. Lima, Kai Strunz

Abstract—With the more recent widespread of real-time simulation technologies, network equivalents received newly interest. Typically, the realization of a Frequency-Dependent Network Equivalent (FDNE) for electromagnetic transients (EMT) studies and its application on computational programs (EMTP) is based on a rational approximation of a given transfer function. One point that has not received much attention is to identify which aspects of the network being represented has a larger impact on the FDNE realization.

In this paper we address some of these issues regarding the frequency dependency of overhead transmission lines parameters and the impact of actual transposition schemes on the realization network equivalents in sequence networks.

Keywords: *FDNE, Simulation Tools, EMTP, VF.*

I. INTRODUCTION

TYPICALLY power flow, harmonic penetration and transient stability studies are carried out using positive sequence networks considering a system with more than a few thousand busbars. Electromagnetic Transient (EMT) analysis on the other hand consider a different approach as the network modeling is more detailed, which demands a reduction on the actual size of the system to be represented. Although some recent efforts [1] tried to include a large portion of an actual network in EMTP-RV, the usual practice consists in deriving network equivalents to reduce the whole dimension of the system to be simulated. Furthermore, with the more recent widespread of real-time simulation technologies, there is a renew interest in improving the integration of the tools used to analyzed steady-state (load flow and harmonic penetration), electromechanical and electromagnetic transients.

The main idea is to detail only a portion of the network in either a real-time simulation environment of EMT-type of program while the remainder of the network is represented using the conventional tools that rely on positive sequence networks modeling. The main reason why EMT studies are made in small parts of a given Power System is due to the relations between propagation functions and transmission line lengths. It is also important to note that the time range of the events considered are discrepant – less than a few milliseconds

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in EMT and more than a few seconds in angle stability or full steady-state in the case of harmonic penetration or load flow studies.

Regardless of the procedure used to identify the rational approximation, one point remains of paramount importance: the impact of the accuracy of the network representation. If in one hand, the use of positive sequence is widely used in load flow and transient stability analysis, on the other hand, power system transients are usually evaluated in phase-coordinates and time-domain with different levels of approximation with respect to line modeling. Figure 1 presents a scheme that summarizes some possible interaction between power system simulation programs. Therefore, prior to the integration of the simulation tools, some investigations are required. In this paper we propose an approach where an Extra High Voltage AC transmission system is studied with different line models and transposition schemes. We obtain the rational approximations of frequency responses from the sequence networks of one terminal bus, i.e., a step before FDNE realization, and discuss its effects on the results.

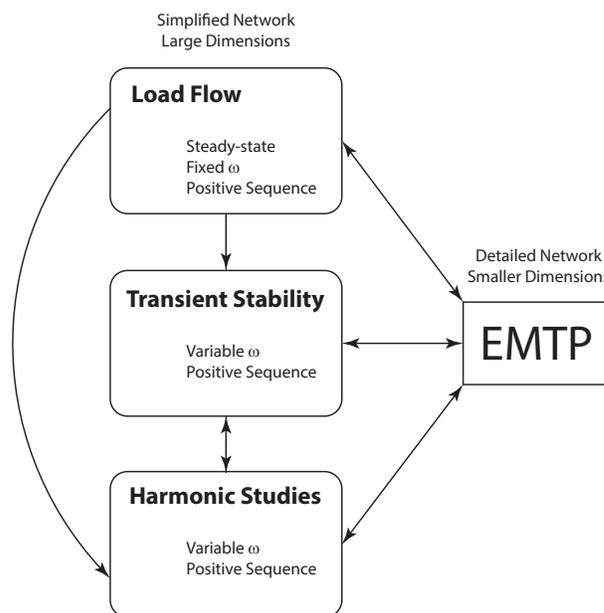


Fig. 1. Simulation characteristics.

The developments of an accurate representation of an equivalent reduced network, date back to 1970. The models then were implemented in a way of representing power system

behavior on Transient Network Analyzers (TNA) or on digital computations. Computational tools were very limited at that time so were some of the mathematical models involved. Consequently, very simple models were incorporated into these types of analysis, specially regarding the representation of network equivalents.

Historically, the first publications aimed to synthesize frequency responses by the identification of resonance points [2]-[5]. In the following, series and/or parallel RLC branches were set in order to match system behavior, only within a specific frequency range. Some simplification assumptions, e.g., symmetry, mainly in transmission lines, were adopted since its effect should not affect significantly the evaluations. Another characteristic from that time was that all network equivalents were single-port. Further developments enhanced fitting techniques with new optimization features and extended the methods to multi-port equivalents [6], [7]. More recently, Gustavsen and Semlyen [8] proposed a robust frequency domain rational fitting technique based on a pole relocation scheme, the so-called Vector Fitting algorithm (VF). This method is a reformulation of Sanathanan-Koerner (SK) [11] iteration using partial fractions basis, which later received improvements [12], [13]. Due its accuracy, its ready-to-use feature and the fact that it is available as a public domain Matlab routine, this technique became widely used. Nevertheless, there are other approaches applied for electromagnetic transient studies. One may approach the problem of rational fitting as an optimization problem using a *Levenberg-Marquardt* algorithm [14]–[16]. The actual poles of a given system may be identified using *Dominant Poles* and *Multiple Dominant Poles Methods* [17]–[19]. After the identification of poles, residues can be easily identified. Another approach that is drawing more attention nowadays is the use of the *Matrix Pencil Method* [20], [21].

II. SYSTEM MODELING

The main objective of this paper is to provide an analysis regarding the importance of frequency dependency inclusion and transposition schemes in actual power systems involving overhead transmission lines (OHL). The system used for the test is shown in Fig. 2. It contains two generation equivalents in buses #1 (500 kV) and #12 (230 kV); four 500 kV and nine 230 kV overhead transmission lines, with lengths from 19 up to 240 km; shunt reactive compensation in buses #2 and #3; and a capacitor bank in bus #4. The loads are connected to voltage levels below 230 kV, except the load on bus #4, which represents a static compensator. The network configuration is based on a part of the actual Extra High Voltage System existing in the North of Brazil. Figures 3(a) and 3(b) show the line profile for the 230 kV and 500 kV circuits.

Typically, for the harmonic penetration analysis of the system in Fig. 2, all OHL would assume ideal transposition and constant parameter line model would be used. However, in fact only the longer circuits (with lengths over 100 km) should be transposed. Furthermore, an actual, instead of ideal, transposition is used. In this work we aim to investigate the impact that this simplification might affect the realization

of a frequency dependent network equivalent (FDNE) in the sequence domain.

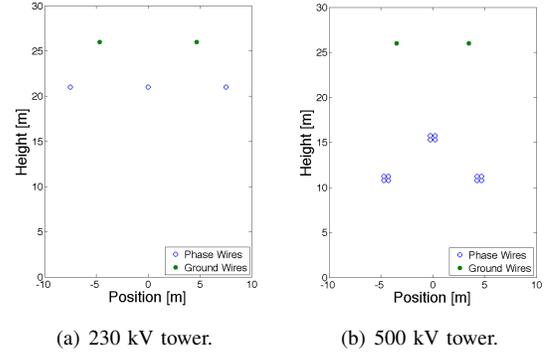


Fig. 3. Profiles for overhead lines.

A. Overhead Transmission Lines

The Bergeron overhead transmission line model is a representation of the traveling wave model, with distributed parameters. It was first introduced on EMTP by Dommel [22]. It also is known as the constant parameter line model. Losses are included by lumped resistances [23] at line ends, see Fig. 4(a). Alternatively, one may use the modal domain frequency dependent line model proposed by Marti [24], also known as JMarti model. It consists on an accurate model for the inclusion of frequency dependency of parameters in electromagnetic transient time-domain simulation as long as the transformation matrix is assumed real and constant. The model is based on a polynomial fitting in the frequency domain of the characteristic impedance $Z_C(\omega)$, Z_{EQ} and the usage of a Thévenin equivalent as depicted in Fig. 4(b), where B_i and F_i are backward and forward waves, and H_c is the wave propagation function.

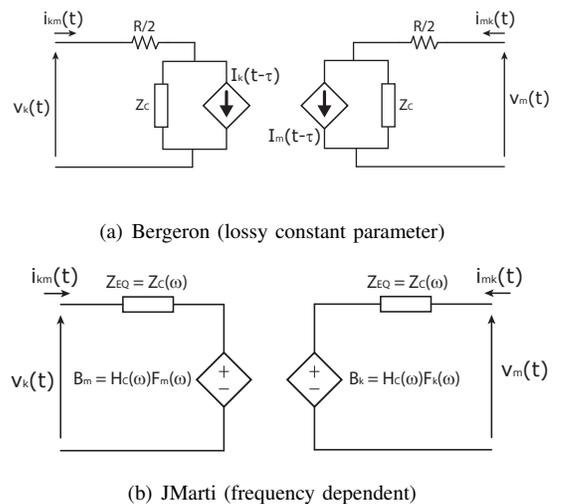


Fig. 4. Basic structure of line models.

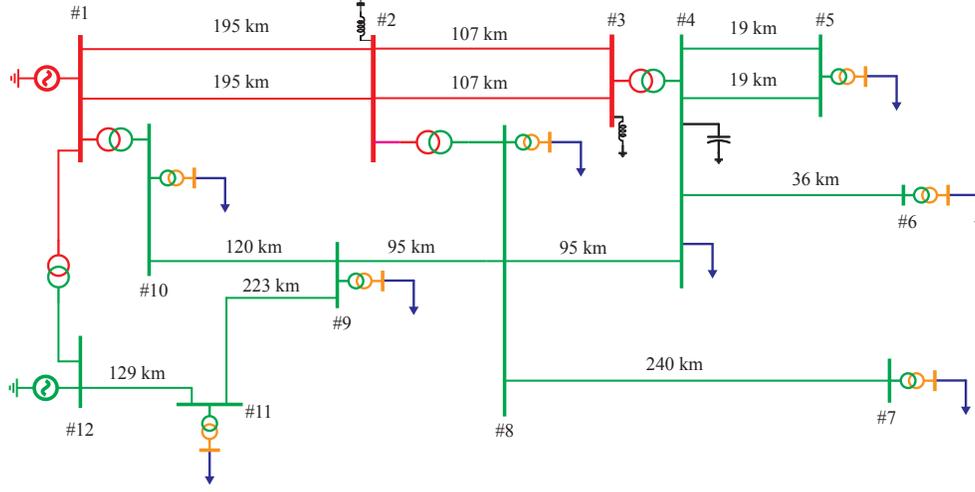


Fig. 2. System under study.

B. Transposition

In an actual transmission line, both impedance and admittance matrices per unit length are symmetrical but not balanced [25]. For instance, the impedance presents the following structure

$$\mathbf{Z} = \begin{bmatrix} Z_{SA} & Z_{mAB} & Z_{mAC} \\ Z_{mAB} & Z_{SB} & Z_{mBC} \\ Z_{mAC} & Z_{mBC} & Z_{SC} \end{bmatrix} \quad (1)$$

where Z_{Si} is the self-impedance of phase i and Z_{mij} the mutual impedances between phases i and j . In the case of an ideally transposed line, i.e., if there are infinitesimal rotation between phase conductors, the impedance can be written as

$$\mathbf{Z}_{approx} = \begin{bmatrix} Z_S & Z_m & Z_m \\ Z_m & Z_S & Z_m \\ Z_m & Z_m & Z_S \end{bmatrix} \quad (2)$$

In actual transmission circuits the structure shown in (2) is approximately obtained by a discrete transposition scheme. The circuit is divided in four parts with the transposition taking place at 1/6, 1/3, 2/3 and 5/6 of the total line length. The discrete transposition scheme creates a small unbalance which might be of importance, specially if harmonic studies are involved.

C. Rational Approximation

The so-called Vector Fitting Algorithm (VF) has become one of the most applied techniques for the FDNE realization. Consider a discrete frequency function $f(s)$, it is possible to write an approximation in terms of a ratio of two polynomial functions [10]:

$$f(s) \approx \frac{a_n s^n + a_{n-1} s^{n-1} + \dots + a_0}{b_m s^m + b_{m-1} s^{m-1} + \dots + b_0} \quad (3)$$

In order to avoid polynomial division, (3) may be rewritten as a sum of partial fractions, i.e., a rational approximation:

$$f(s) \approx \sum_{n=1}^N \frac{r_n}{s - p_n} + d + se \quad (4)$$

where N is the order of approximation, r_n are the residues, p_n the poles, d is the independent term and e is the term proportional to s . VF identifies the poles of $f(s)$ by solving (5) as a linear least squares problem

$$\begin{aligned} \sigma(s)f(s) &= \sum_{n=1}^N \frac{r_n}{s - \tilde{p}_n} + d + se \\ \sigma(s) &= \sum_{n=1}^N \frac{\tilde{r}_n}{s - \tilde{p}_n} + 1 \end{aligned} \quad (5)$$

where \tilde{p}_n are the chosen initial poles. It can be shown [10] that the poles of $f(s)$ are equal to the zeros of $\sigma(s)$ which are calculated as the eigenvalues of the matrix

$$p_n = \text{eig}(\mathbf{A} - \mathbf{bc}^T) \quad (6)$$

where \mathbf{A} is a diagonal matrix containing the initial poles \tilde{p}_n , \mathbf{b} is a row vector of ones and \mathbf{c}^T is a column vector with the residues \tilde{r}_n . The procedure runs always with a known set of poles, since the new poles p_n take the place of the old ones \tilde{p}_n in the following iteration. Once the poles are identified, the residues are calculated by solving (5).

The advantage of VF usage lies with its capability of approximating the given function with no knowledge of the system itself, i.e., only approximating the frequency response, which speeds computation. Although the calculated poles may not be the actual system poles, it is not always possible to know all of them, hence, the approximation may be the only way to describe the system under study. In this paper, we assumed that there are not coupling between sequence networks then only single-phase impedances or admittances are involved and there is no need to enforce passivity as in the case of rational approximation of admittance matrices [27], [28].

III. SIMULATION RESULTS

The rational approximation in terms of Eq. (4) parameters lead to the conclusion that variable e may be interpreted as a capacitance, if $f(s)$ is the admittance of the measured bus [29].

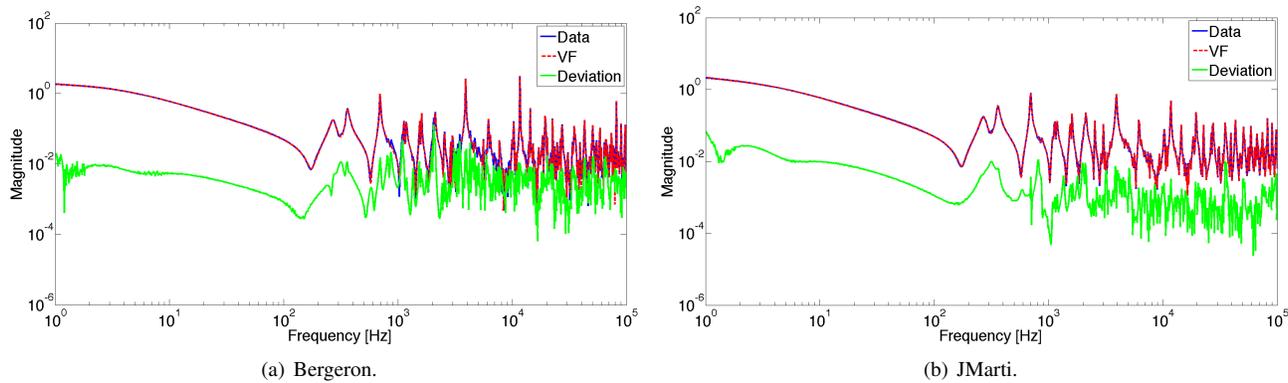


Fig. 5. Y_1 assuming all lines ideally transposed.

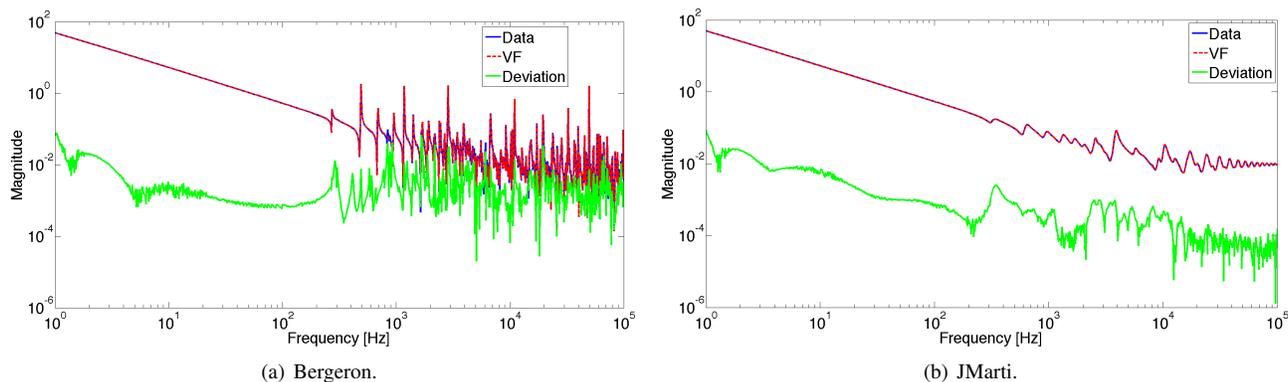


Fig. 6. Y_0 assuming all lines ideally transposed.

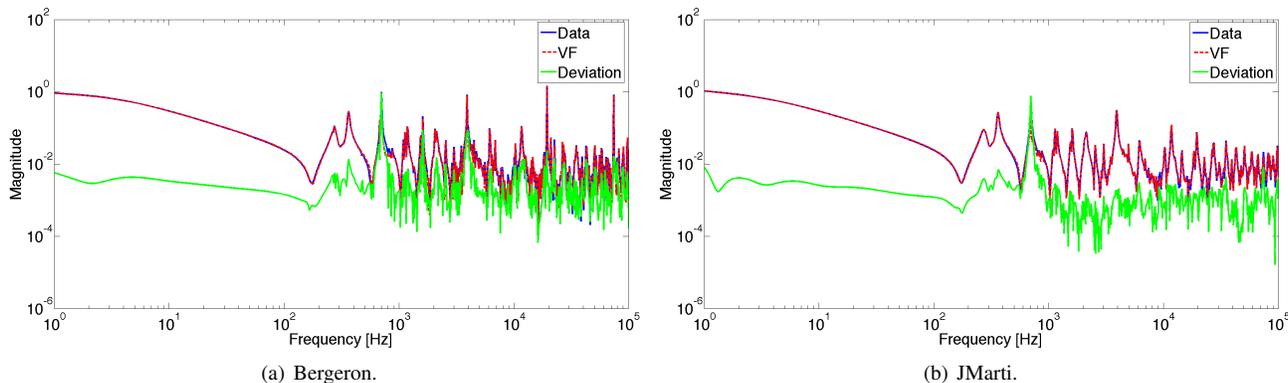


Fig. 7. Y_1 with untransposed (shorter) lines, longer lines ideally transposed.

Therefore, in this paper we will apply this information to consider $e = 0$ and apply VF algorithm to obtain a FDNE from the admittance response of bus #4. It should be highlighted that transmission lines directly connected to bus #4 have less than 100 km. To obtain the frequency response of bus #4, we used the Frequency Scan from ATP. In appendix A we review the basic procedure to obtain the sequence domain impedances.

The frequency responses and VF approximation of positive- and zero-sequence admittance simulated at bus #4 are shown in this Section. To determine the order of approximation, we

have chosen the lowest number of poles that produced an RMS error below 10^{-2} , based on the absolute minimum value of each curve.

Frequency responses obtained with the transmission system modeled by Bergeron have a noticeable oscillatory behavior, with peaks way higher than with JMarti model, for higher frequencies. For positive-sequence and, specially, for zero-sequence responses, the attenuation of admittance magnitude was significant.

Based on several evaluations on impedance and admittance

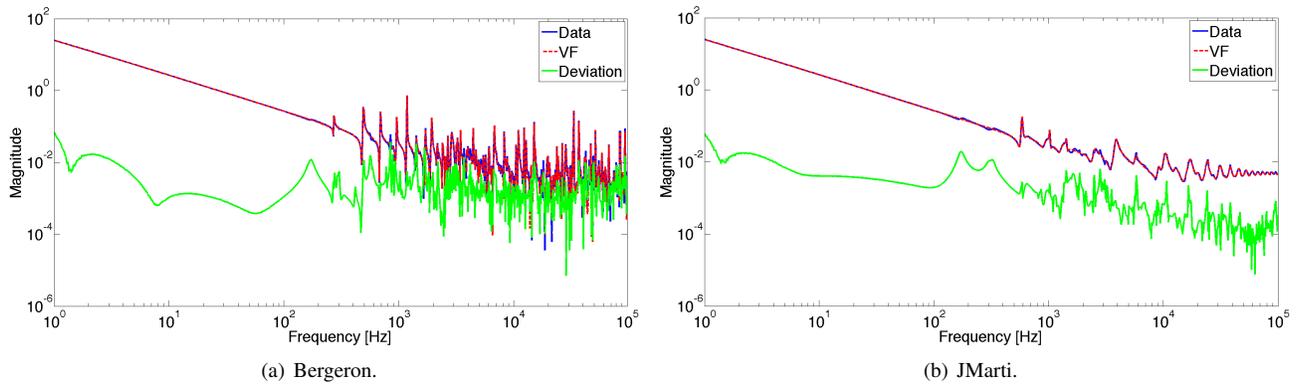


Fig. 8. Y_0 with untransposed (shorter) lines, longer lines ideally transposed.

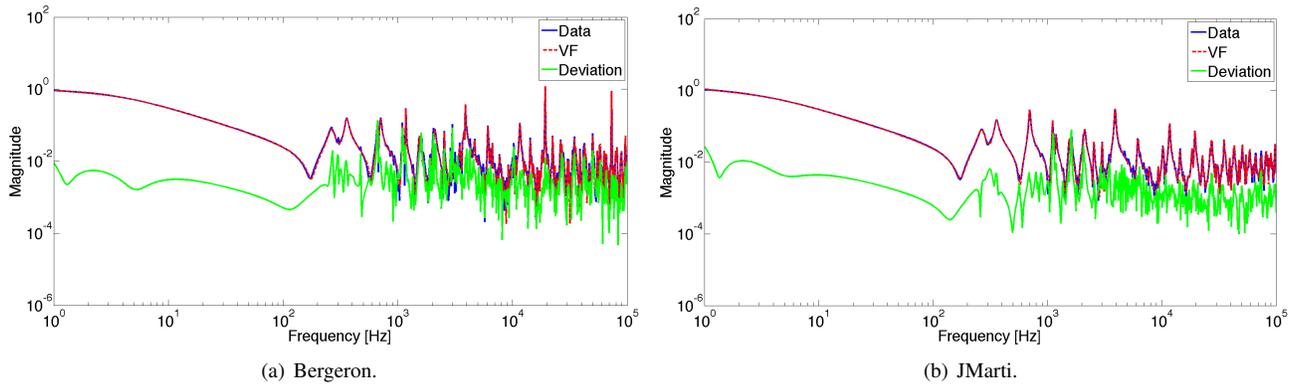


Fig. 9. Y_1 using actual transposition scheme for longer lines, shorter lines untransposed.

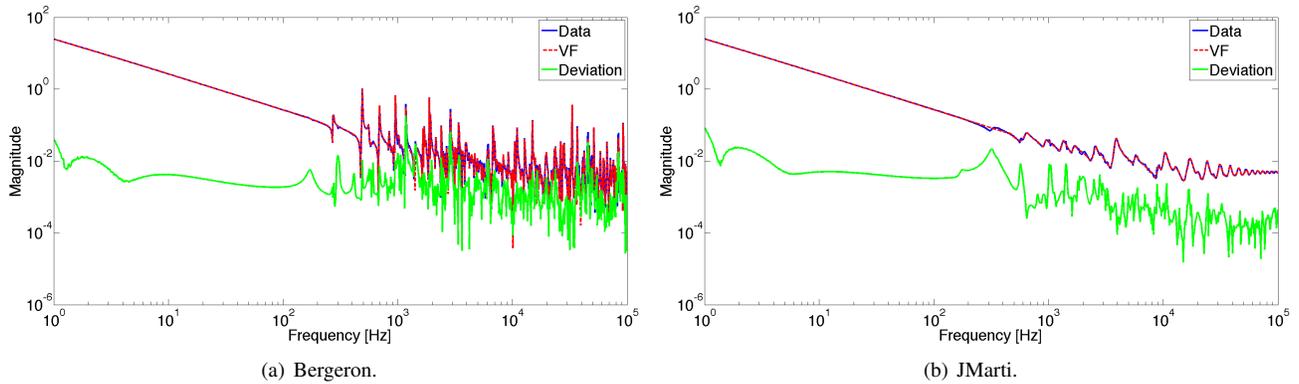


Fig. 10. Y_0 using transposition scheme for longer lines, shorter lines untransposed.

fitting [30], results showed that since admittance frequency responses presented more attenuate tendencies, they become more suitable for rational fitting than the impedance frequency responses.

For the ideally transposed transmission system, Figs. 5(a) and (b) depict the positive-sequence admittance calculated for each line model and Figs. 6(a) and (b), the zero-sequence. For the entire frequency range, it is possible to notice the attenuation of the admittance behavior, considering JMarti model, especially for zero-sequence response.

Considering the untransposed transmission system, Figs. 7(a) and (b) depict the positive-sequence and Figs. 8(a) and (b), the zero-sequence responses. We were unable to achieve the criterion of RMS error for positive-sequence responses, even though were they quite similar to those of Figs. 5(a) and (b). Zero-sequence responses kept its behavior.

Finally, the influence of an actual transposition scheme applied to the transmission system can be evaluated in Figs. 9(a) and (b) for the positive-sequence and in Figs. 10(a) and (b) for zero-sequence. Tab. I summarizes the results of the rational

approximation using VF, including the number of poles and RMS error calculated for each case.

TABLE I
RESULTS OF VF.

Sequence Admittance	OHL Model	Transp. Scheme	Number of Poles	RMS-Err	
Positive	Bergeron	Ideal	208	8.152×10^{-3}	
		Un	424	2.219×10^{-2}	
		Actual	256	8.456×10^{-3}	
	Y ₁	JMarti	Ideal	282	6.659×10^{-3}
			Un	418	2.490×10^{-2}
			Actual	150	6.271×10^{-3}
Zero	Bergeron	Ideal	292	9.840×10^{-3}	
		Un	284	7.866×10^{-3}	
		Actual	350	9.676×10^{-3}	
	Y ₀	JMarti	Ideal	78	8.816×10^{-3}
			Un	60	9.839×10^{-3}
			Actual	52	9.833×10^{-3}

IV. CONCLUSIONS

This work has focused on the rational approximations of network equivalents using sequence domain. It was found that regardless of the inclusion of frequency dependency and transposition scheme, an accurate representation of the system's frequency response was obtained using the Vector Fitting technique. The inclusion of the frequency dependency was of paramount importance for a lower order realization. The usage of conventional line models such as the Bergeron model implied in a significant increase in the number of the poles to achieve a precise fitting function.

The evaluation of the RMS error for the fitted admittance presented a smoother behavior when compared with fitted impedance. The inclusion of the transposition scheme produced a noticeable impact in the behavior of both the equivalent impedance and the admittance in the higher frequencies. This did impact the quality of the rational model. It is important to mention that more accurate representation of the network provided functions that were more easily fitted, i.e., considering a frequency dependent line model with an actual transposition scheme provided equivalents that were more easily fitted using a rational model. This result emphasizes that a system that has a modeling closer to its actual behavior can be approximated with rational functions.

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APPENDIX

A. Sequence Networks

The procedure is based on calculating the Thévenin equivalent impedance from the terminal of interest. By injecting a three-phase current source with magnitude 1.0 A and phase angles shown in Tab. II, we establish a simplification where the only measured quantity is the voltage in phase A. When considering an ideally transposed transmission system, the symmetric current source provokes symmetric voltage measurements, which gives a unique sequence network response. Hence, we may account, e.g., that the positive-sequence impedance is the ratio of the voltage measured over phase A with respect to the current applied on phase A, i.e., 1.0 A, for they are equal do the positive-sequence voltage and current, respectively. Equation (7) show this evaluation, also valid for negative- and zero-sequence networks.

TABLE II
ANGLES OF THE THREE-PHASE SOURCE.

Sequence	θ_A	θ_B	θ_C
Zero	0°	0°	0°
Positive	0°	-120°	$+120^\circ$
Negative	0°	$+120^\circ$	-120°

$$\begin{bmatrix} V_A \\ V_B = \alpha^2 V_A \\ V_C = \alpha V_A \end{bmatrix} \implies \begin{bmatrix} V_0 = 0 \\ V_1 = V_A \\ V_2 = 0 \end{bmatrix}$$

$$\begin{bmatrix} I_A = 1\angle 0^\circ \\ I_B = 1\angle -120^\circ \\ I_C = 1\angle +120^\circ \end{bmatrix} \implies \begin{bmatrix} I_0 = 0 \\ I_1 = I_A \\ I_2 = 0 \end{bmatrix} \quad (7)$$

$$\begin{bmatrix} 0 \\ V_A \\ 0 \end{bmatrix} = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} 0 \\ 1\angle 0^\circ \\ 0 \end{bmatrix}$$

$$\implies Z_1 = \frac{V_A}{1.0\angle 0^\circ} \therefore Z_1 = V_A \quad \square$$

However, in the cases where the transmission system is not ideally transposed, there exists mutual components between positive- and zero-sequence networks. For the results we show in this paper, we bypass the mutual relations of sequences and only consider $Y_1 = I_1/V_1$ and $Y_0 = I_0/V_0$ as sequence networks, due to the little influence these mutual components impose over the responses.

As it consists on a single curve, the sequence admittance frequency responses can be found simply by calculating $Y_{seq}(s_k) = 1/Z_{seq}(s_k)$ for each point of frequency s_k . For the power system we consider in this paper, we use admittance instead of impedance. In this paper, the generators are modeled as three-phase voltage sources behind impedances, thus negative-sequence behavior is identical to the one presented by positive-sequence.