Rational Approximation to Full-Wave Modeling of Underground Cables

A. P.C. Magalhães, A. C. S. Lima and M. T. Correia de Barros

***Abstract*-- The full-wave model of underground cables is presented. The solutions of this model in frequency domain, applied in underground insulated, cable is evaluated. To allow the inclusion of this formulation in EMT programs a rational vector fitting of the propagation constant, characteristic admittance Yc and the propagation function H in frequency domain is presented. The simulations in time domain is carried out considering different soil models. The results are compared with those obtained using a conventional quasi-TEM (Transverse Electro-Magnetic) approximation. This comparison shows that the quasi-TEM approximation can provide a good approximation for all the considered soil models.**

***Keywords*: Electromagnetic transients, Full-wave formulation, Underground Cables**

I. Introduction

THE first attempts to describe the influence of external media in propagation characteristic of transmission line and underground cable was proposed, respectively, by Carson [1][2] and Pollaczek [3] in the 1920s. These works are based on the quasi-TEM mode of propagation, i.e., the current density vector does not depend on the yet unknown propagation constant. Furthermore, the expressions of ground return impedance assume the ground as a good conductor, i.e. neglecting displacement currents and assuming ground permeability the same as in the air. These limitations motivated the development of a full-wave model to provide a better understanding of the influence of a dispersive ground on the parameters per unit length (p.u.l.), i.e. series impedance and shunt admittance of the line. Kikuchi [4] was the first to obtain a solution to the propagation of a full-wave model using the magnetic vector and the electric scalar potentials. Wait [5] using both electric and magnetic Hertz vectors developed a solution for a full-wave model. Wedepohl and Efthymiadis [6] proposed to use Transverse Electric (TE) and Transverse Magnetic (TM) propagation decomposition with magnetic and electric vector potentials to achieve a solution to the full-wave model. In all these works, only single-phase overhead conductors are considered, an extension of the full-wave model to an underground cable was proposed by Wait in [7].

One of the key aspects in the development of a full-wave model is the solution of the so-called modal equation, i.e., solve the integration equation that derives the propagation function and then evaluate the line parameters using infinite integrals. Furthermore, full-wave models have been used to investigate the behavior of the electromagnetic field surrounding an underground cable [8]. That is the main reason why approximated formulas were proposed in the past.

For the analysis of electromagnetic transients where the line model must be adapted to two Norton equivalents representing the sending and receiving end of the underground cable. This modeling of a distributed and frequency dependent line or cable is known as the Method of Characteristics (MoC) or the travelling wave method. In the time-domain modeling of a full-frequency dependent line or cable one must resort to the fitting of the characteristic admittance $\mathbf{Y}_c$ and propagation function $\mathbf{H}$ in the frequency domain [15]-[12]. If we determine the propagation constant and then obtain the p.u.l parameters the procedure of a full-wave model could be straight forward adapted. Therefore, the main goal of this paper is to proposed and analyze a full-wave model in order to assess if significant differences are found, considering the scenarios were the quasi-TEM approach is typically used.

This work is organized as follows. In section II we present the full-wave model of an underground cable. In section III we present the frequency responses of impedance and admittance p.u.l. and the frequency responses of $\mathbf{Y}_c$ and $\mathbf{H}$ and in section IV we present their rational approximation. Time domains simulation of a 300 m cable are shown section V and the main conclusions of this paper are presented in section VI.

II. Full-Wave Underground Cable

The full-wave model presented here was originally formulated by Wait in [5] and extended in [13][14] to include the effect of conductor losses. For this formulation, we consider two semi-infinite media respectively named "1" and "2" as depicted in Fig 1. In medium "1" there is an infinitely long conductor where the injected current has the form $I = I_{\max}\exp(-\gamma z + j\omega t)$ where γ is the unknown propagation constant to be determined. The electric and magnetic field in both media can be expressed by two Hertz Vectors, one of the electric type, $\mathbf{\Pi}_E$ and other of the magnetic type, $\mathbf{\Pi}_M$. So the fields in media i are given from

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A. P. C. Magalhães and A. C. S. Lima are with Federal University of Rio de Janeiro, COPPE/UFRJ, P.O. Box 68504, 21094-090, Rio de Janeiro, RJ, Brazil (e-mails: antoniopaulo@coep.ufrj.br, acsl@dee.ufrj.br)

M.T. Correia de Barros is with IST, Universidade de Lisboa, Av. Rovisco Pais, 1049-001 LISBOA, Portugal, teresa.correiadebarros@tecnico.ulisboa.pt

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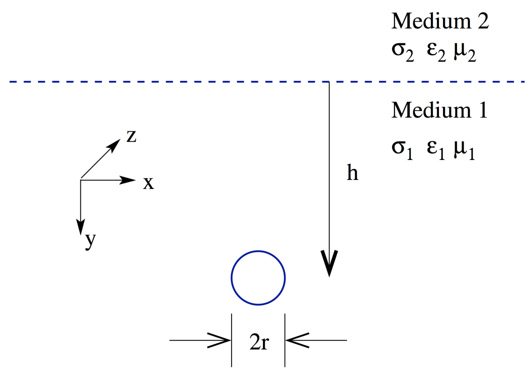


Fig. 1. Conductor configuration

$$E_i = \nabla\times\nabla\times\Pi_{E_i} - j\omega\mu_i\nabla\times\Pi_{M_i}$$
$$H_i = (\sigma_i + j\omega\varepsilon_i)\nabla\times\Pi_{E_i} + \nabla\times\nabla\times\Pi_{M_i} \quad (1)$$

Both Hertz vectors have a single component in the direction of propagation and its relation with vector potential, **A**, and electric scalar potential φ can be obtained by

$$A = \mu(\sigma + j\omega\varepsilon)\Pi_E + \mu\nabla\times\Pi_M \ , \ \phi = -\nabla.\Pi_E \quad (2)$$

Assuming the continuity of tangential components of $\mathbf{E}_i$ and $\mathbf{H}_i$ is possible to obtain an expression for the electric field in both media. The modal equation is found considering the electric field null at the surface of the conductor. This procedure is explained next for the case of a buried insulated conductor.

A. Full-Wave Modal Equation

For a solid buried insulated cable with inner radius r_0 and an insulation layer between r_0 and the outer radius r with a permittivity ε_d, buried at depth *h*, the modal equation that defines the behavior of the unknown propagation constant γ is written as

$$M = \frac{2\pi}{j\omega\mu_0} z_a + \left(1 - \frac{\gamma^2}{\gamma_1^2}\right)\Lambda + \left(S_1 - \frac{\gamma^2}{\gamma_1^2} S_2\right) = 0 \quad (3)$$

$$z_a = z_{\text{int}} + z_d - \gamma^2 y_d^{-1} \quad (4)$$

where z_{int} is the conductor internal impedance given by

$$z_{\text{int}} = \frac{1}{2\pi r_0}\left(\frac{j\omega\mu_c}{\sigma_c}\right)^{1/2} \frac{I_0(\gamma_c r_0)}{I_1(\gamma_c r_0)} \quad (5)$$

where μ_c and σ_c are the magnetic permeability and conductivity of the conductor, respectively, and I_0 and I_1 are modified Bessel functions of the first kind, zero and first order, respectively. $\gamma_c \approx \sqrt{j\omega\mu_c\sigma_c}$ is the conductor propagation constant, z_d and y_d are, respectively, the impedance and admittance per unit length of the cable insulation layer is given by

$$z_d = \frac{j\omega\mu_0}{2\pi}\ln\frac{r}{r_0} \qquad y_d = 2\pi j\omega\varepsilon_d\left(\ln\frac{r}{r_0}\right)^{-1} \quad (6)$$

S_1 and S_2 are the Sommerfeld integrals given by

$$S_1 = \int_{-\infty}^{\infty} \frac{\exp(-2hu_1)}{u_1 + u_2}\exp(-jr\lambda)d\lambda \ ,$$
$$S_2 = \int_{-\infty}^{\infty} \frac{\exp(-2hu_1)}{n^2 u_1 + u_2}\exp(-jr\lambda)d\lambda \quad (7)$$

with, $u_1 = \sqrt{\lambda^2 + \gamma_1^2 - \gamma^2}$, $u_2 = \sqrt{\lambda^2 + \gamma_2^2 - \gamma^2}$, and $\gamma_2 = \gamma_0 = j\omega\sqrt{\mu_0\varepsilon_0}$ is the propagation constant of air. $\Lambda = K_0(r\eta_1) - K_0(d\eta_1)$ where K_0 is the modified Bessel function of the second kind, zero order with $\eta_1 = \sqrt{\gamma_1^2 - \gamma^2}$ and $d = \sqrt{4h^2 + r^2}$.

B. Impedance and Admittance

To define the propagation characteristics we must solve the wave equation

$$\frac{d^2U}{dz^2} = ZYU \ , \ \frac{d^2I}{dz^2} = YZI \quad (8)$$

where *I* is the underground cable current and *U* is the wire voltage to ground given by

$$U = -\int_0^{h-r} E_y dy = \phi(0, h-r) - \phi(0,0) + j\omega\int_0^{h-r} A_y(0,\xi)d\xi \quad (9)$$

and φ is the electric scalar potential and A_y is the *y*-component of the magnetic vector potential.

For an insulated buried cable, the impedance per unit length is given by

$$Z = z_{\text{int}} + z_d + z_{ext} \quad (10)$$

The admittance per unit length is defined as

$$Y = \left(y_d^{-1} + Y_{ext}^{-1}\right)^{-1} \quad (11)$$

In the evaluation of both *Z* and *Y*, only the terms z_{ext} and Y_{ext} are dependent on the propagation constant γ, which may be obtained from the solution of the modal equation in expression (3). Thus, z_{ext} and Y_{ext} are the ground return impedance and admittance based on full wave, respectively,

$$z_{ext} = \frac{j\omega\mu_0}{2\pi}\left[\Lambda + S_1 - \left(\frac{\gamma}{\gamma_1}\right)^2 (T + S_2)\right] \quad (12)$$

$$Y_{ext} = 2\pi(\sigma_1 + j\omega\varepsilon_1)[\Lambda - T]^{-1} \quad (13)$$

being

$$T = \int_{-\infty}^{\infty} \frac{u_2}{u_1}\frac{[\exp(-hu_1) - \exp(-2hu_1]}{n^2 u_1 + u_2}\exp(-jr\lambda)d\lambda \quad (14)$$

C. Characteristic Admittance and Propagation Function

In the full-wave formulation, as well as *Z* and *Y*, both $\mathbf{Y}_c$ and **H** are dependent of unknown propagation constant. For a single-phase cable both quantities are scalars. The relation between the voltage and current waves is given by the characteristic admittance

$$\mathbf{Y}_c = Z^{-1}\gamma \quad (15)$$

and the delay and distortion of the these waves propagating between the line ends of length l is given by the propagation function

$$\mathbf{H} = e^{-\gamma l} \tag{16}$$

III. Frequency Domain Responses

A. Propagation Constant

Consider a conductor buried at $h = 1\text{ m}$ in a dispersive ground with $\sigma_g = 0.01$ S/m and $\varepsilon_{rg} = 10$. The ground propagation constant is $\gamma_g = \sqrt{j\omega\mu_g\left(\sigma_g + j\omega\varepsilon_{rg}\varepsilon_0\right)}$ with μ_g and ε_0 being the vacuum magnetic permeability and electric permittivity, respectively. We consider that the insulation layer is lossless with a relative permittivity $\varepsilon_{rd} = 3$. The real part of γ_g (attenuation constant) is shown in Fig. 2 together with the results for the full-wave model considering a a frequency range of 1 kHz to 10 MHz. For the same frequency range, Fig. 3 depicts the imaginary part of ground propagation constant, γ_g, (phase constant), the air propagation constant and the results for the imaginary part of the propagation constant obtained using the full-wave model.

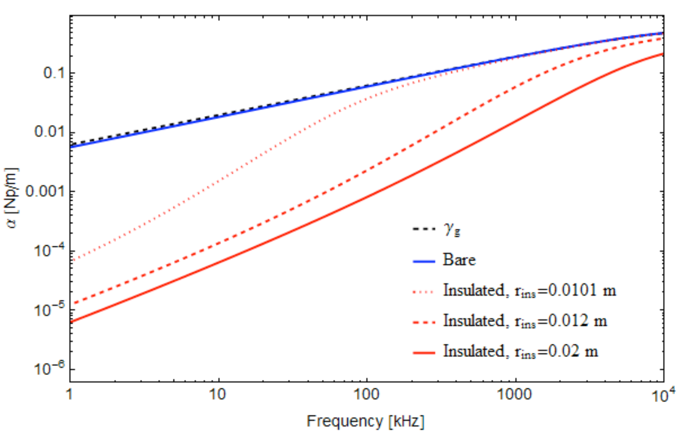


Fig. 2. Attenuation constant

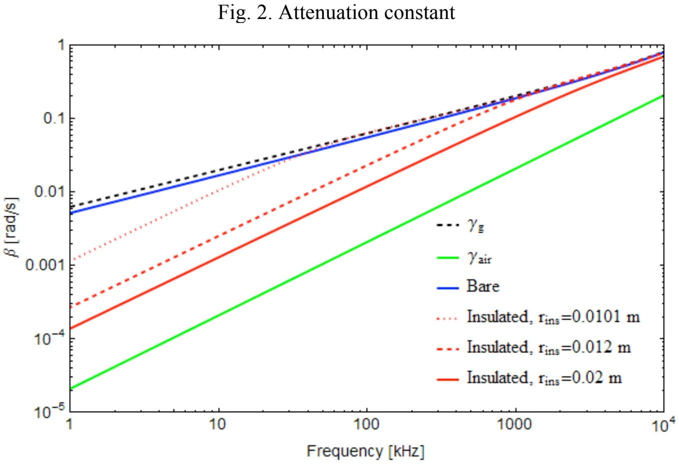


Fig. 3. Phase constant

For the bare cable, it was observed that the propagation constant is almost the same as the ground propagation constant. For the insulated cable, regardless of the thickness of the insulation layer, the same behavior at the higher frequency was found in all cases, they all tend to ground propagation constant for frequencies higher than 1 MHz. Although not shown here, the same behavior was found for bare and insulated cables buried at different depts.

B. Impedance and Admittance

For the evaluation of the pul parameters, we have considered different soils models. The first soil model is a frequency independent soil model considering the ground conductivity and permittivity, i.e. considering ground displacement currents. The second soil model is a frequency dependent model proposed by Portela in [18] and is characterized by the propagation constant in (17)

$$\gamma_g = \sqrt{j\omega\mu\left(\sigma_0 + \Delta_1\left(\frac{f}{10^6}\right)^{\alpha_i}\left(\cot\left[\frac{\pi}{2}\alpha_i\right] + j\right)\right)} \tag{17}$$

where $\sigma_0 = 0.01$ S/m, $\Delta_1 = 11.71$ and $\alpha_i = 0.706$. The last soil model considered is the one proposed by Visacro and Alípio in [19] where propagation constant is given below

$$\gamma_g = \sqrt{j\omega\mu\left(\sigma(f) + j\omega\varepsilon_r(f)\varepsilon_0\right)} \tag{18}$$

where for a frequency f and with $\sigma_0 = 0.01$ S/m we have

$$\sigma(f) = \sigma_0\left(\frac{f}{100}\right)^{0.072}$$

$$\varepsilon_r(f) = 2.34\cdot 10^6\left(\frac{1}{\sigma_0}\right)^{-0.535} f^{-0.597}$$

A comparison between impedance admittance responses for these soil models is presented in Figs. 4 and 5. In the aforementioned figures we have only presented the module of the impedance and admittance per unit of length. In these figures, the label C indicates the frequency independent soil model considering both conductivity and permittivity, the soil model based on (17) has the label P and the one based on (18) has the label V. Although not shown here in detail, there are small differences between the phases of the pul parameters for the frequency dependent soil models that can cause differences in both Yc and **H** as it will be shown next.

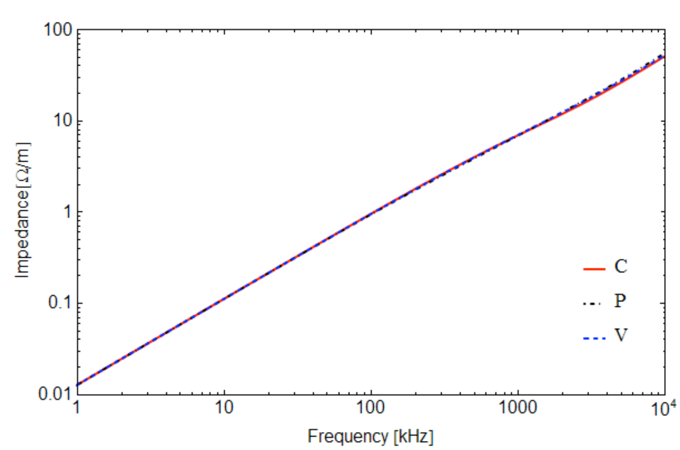


Fig. 4. Comparison of impedance magnitude p.u.l

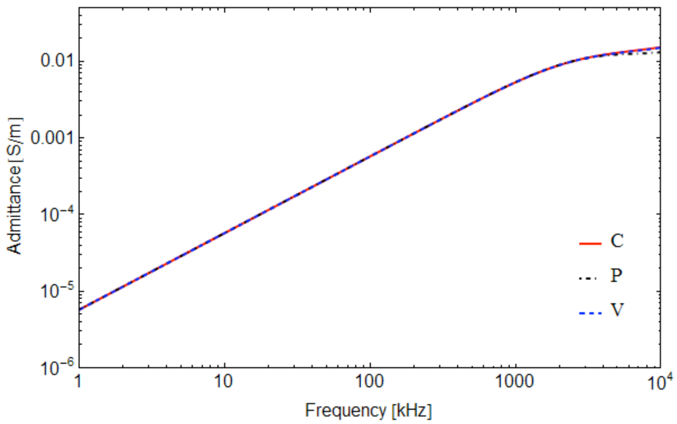


Fig. 5. Comparison of admittance magnitude p.u.l.

C. Characteristic Admittance and Propagation Function

For the same frequency range Figs. 6 and 7 show the responses of characteristic admittance, $\mathbf{Y}_c$, and propagation function, $\mathbf{H}$.

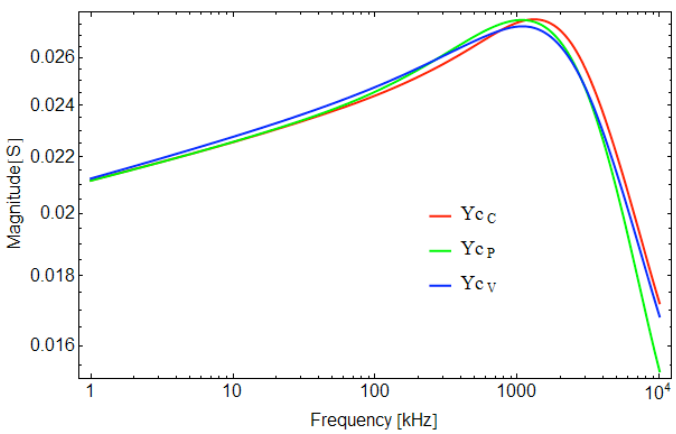


Fig. 6. Characteristic admittance

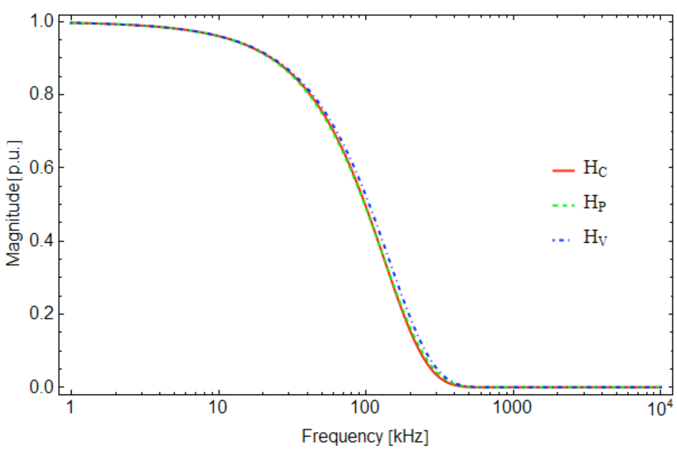


Fig. 7. Propagation Function

IV. Rational Approximations in Frequency Domain

For the rational approximation we used the Vector Fitting routine [15][16][17]. Typically, it can provide an accurate rational approximation of a given frequency response using real and complex poles that come in conjugate pairs. The approximation has the following form:

$$f(s) = \sum_{1}^{N} \frac{c_n}{s - a_n} + se + d \tag{19}$$

where a_n are the poles and c_n are the residues while the terms e and d are real and constant and N is the total number of poles.

For the full-wave time domain simulations the quantities of interest that need to be synthesized are the characteristic admittance $\mathbf{Y}_c$ and the propagation function $\mathbf{H}$. Nevertheless these quantities are implicit functions of the full-wave propagation constant. In this work we propose the following: fit we obtain the full-wave propagation function using a few frequency samples, say 100 frequencies. We subject this result to a rational approximation using VF. Now using the fitted propagation function in a larger sample of frequencies we then evaluate the infinite integrals (7) and (14) and obtain the pul parameters which in turn are used to evaluate both $\mathbf{Y}_c$ and $\mathbf{H}$. These are then subject to a rational approximation.

A. Synthesis of Propagation Constant

The propagation constant response in frequency domain was obtained by solution of the full-wave modal equation that satisfies the zero equal identity. This result was subjected to vector fitting routine for all ground models. The Table I shows the poles number and the RMS error found in synthesis of propagation constant for each ground model.

TABLE I
Number of Poles and RMS error of adjust of Propagation Constant

Soil Model	Nº of Poles	RMS Error
Conventional “C”	36	$1.5\ 10^{-3}$
Portela “P”	36	$9.41\ 10^{-4}$
Visacro “V”	36	$6.86\ 10^{-4}$

The Fig. 8 depicts the fitting results for the propagation constant considering the three soil models. Fitted quantities have an index AD in this figure. We separate the deviation results from the fitted results to better visualize the deviation associated with the synthesis of each soil models and we adopt this strategy in all figures of this work. The differences between the original and adjusted functions are shown in Fig. 9. The deviation was calculated by $\Delta\gamma = \left|\gamma - \gamma_{fit}\right|$. Although there are some noticeable differences between the fitted and the original parameters, as it will be shown here, these mismatches do not affect significantly the evaluation of the $\mathbf{Y}_c$ and $\mathbf{H}$.

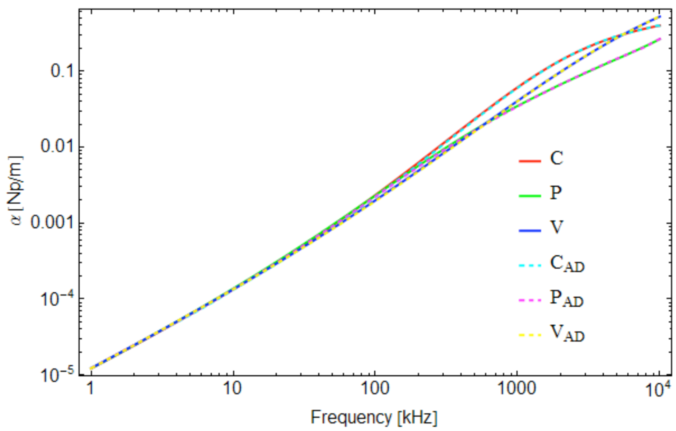
B. Synthesis of Characteristic Admittance

The synthesizing process of characteristic admittance was the same for the propagation constant. The number of poles and the related RMS error is shown in Table II.

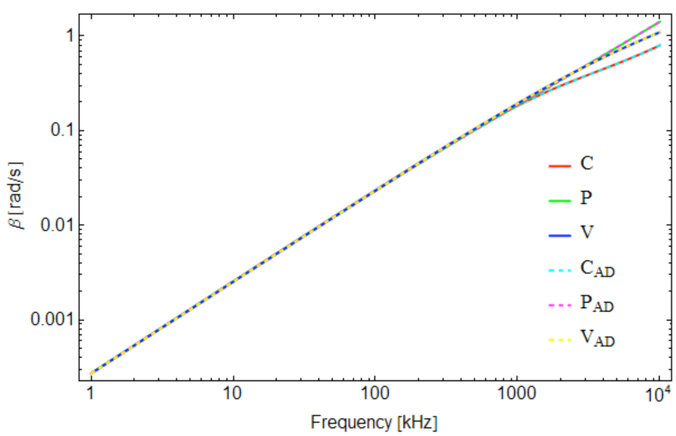
The original results of characteristic admittance and the results of adjust (index AD) are shown in Fig. 10 and the differences between them are sown in Fig. 11.

TABLE II
NUMBER OF POLES AND RMS ERROR OF ADJUST OF CHARACTERISTIC ADMITTANCE

Soil Model	Nº of Poles	RMS Error
Conventional "C"	12	$1.957x10^{-4}$
Portela "P"	12	$1.009x10^{-4}$
Visacro "V"	12	$1.608x10^{-4}$



a. Attenuation constant



b. Phase constant

Fig. 8. Fitted Propagation constant

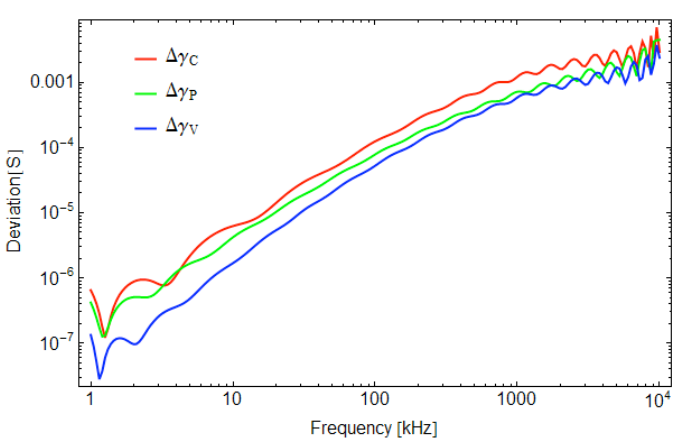


Fig. 9. Deviation of fitted propagation constant

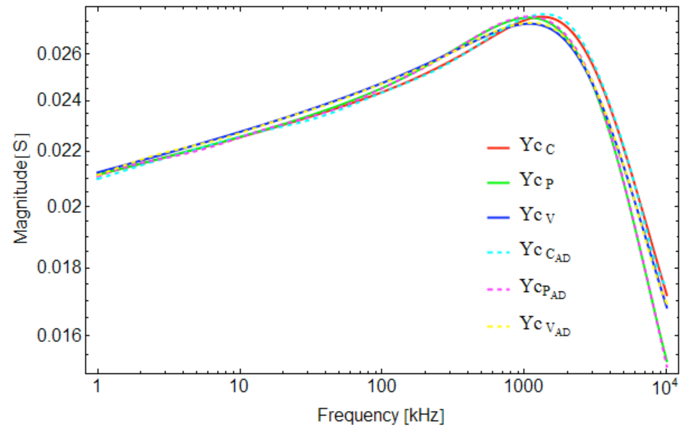


Fig. 10: Fitting result for the characteristic admittance.

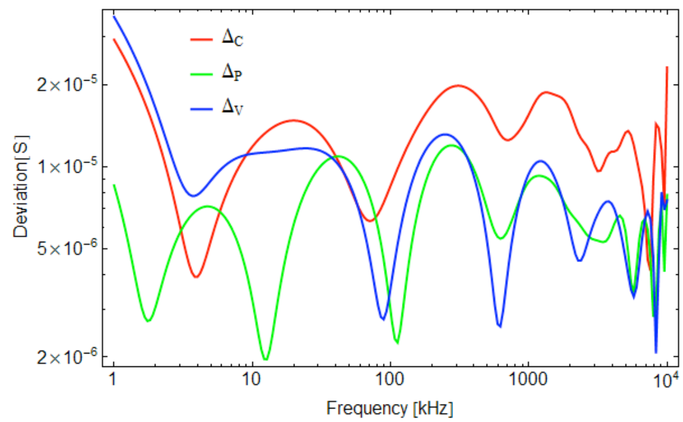


Fig. 11. Deviation of fitted characteristic admittance

C. Synthesis of Propagation Function

The synthesis of propagation function, **H**, was different of the synthesis of propagation constant and characteristic admittance. In this case is necessary to determine and extract the time delay before submitting the propagation function to the vector fitting routine, so H can be approximated by

$$\mathbf{H} \approx H_{fit} = \sum_{1}^{N} \frac{c_n}{s - a_n} e^{-s\tau} \tag{20}$$

where τ is the time delay.

For the time delay identification we use the Brent`s method described in [12]. This method consist in specification of an time delay interval $\tau_{\min} \le \tau \le \tau_{\max}$ where the propagation function is subject to the vector fitting and the time delay is the value of τ that provides the lowest RMS error. The minimal time delay was defined as the lossless time delay $\tau_{\min} = {}^{l}\!/_{v_c}$ being l the cable length and v_c the velocity of the fastest mode. For the maximum value we have

$$\tau_{\max} = \frac{\omega(\Omega)}{\mathrm{Im}[\gamma(\Omega)]} \tag{21}$$

where Ω is the highest frequency considered. With the calculated value of time delay the propagation function was subject to the vector fitting routine as

$$h = \mathbf{H} \cdot e^{s\tau} \tag{22}$$

The results of the number of poles, RMS error and the time delay found are shown in Table III for each ground model.

TABLE III
NUMBER OF POLES AND RMS ERROR OF ADJUST OF PROPAGATION FUNCTION

Soil Model	Nº of Poles	RMS Error	Time Delay [s]
Conventional "C"	12	$7.172x10^{-4}$	$6.493x10^{-6}$
Portela "P"	12	$5.642x10^{-4}$	$6.442x10^{-6}$
Visacro "V"	12	$9.946x10^{-4}$	$7.693x10^{-6}$

The originals results of **H** and the results of synthesis (index AD) are depicted in Fig. 12 and the respective deviations are shown in Fig. 13.

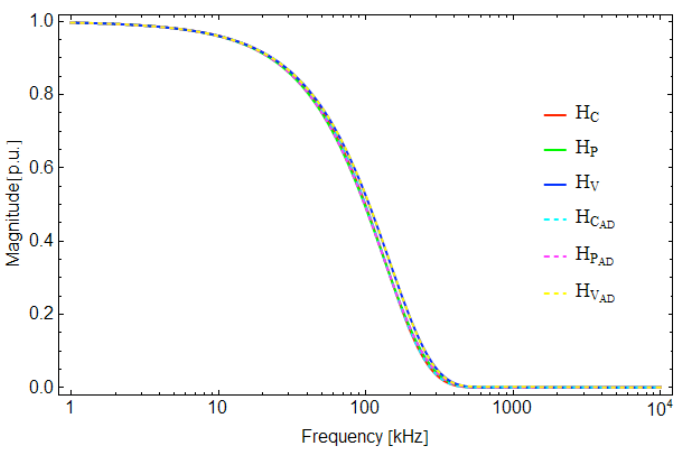


Fig. 12. Rational fitting of propagation function **H**

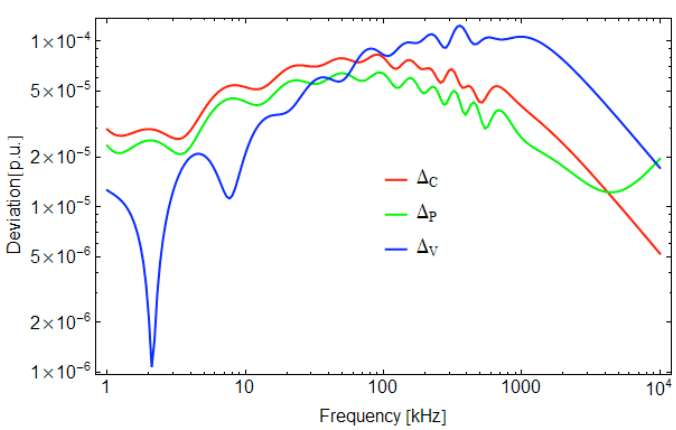


Fig. 13. Error in the rational approximation of **H**

V. TIME DOMAIN SIMULATIONS

For the time domain simulations we consider an insulated buried cable according to the Fig. 14 where $r_c = 0.01$ m is the conductor radius and $r_d = 0.012$ m is the insulation layer outer radius. The different ground models are considered. Furthermore, we also consider the quasi-TEM approximation of the external impedance and admittance. The formulation of this approximation is presented in Appendix A.

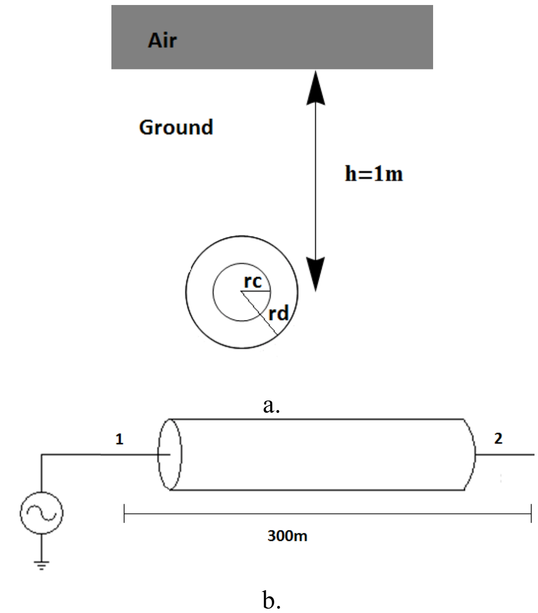


Fig. 14: a. Underground Cable System and b. Circuit Configuration.

A. Voltage Step Response

The results of voltage step response are shows in Figures (15) and (16)

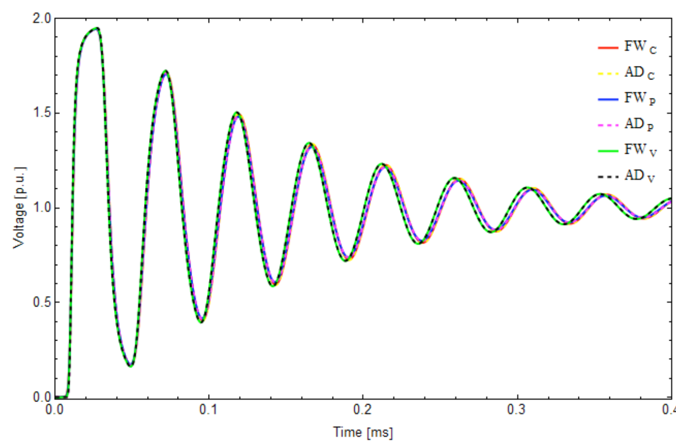


Fig. 15. Voltage step responses: FW- full-wave model and AD- rational approximation of full-wave model

These results show that except the intrinsic differences of each ground model, there are no differences between the full-wave model with and without synthesis. Furthermore the differences between the results of the full-wave model and the results of a conventional quasi-TEM approximation are very small and for a practical cases do not result in any significant loss of information for the system in study.

B. Heidler Voltage impulse Responses

For the time-domain simulations we consider that an impulse voltage is injected at the conductor in terminal "1" (see Fig. 14), while the terminal "2" is open. The impulse voltage using the Heidler function as shown in expression

(23), where $\eta = \left(\frac{t}{\tau_1}\right)^n e^{-\frac{\tau_1}{\tau_2}\left(\frac{n\tau_2}{\tau_1}\right)^{1/n}}$, rising time τ_1=1.8 μs, decreasing time τ_2=95 μs and n=2.

$$V_H = \frac{V_{\max}}{\eta}\left(1+\left(\frac{t}{\tau_1}\right)^n\right)e^{\frac{-t}{\tau_2}} \tag{23}$$

The time domain results are shown in the following figures.

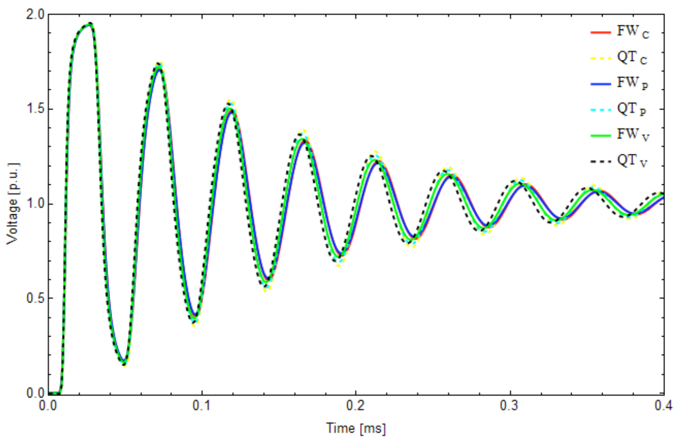


Fig. 16. Voltage step responses: FW - full-wave model and QT- quasi-TEM approximation

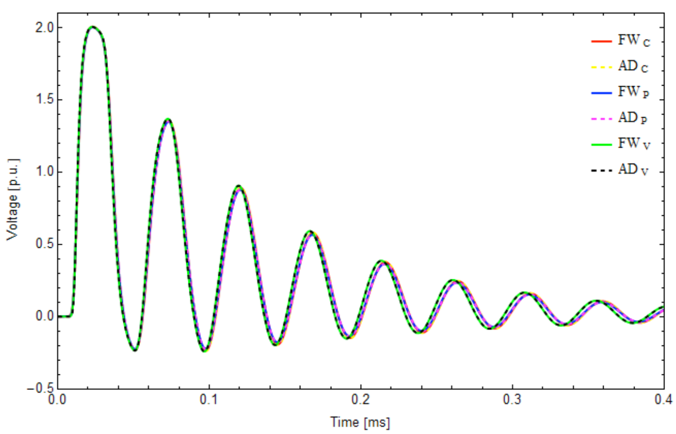


Fig. 17. Impulse responses: FW- full-wave model and AD – rational approximation of full-wave model

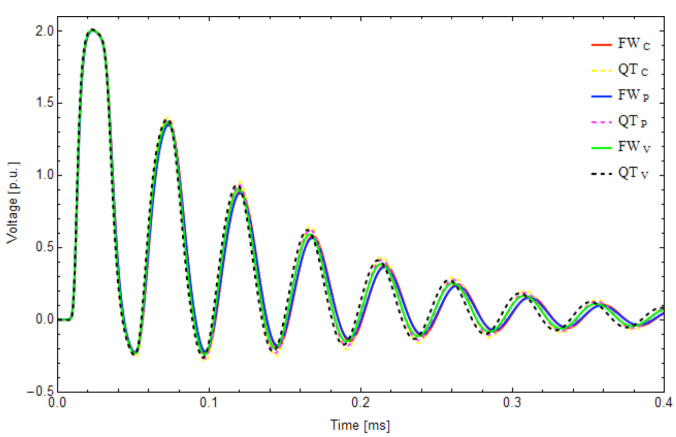


Fig. 18. Impulse responses. FW - full-wave model and QT - quasi-TEM approximation

VI. Conclusions

The main contribution in this work is evaluating the accuracy of the quasi-TEM approximation in modelling an underground cable for EMT simulations, by comparison with the full-wave model results.

It is shown that, in fact, we need not to resort to a full-wave model, which is very hard to solve, since the quasi-TEM approximation is much easier to solve in addition to having the extension to the tree-phase systems. For the required accuracy, the quasi-TEM approximation is able to represent completely the influence of external media in propagation characteristic of underground cables.

The full-wave model for an insulated buried cable was presented. The parameters per unit length (impedance and admittance) were derived directly from the full-wave modal equation. These parameters were evaluated, in frequency domain, for different soil models. The results were important to determine the behavior of the characteristic admittance and the propagation function that defines the behavior of the cable.

For the time domain simulations, the rational approximation in frequency domain is necessary, and the propagation constant, characteristic admittance and propagation function were synthesized using the vector fitting routine. The results of synthesis in time domain were compared with the full-wave without synthesis and with a conventional quasi-TEM approximation.

Results show that the quasi-TEM approximation can provide a good approximation for all the considered soil models.

VII. Appendix A – Quasi-TEM Approximation.

For the Quasi-TEM Approximation we assume that for the frequency range of interested (100Hz to 30MHz) the value of propagation constant can be neglected in the calculation of ground return impedance and admittance. This fact can be seen by the response of propagation constant of insulated cable in frequency domain in Fig. 2 and Fig. 3. This approximation we consider $|\gamma| = |\gamma_1|$ thus the following holds $u_1 = \sqrt{\lambda^2+\gamma_1^2} = \bar{u}_1$; $u_2 = \sqrt{\lambda^2+\gamma_2^2} = \bar{u}_2$; $\eta_1 = \gamma_1$ and the ground return impedance and admittances are given by

$$\bar{z}_{ext} = \frac{j\omega\mu_0}{2\pi}\left[\bar{\Lambda}+\bar{S}_1\right] \tag{A.1}$$

$$\bar{Y}_{ext} = 2\pi\left(\sigma_1 + j\omega\varepsilon_1\right)\left[\bar{\Lambda}-\bar{T}\right]^{-1}$$

where $\bar{\Lambda} = K_0(r\gamma_1) - K_0(d\gamma_1)$ and

$$\bar{S}_1 = \int_{-\infty}^{\infty}\frac{\exp(-2h\bar{u}_1)}{\bar{u}_1+\bar{u}_2}\exp(-jr\lambda)d\lambda \tag{A.2}$$

$$\bar{T} = \int_{-\infty}^{\infty}\frac{\bar{u}_2}{\bar{u}_1}\frac{[\exp(-h\bar{u}_1)-\exp(-2h\bar{u}_1)]}{n^2\bar{u}_1+\bar{u}_2}\exp(-jr\lambda)d\lambda$$

and where r is the external radius of the conductor, i.e., considering the insulation layer, $d = \sqrt{(2h)^2 + r^2}$. The propagation constant is then calculated as:

$$\gamma = \sqrt{(z_{\text{int}} + z_d + \bar{z}_{ext}).(y_d^{-1} + \bar{Y}_{ext}^{-1})^{-1}} \qquad \text{(A.3)}$$

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