Damping of Oscillations Related to Lumped-Parameter Transmission Line Modeling

Andreas I. Chrysochos, Georgios P. Tsolaridis, Theofilos A. Papadopoulos, and Grigoris K. Papagiannis

***Abstract*--Although PI-equivalent transmission line modeling is simple in use and offers various advantages, its main drawback is the emergence of artificial numerical oscillations in transient responses caused by the parameter lumpiness. In this paper, two different low-pass filters are proposed to damp out the artificial numerical oscillations. The presented post-processing methodology is first demonstrated on a single-phase overhead line and its performance is compared to the corresponding of a traditional method with the inclusion of damping resistances. Results show the successful alleviation of the related oscillations, whereas the proposed formulation is further generalized to multiconductor overhead transmission lines without considerable loss of accuracy.**

***Keywords*: artificial oscillations, electromagnetic transients, filters, lumped-parameter modeling, PI-equivalents.**

I. Introduction

THE development of time-domain (TD) simulation models for the accurate calculation of electromagnetic transients in transmission lines has been considerably increased again in recent years. Despite the progress that has been made in the optimization of fully frequency-dependent TD models and their extensive use, there are still open issues regarding their accuracy and numerical stability [1]-[3]. In such cases, as well as when the need for numerical efficiency outweighs the importance of high accuracy, more simplified approaches can be considered, such as frequency-dependent models with constant modal transformation matrices [4] or constant-parameter models [5].

Among these models, the lumped-parameter PI-equivalent approach can be selected, which is simple in use and offers various advantages over other methods [6]. Specifically, the calculation routine of the PI-equivalent model is explicitly expressed in phase-domain, avoiding the transformation to modal-domain and the resulting approximations to propagation characteristics and modal transformation matrices [7]. Therefore, the PI-equivalent approach yields accurate enough results in both overhead lines and underground cables, examining not only steady-state conditions but also transient phenomena with the per-unit-length (pul) parameters calculated at the dominant resonance frequency of the examined configuration [8]-[10].

However, in order to take into account the distributed nature of the transmission line, a large number of PI-equivalents must be considered in cascade connection. This often leads to artificial numerical oscillations, which are caused by the application of several lumped elements associated to the commonly-used trapezoidal integration. As a consequence, the PI-equivalent model results in transient responses and voltage profiles along transmission lines which are heavily distorted by the spurious oscillations, and thus often inaccurate [11].

Several solutions have been proposed in the literature to alleviate the problem of the lumped-element cascade connection. The most simplified approach is the introduction of damping resistances parallel to the series R-L branches of the PI-equivalents [12]. This method is quite effective but also introduces steady-state and transient response errors, due to the modification of the examined configuration resulting by the additional resistive elements. Other more sophisticated solutions interfere with the state-space expression of the PI-equivalent model, resulting in the damping of the numerical oscillations but with the simultaneous emergence of numerical instabilities for the whole calculation procedure [11], [13].

In this paper, a post-processing method is proposed to tackle the numerical oscillations caused by the lumped-parameter modeling of transmission lines. Specifically, the transient responses are subject to user-defined low-pass filters (LPFs) in order to remove the resonance frequencies of the artificial oscillations. The ripple and the order of the filters are selected in such a way that simulation accuracy is not significantly degraded, while the cutoff frequency is calculated by an empirical rule, taking into account the length and the resonance frequency of the examined transmission line.

The performance of both infinite and finite impulse response (IIR and FIR) filters are examined in overhead line configurations. Transient responses and voltage profiles along transmission lines are compared to the corresponding obtained by the damping method with variable parallel resistances, as well as by other TD and frequency-domain (FD) models. Results verify the performance of the proposed method in

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damping out oscillations related to lumped-parameter modeling without considerable loss of accuracy.

II. Problem Formulation

A. *Artificial Numerical Oscillations*

Despite the assumption of frequency-independent pul parameters, the PI-equivalent approach is often preferred over other models in power engineering studies, due to its modeling simplicity and highly accurate phase-domain calculation routine. However, in order to accurately represent the distributed nature of power lines, the PI-equivalent method requires the cascade connection of a large number of lumped segments, resulting in spurious oscillations of the corresponding TD transient responses.

This is demonstrated in the energization test case of the 100-km single-phase overhead line of Fig. 1, where a 1 pu, 50 Hz sinusoidal voltage source is applied to the sending end (*S*). In Fig. 2, the switching transient response at the receiving end (*R*) is shown, assuming the homogeneous earth formulation of [14] with earth resistivity equal to 100 Ω·m. The open-ended response using 100 cascaded PI-equivalents is compared to the corresponding obtained from the frequency-independent TD travelling wave model of Bergeron [5] and the FD model of Numerical Laplace Transform (NLT) [15], the latter of which is considered as reference. Results follow the same trend, although significant oscillations are evident on the transient response of the PI method.

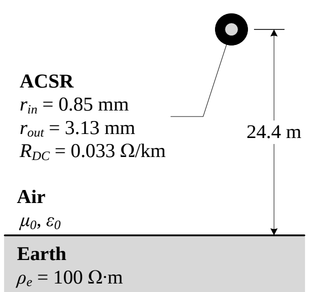

Fig. 1. Cross-section of the single-phase overhead line.

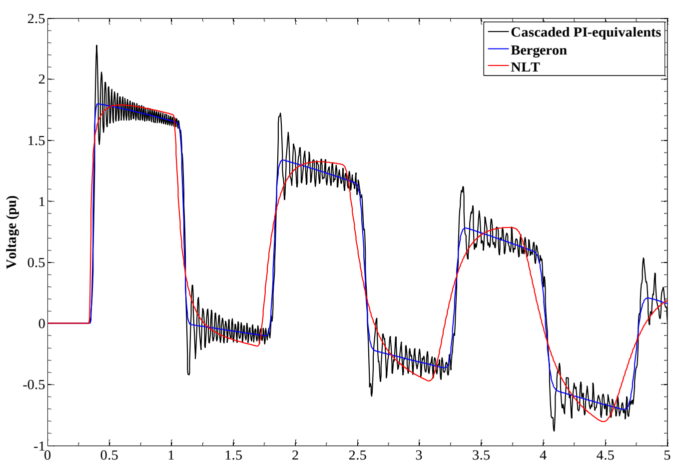

Fig. 2. Transient response at the end *R*.

The same problem is also observed in Fig. 3, illustrating the peak voltage profile along the line for the energization case. The higher voltage levels using the cascaded PI-equivalents are mainly caused by the artificial numerical oscillations, which can generally lead to overestimated results in insulation coordination and surge protection studies [16].

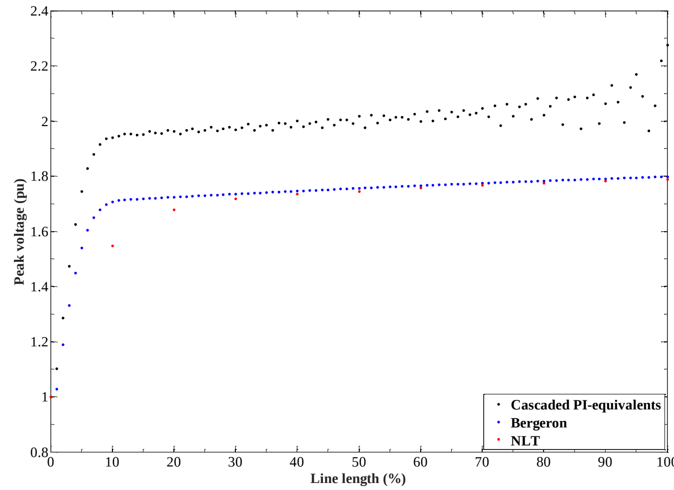

Fig. 3. Peak voltage profile along line length.

B. *Sensitivity Analysis*

In Figs. 4 and 5 a sensitivity analysis of the emerging numerical oscillations is carried out, by varying the number of cascaded PI-equivalents and the simulation time step, respectively. Results show that the spatial resolution significantly affects the transient response and the resulting oscillations, whereas the change of time integral has almost no effect.

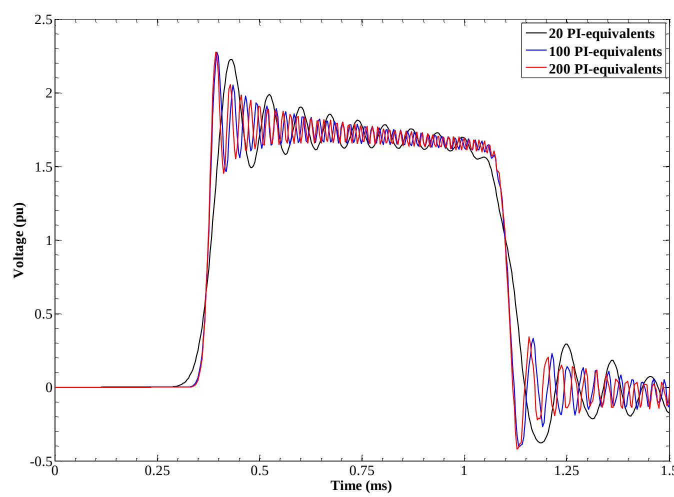

Fig. 4. Numerical oscillations for different number of PI-equivalents.

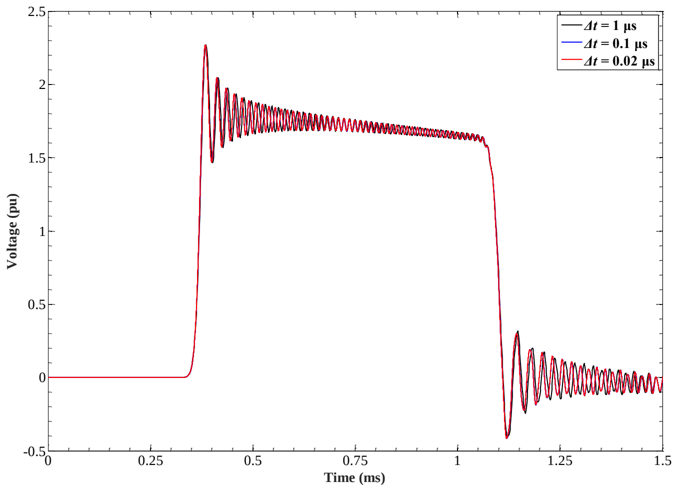

Fig. 5. Numerical oscillations for different time interval.

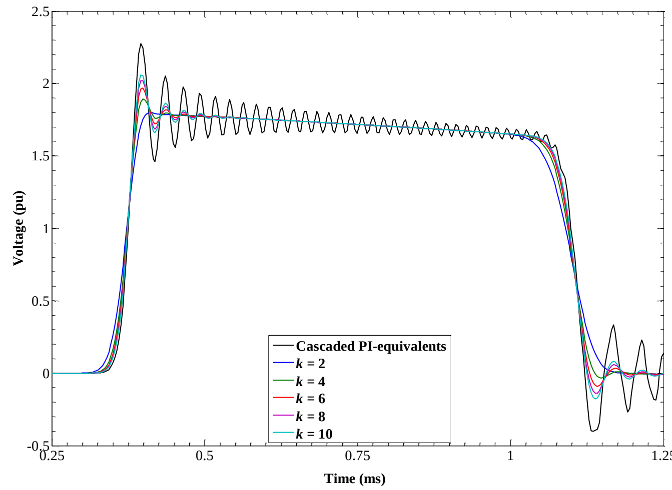

Fig. 7. Numerical oscillations for different values of damping resistance.

III. Traditional Method

To date, the introduction of damping resistances parallel to the series R-L branches of the PI-equivalents is the simplest method to alleviate the spurious oscillations caused by the lumpiness. The insertion of the damping resistance R_d on the i-th PI-equivalent is presented in Fig. 6. The R_d value is given by (1), where L is the inductance of the i-th PI equivalent, Δt is the simulation time interval, and k is an adjustable factor typically varying between 2 and 10 [12].

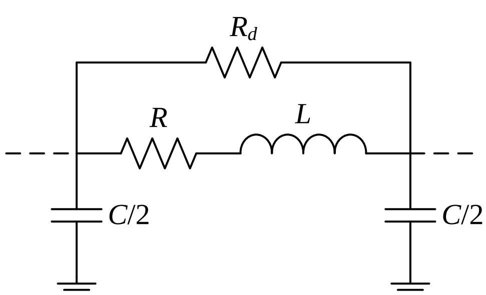

Fig. 6. Damping resistance R_d connected to the i-th PI-equivalent.

$$R_d = k \cdot \frac{2L}{\Delta t} \tag{1}$$

In Fig. 7 the open-ended response of Fig. 2 is recalculated with the inclusion of the damping resistances, assuming different values of parameter k. Results show that this method is quite efficient for small values of k, since the spurious oscillations are almost fully alleviated. However, such values of k may also introduce steady-state and transient response errors, due to the possible modification of the examined configuration resulting by the additional resistive elements. As a result, a tradeoff between the simulation accuracy and the damping of oscillations should be considered, with a value of $k = 6$ generally providing acceptable results in most cases.

IV. Proposed Methodology

A. Flowchart

The proposed methodology for damping out the numerical oscillations associated with the lumped-parameter PI-equivalent model is summarized in the following steps:

Step 1: Calculation of the first order resonance frequency (f_{res}), dominating the corresponding transient response.

Step 2: Derivation of the upper frequency limit (f_{lim}), above which the spectrum content of the transient response can be neglected without introducing significant calculation error.

Step 3: Selection of the spatial and time intervals (Δx, Δt), and simulation of the transient response using the cascaded PI-equivalent model.

Step 4: Application of the user-defined LPFs to damp out the spurious oscillations and extraction of the distortionless TD transient responses.

B. Calculation of First Order Resonance Frequency

The derivation of f_{res} can be performed using the iterative method of [17] in a fast and highly accurate way. The resonance frequency of any order and propagation mode can be calculated for both overhead and underground configurations, taking also into account possible transposition or cross-bonding schemes as well as any homogeneous or stratified earth formulation.

However, in order to simplify the proposed procedure, f_{res} is approximately calculated by the simplified expression of (2), where the asymptotic velocity υ_{asympt} is defined by (3) and depends on the examined configuration as follows [16]:

- For overhead configurations, (3) is assumed to be equal to the air propagation constant by setting $\varepsilon = \varepsilon_0$ and $\mu = \mu_0$, where ε_0, μ_0 are the permittivity and permeability of air, respectively.

- For overhead and underground cable configurations, (3) is assumed equal to the propagation velocity through the inner insulation medium by setting $\varepsilon = \varepsilon_{ins}$ and $\mu = \mu_{ins}$, where ε_{ins}, μ_{ins} are the permittivity and permeability of the inner insulation layer, respectively.

$$f_{res} = \begin{cases} \upsilon_{asympt} / 4\ell, & open-circuit\ case \\ \upsilon_{asympt} / 2\ell, & short-circuit\ case \end{cases} \quad (2)$$

$$\upsilon_{asympt} = \frac{1}{\sqrt{\varepsilon \cdot \mu}} \quad (3)$$

C. Calculation of Upper Frequency Limit

Although transient responses generally contain a wide range of frequencies, an upper frequency limit f_{lim} can be empirically determined, above which the spectrum content does not significantly contribute to the resulting TD response. This limit is equal to multiple times the first order resonance frequency and can be practically defined as the frequency at which the normalized spectrum magnitude is equal to 3 % of the corresponding magnitude at f_{res}.

In Table I the upper frequency limit is presented for the overhead line of Fig. 1, assuming both the open- and short-circuit cases as well as line lengths from 10 km to 200 km.

TABLE I
UPPER FREQUENCY LIMIT FOR BOTH LINE TERMINATIONS

Line length (km)	Open-circuit (kHz)	Short-circuit (kHz)
10	51.38	117.60
20	25.50	58.52
50	12.98	23.37
80	8.06	16.30
100	7.86	14.43
150	6.18	10.60
200	5.32	8.58

Results show that f_{lim} can be directly related to f_{res} of (2), since it decreases as line length increases for both termination cases. Equation (4) accurately fits the results of Table I for both cases, expressing the relation of the upper frequency limit with the first order resonance frequency and the line length for overhead lines.

$$f_{lim}[kHz] = (0.046 \cdot \ell[km] + 6) \cdot f_{res}[kHz] \quad (4)$$

D. Selection of Spatial and Time Intervals

A minimum number N of cascaded PI-equivalents must be used, in order to adequately simulate the distributed nature of the examined power transmission line. The minimum number should be sufficient enough to accurately simulate the examined transmission line for frequencies up to f_{lim}. This is readily performed by comparing the FD behavior of the cascaded PI-equivalents to the corresponding of the exact longitudinal line model for each considered length [18].

From this analysis the minimum required number of cascaded PI-equivalents is presented in Fig. 8 for different line lengths, assuming both the open- and short-circuit cases of Fig. 1. Results present a linear pattern with line length, which can be best approximated by (5). In order to further simplify the analysis, a fixed number of 100 and 200 PI-equivalents can be considered for the open- and short-circuit case, respectively, yielding results on the safe side for all line lengths.

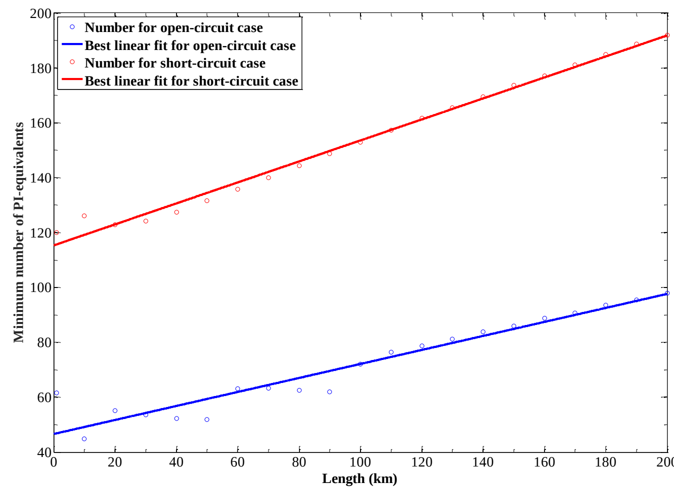

Fig. 8. Minimum required number of cascaded PI-equivalents and best linear fit for both termination cases.

$$N = \begin{cases} 0.26 \cdot \ell[km] + 47, & open-circuit\ case \\ 0.38 \cdot \ell[km] + 115, & short-circuit\ case \end{cases} \quad (5)$$

After N has been defined, the spatial interval Δx can be directly calculated by (6), while the corresponding time resolution Δt can be defined by the one-dimensional Courant-Friedrichs-Lewy condition shown in (7) [19], assuming the asymptotic velocity of (3).

$$\Delta x = \frac{\ell}{N} \quad (6)$$

$$\Delta t \leq \frac{\Delta x}{\upsilon_{asympt}} \quad (7)$$

E. Application of LPFs

Since TD simulations have been carried out using the cascaded PI-equivalent model and the parameters of (5)-(7), the transient responses are subject to user-defined LPFs in order to damp out the resulting artificial oscillations. In this paper two different filters are examined, namely the IIR and FIR filters, both characterized by zero phase shift, and thus do not introduce any time delay in the filtered transient responses [20].

More specifically, the IIR filter is based on a Chebyshev type I filter with its typical transfer function magnitude shown in Fig. 9. The passband ripple is set to 0.05 dB, the cutoff frequency is set to f_{lim} of (4), while the stopband frequency and its attenuation are set to $4 \cdot f_{lim}$ and 20 dB, respectively.

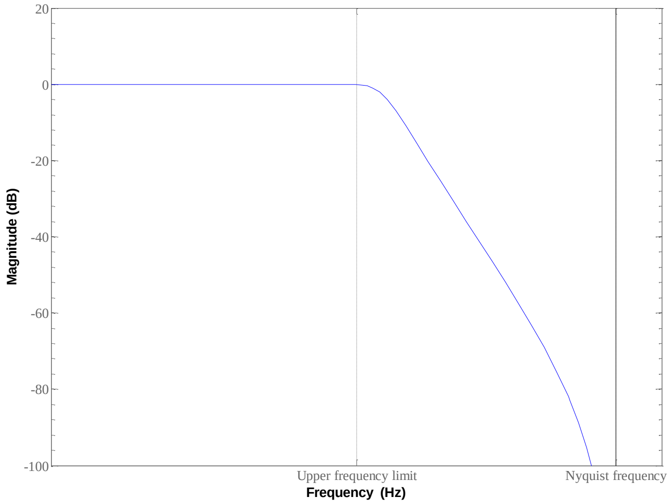

Fig. 9. Bode magnitude plot of IIR Chebyshev type I filter.

The FIR filter is based on the equiripple design scheme with its typical transfer function illustrated in Fig. 10. Once again, the cutoff frequency is set to the upper frequency limit of (4), while the frequency limits of the passband and stopband are set to $0.4 \cdot f_{lim}$ and $1.6 \cdot f_{lim}$, respectively. Furthermore, the passband ripple and the stopband attenuation are set to 0.05 dB and 10 dB, respectively.

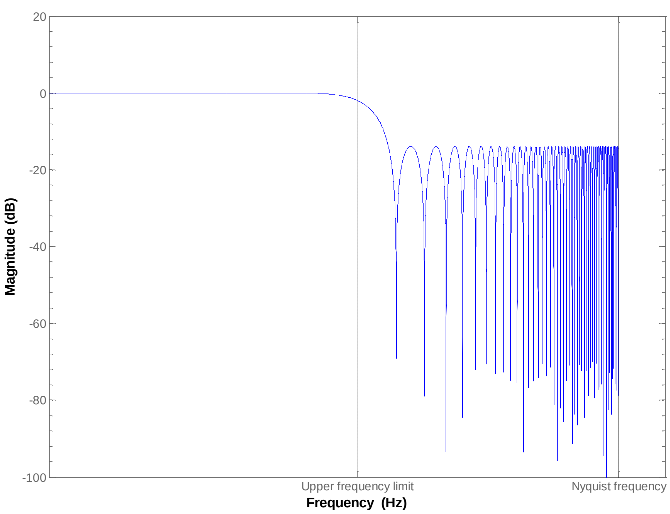

Fig. 10. Bode magnitude plot of FIR Equiripple filter.

V. Results

A. FD Voltage Ratio

The performance of the proposed methodology using both types of LPF is first demonstrated in the FD calculation of the open-ended voltage ratio for the line configuration of Fig. 1. Results of Fig. 11 show the difference between the exact longitudinal line model and the lumped-parameter cascaded PI-equivalent approach, which is observed above the upper frequency limit of 7.95 kHz. The filtered FD responses significantly attenuate the spectrum context above f_{lim}, without affecting peaks in the passband range. However, this is not the case for the traditional method, where the inclusion of damping resistances slightly modify the spectrum context below the upper frequency limit and thus leads to calculation errors.

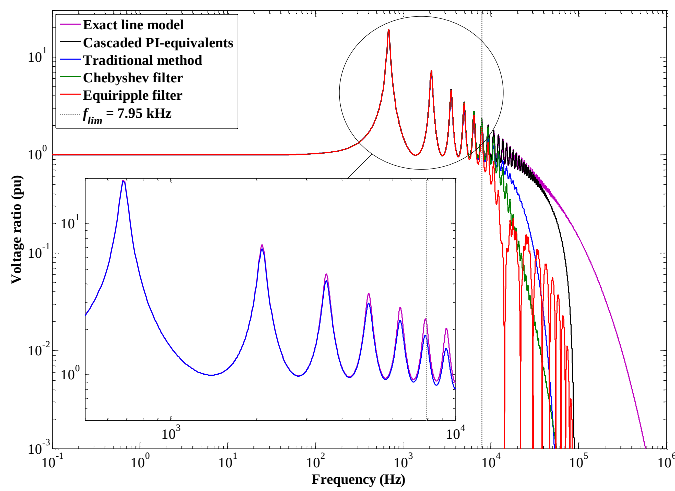

Fig. 11. FD voltage ratio for different line realizations.

B. TD Transient Response

Additionally, both filters are applied to the fast front transient response of Fig. 2. Results, presented in Fig. 12, are compared to the corresponding of the Bergeron and NLT models. The spurious oscillations caused by the cascaded PI-equivalents are significantly damped by both filters as well as by the traditional method.

The performance of the proposed methodology is even more pronounced in the peak voltage profile of Fig. 13, where a significant difference is observed compared to the result of the cascaded PI-equivalents, without applying any damping method. Although all results are on the safe side compared to the corresponding of Bergeron and NLT, the FIR filter seems to yield more accurate results than the IIR filter and the traditional method.

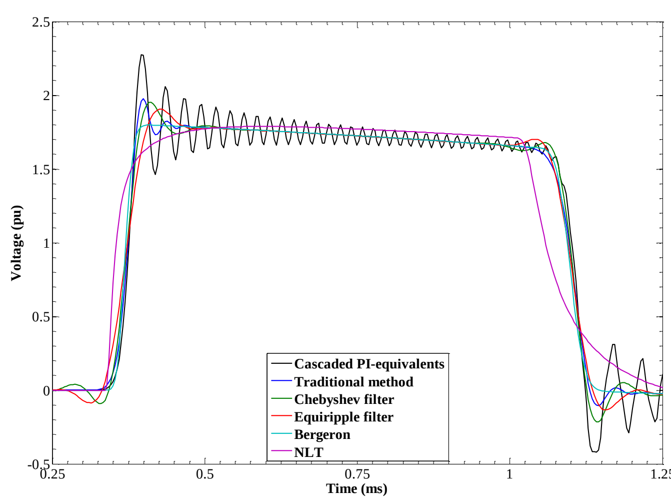

Fig. 12. TD transient response for different line realizations.

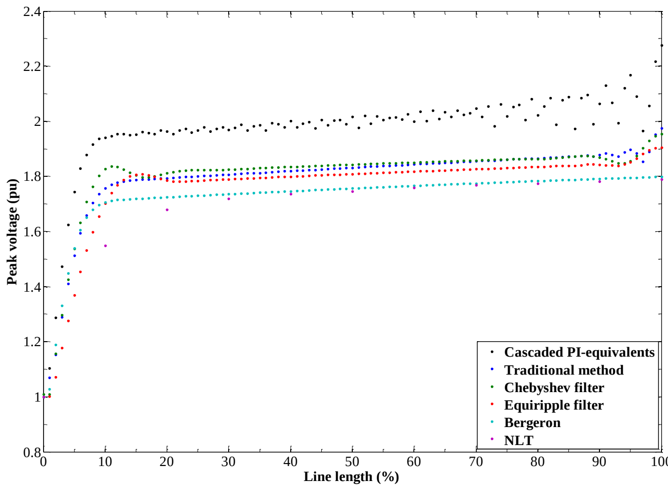

Fig. 13. Peak voltage profile along line length for different line realizations.

VI. Generalization of Proposed Methodology

The proposed methodology can be readily applied to multiconductor overhead lines by adopting the same equations. Three conductors similar to the one of Fig. 1 are placed in a typical flat configuration, forming a three-phase overhead line. Assuming the same geometrical and electrical data, a three-phase switching transient phenomenon is simulated using the cascaded PI-equivalents approach, the proposed methodology, the traditional method with damping resistances as well as both Bergeron and NLT models. Results, shown in Fig. 14, present a similar behavior to the corresponding of Fig. 12 for the single-phase overhead line. It is evident that both post-processing filters present a more efficient attenuation of the spurious oscillations compared to the traditional method.

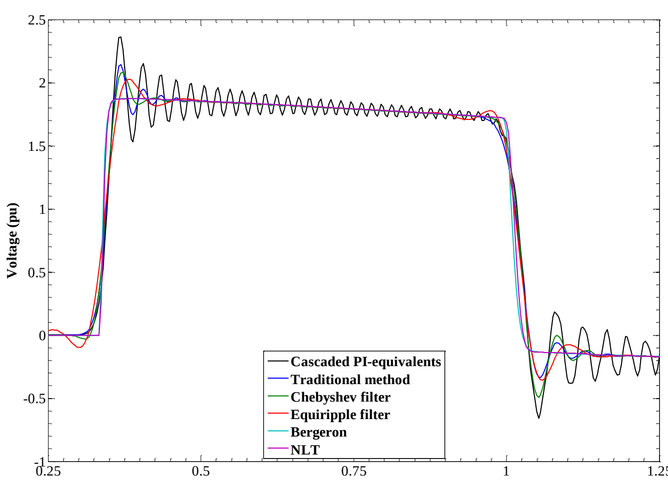

Fig. 14 TD transient response of phase *a* for the three-phase overhead line with different line realizations.

VII. Conclusions

In this paper a post-processing method has been presented to damp out the numerical oscillations caused by the lumped-parameter modeling of overhead transmission lines. Specifically, the performance of two different LPFs has been examined and compared to a traditional method for alleviating oscillations as well as to TD and FD models. The main remarks indicate that:

- Both LPFs are quite efficient, significantly damping out the spurious oscillations. This is indicated in both TD transient responses and peak voltage profiles.
- The performance of FIR filter seems to be better compared to the corresponding of IIR filter and of the traditional method. Using the FIR filter, a smaller overshoot is observed on TD transient responses, while the peak voltage profile levels are also closer to the corresponding of TD and FD models.
- The formulation and application of the proposed methodology is very straightforward, since it is a post-processing method without affecting the time-domain simulation routine.
- The proposed methodology can be readily generalized for multiconductor transmission lines, without altering the proposed empirical formulas of f_{lim} and N.

Finally, as a topic for future research, the proposed methodology could be also extended on underground configurations, by modifying accordingly the equations calculating the upper frequency limit and the required minimum number of cascaded PI-equivalents.

VIII. References

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