Definition of a new formula for the characteristic impedance of vertical conductors for lightning transients

Pablo Gómez

***Abstract*-- A simple and accurate formula to compute the characteristic impedance of vertical conductors for lightning transient studies is introduced in this paper. For such purpose, existing formulas are initially compared in a parametric manner using FEM as base solution. Conversely to existing formulas, the formula proposed in this paper can take into account ground resistivity and non-uniformity of electrical parameters of a vertical conductor. It is also frequency independent, making it suitable for direct implementation in EMTP-type programs using a simple Bergeron model. Comparisons with previously published measurements and with the results from a detailed non-uniform frequency domain model demonstrate the accuracy and simplicity of the proposal.**

***Keywords*: characteristic impedance, lightning transients, parametric comparison, vertical conductors.**

I. Introduction

SIMULATION of lightning transients in complex conductive structures such as transmission towers, grounding grids, lightning protection systems of buildings (LPS), wind turbines, etc., has been the topic of several studies (see for instance [1]-[6]). These structures are commonly modeled by means of vertical conductors or the combination of vertical and horizontal conductors, applying one of the following approaches: a) rigorous electromagnetic-field theory, b) electromagnetic field simulations (finite differences, method of moments, FEM, etc.), c) distributed parameter models based on transmission line theory, d) lumped parameter models from circuit theory. The first two approaches can provide very accurate results, but they can also be very time consuming, particularly when 3D simulations are required; therefore, their application is limited. Regarding the last two approaches, distributed parameter models are preferred from lumped parameter ones for high frequency transients such as those due to lightning.

EMTP-type software can be used for the simulation of these structures when a distributed parameter modeling approach is chosen. However, vertical conductor models are not directly available to date in these time domain simulation programs.

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This paper introduces a simple formula to compute the characteristic impedance of vertical conductors for lightning transient studies. In order to propose such formula, an initial parametric comparison of existing formulas is performed, using FEM as base solution.

Conversely to existing formulas (Wagner, Sargent, Jordan, Hara, Chisholm, Ametani, etc.), the proposed formula can take into account the ground resistivity and non-uniformity of electrical parameters, and it is also frequency independent. Therefore, it can be directly implemented in EMTP simulations via a simple Bergeron model. Comparisons with previously published measurements [7] and with the results from a detailed non-uniform frequency domain model [8] demonstrate the accuracy and simplicity of the proposed formula. This formula can provide good results for conductors with a height ranging from 1 m to 100 m, provided that the height/radius ratio is in the order of 40 or larger. This makes the formula useful for most practical applications.

II. Existing Formulas for the Characteristic Impedance of Vertical Conductors

For the formulas listed in this section, the following assumptions are made:

1. Perfectly conducting ground
2. Cylindrical conductor completely perpendicular to the ground plane
3. Lossless conductor

From the basis of electromagnetic theory, Wagner and Hileman [9] analyzed the response of a cylindrical conductor excited by a step current waveform. They obtained the following definition for the characteristic impedance of a vertical conductor:

$$Z_0^W = 60\ln\left(\frac{2\sqrt{2}h}{r}\right) \tag{1}$$

where h is the height of the cylinder and r is its radius.

From (1) and the application of Duhamel's integral, Sargent and Darveniza [10] obtained an expression that considers a linear-ramp type excitation:

$$Z_0^S = 60\left[\ln\left(\frac{2\sqrt{2}h}{r}\right) - 1\right] \tag{2}$$

Sargent and Darveniza showed that this definition is very similar to the one obtained by applying a double-exponential excitation, which is commonly used to represent a lightning stroke.

On the other hand, Hara et al. [11] found an empirical formula from the modification of the aforementioned expressions. This formula is given by

$$Z_0^H = 60\left[\ln\left(\frac{2\sqrt{2}h}{r}\right) - 2\right] \qquad (3)$$

This expression has proven good accuracy when compared to lab tests for specific geometries.

Using the method of images, Jordan [12] obtained the following formula:

$$Z_0^J = 60\left[\ln\left(\frac{h}{r}\right) - 1\right] \qquad (4)$$

Also applying the method of images and starting from Neumann's integral, Ametani et al. [13] derived the following equation:

$$Z_0^A = 60\left[\ln\left(\frac{\left(h+\sqrt{h^2+r^2}\right)^2}{r\left(2h+\sqrt{4h^2+r^2}\right)^2}\right) + \frac{3r+\sqrt{4h^2+r^2}-4\sqrt{h^2+r^2}}{2h}\right] \qquad (5)$$

Formulas (1)-(5) consider that the lightning strikes the conductor in a vertical manner. Chisholm et al. [14] recommended a different expression, considering that the lightning stroke impacts the conductor horizontally:

$$Z_0^C = 60\left[\ln\left(\frac{h+\sqrt{h^2+r^2}}{r}\right) - 1\right] \qquad (6)$$

III. Parametric Comparison of Existing Formulas Against FEM

In order to compare the results from the formulas listed in Section II, FEM-based software COMSOL Multiphysics is used. A 2D axisymmetric geometry is considered. AC/DC module and magnetic-fields physics are selected. A current excitation of 1 A in the –*z* direction is applied (from conductor's top to bottom). The ground effect at high frequencies (related to the lightning phenomenon) is approximated by replacing the ground plane with a magnetic insulation boundary, such that the magnetic field does not penetrate the ground. Furthermore, open boundaries corresponding to the air surrounding the conductor are included by means of "infinite element" subdomains included in COMSOL for such purpose. The corresponding geometry is shown in Fig. 1.

Once the magnetic fields simulation is performed, the vertical conductor's inductance is obtained from the magnetic energy W_m in the surrounding air:

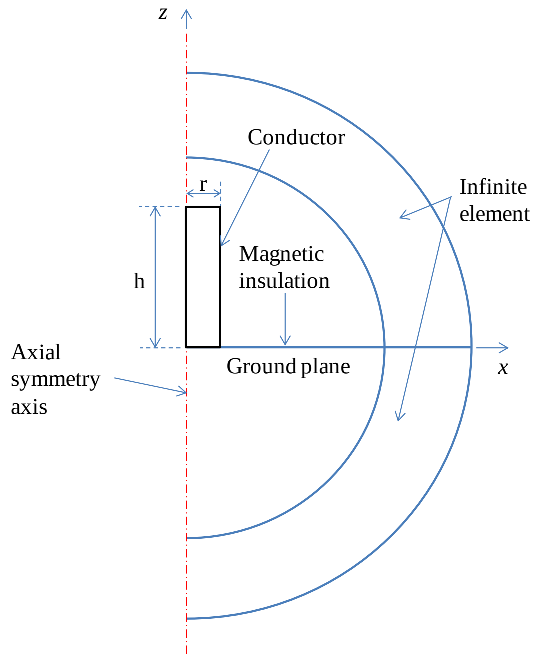

Fig. 1. Geometrical setup for FEM simulation

$$L = \frac{2W_m}{I^2} \qquad (7)$$

From (7), neglecting losses and assuming that the propagation of the lightning stroke along the conductor occurs in a transversal electromagnetic (TEM) mode, the characteristic impedance of the conductor can be computed as follows:

$$Z_0 = \frac{cL}{h} \qquad (8)$$

where *c* is the velocity of light in free-space.

The parametric comparison is performed considering a vertical conductor with a height *h* = 10 m and a radius *r* varying from 1 cm to 1 m, such that the ratio *h/r* is in the range 10-1000. The corresponding results are shown in Fig. 2. It can be noticed that the formulas from Ametani, Hara and Jordan are the closest ones to the results from FEM. On the other hand, the formulas from Wagner, Sargent and Chisholm clearly overestimate the characteristic impedance values for the range under consideration.

Once the most accurate formulas (for the values of *h* and *r* previously defined) have been detected, a new comparison by means of the computation of relative differences against FEM is performed. This is shown in Fig. 3. It can be observed that Hara's formula yields the lowest differences as the ratio *h/r* increases, this is, when *h* >> *r*. For the same condition (which is very common in vertical structures) it can be noticed that the results from Ametani's and Jordan's formulas tend to match. In fact, Ametani proved that, for *h* >> *r*, his formula is identical to Jordan's formula [13].

The opposite case is also analyzed, this is, a low *h/r* ratio. Fig. 4 shows a zoom-in of the relative difference for an *h/r* ratio in the range 10-100. It is observed that the results from all 3 formulas tend to differ from the FEM results as the *h/r* ratio decreases. This plot shows that, with any of the 3 formulas considered, the conductor's height has to be at least 40 times larger than its radius in order to obtain characteristic impedance values with a relative difference below 5% against FEM. This gives a good indication of the validity range of the formulas.

IV. Definition of a New Formula

To date, there are no models available to simulate the transient behavior of vertical conductors in EMTP-type programs. An alternative is to use an existing transmission line model and compute the characteristic impedance from one of the formulas listed in Section II. However, there are two important omissions in such formulas:

1. Ground resistivity
2. Non-uniformity of electrical parameters

There are different approaches to take these matters into account. However, none of these approaches can be used directly in existing transmission line models from EMTP-type programs, precluding their extensive application.

In this section, a formula is introduced that takes into account ground resistivity and non-uniformity of electrical parameters in a simple manner. This formula is based on a modification of Hara's formula. In order to define such modification, a frequency domain model of a vertical conductor was implemented. This model uses the concept of complex penetration depth to take into account ground resistivity and cascaded connection of chain matrices to take into account the non-uniformity of electrical parameters of the vertical conductor [8]. The characteristic impedance of the vertical conductor is extracted from the model according to

$$Z_0^{model} = \sqrt{\Phi_{12} / \Phi_{21}} \qquad (9)$$

where Φ_{12} and Φ_{21} are the elements (1,2) and (2,1) from the chain matrix of the complete non-uniform conductor (obtained after applying cascaded connection). The behavior of (9) at different frequencies, resistivities and *h/r* ratios is observed. Then, curve fitting (function `fit` from MATLAB) is applied to obtain a formula for the characteristic impedance of a vertical conductor as a function of its height *h*, radius *r* and ground resistivity ρ:

$$Z_0^P = Z_0^H + Z_\rho + Z_{NU} \qquad (10)$$

where Z_0^H is the characteristic impedance given by Hara's formula (eq. (4)), Z_ρ is the impedance modification due to the ground resistivity, given by

$$Z_\rho = (54.8 - 33.4 h^{0.062}) \rho^{0.2} \qquad (11)$$

and Z_{NU} is the impedance modification due to non-uniformity of electrical parameters, given by

$$Z_{NU} = 36.6 - 110.2 (h/r)^{-0.48} \qquad (12)$$

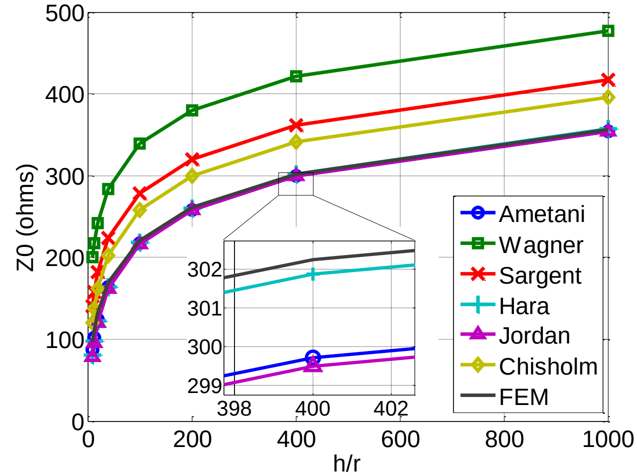

Fig. 2. Characteristic impedance of vertical conductor as a function of the *h/r* ratio

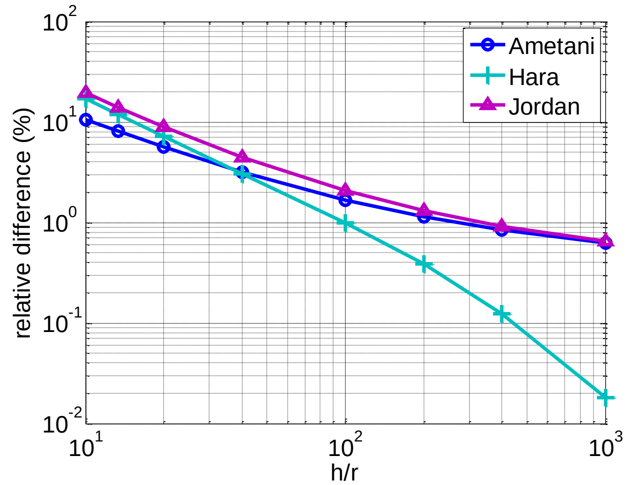

Fig. 3. Relative difference between analytical formulas and FEM

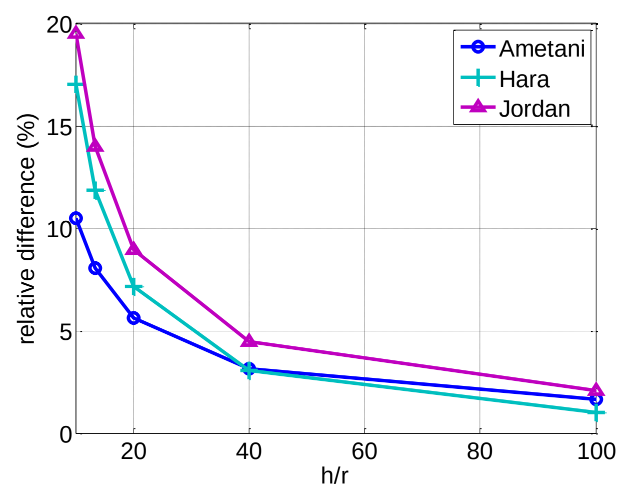

Fig. 4. Zoom-in of Fig. 3 for a conductor' height between 10 and 100 times larger than its radius.

Ranges for the curve fitting used to obtain formula (10) are as follows:

$$1\,\Omega\cdot\text{m} \leq \rho \leq 1000\,\Omega\cdot\text{m} \tag{13a}$$

$$1\,\text{m} \leq h \leq 100\,\text{m} \tag{13b}$$

$$h/r \geq 40 \tag{13c}$$

Also, mean values of impedance for a frequency range from 10 kHz to 10 MHz are considered.

Tables I to III show the accuracy of the proposed formula when compared to the more complex frequency domain model for heights of 10 m, 50 m and 1 m, respectively. Variation of resistivity and *h/r* ratio is also considered. Relative differences are very low for all cases, reaching their largest values (below 7%) at the limits of the validity ranges.

TABLE I
RELATIVE DIFFERENCE BETWEEN Z_0 COMPUTED WITH THE PROPOSED FORMULA AND WITH THE BASE SOLUTION, h = 10 m

Resistivity Ω·m	*h/r*			
	40	100	1000	10000
	Relative difference (%)			
1	4.05	2.52	1.10	0.97
10	3.55	1.90	0.57	0.61
100	3.78	1.73	0.25	0.33
1000	6.32	3.35	1.09	0.88

TABLE II
RELATIVE DIFFERENCE BETWEEN Z_0 COMPUTED WITH THE PROPOSED FORMULA AND WITH THE BASE SOLUTION, h = 50 m

Resistivity Ω·m	*h/r*			
	40	100	1000	10000
	Relative difference (%)			
1	3.28	2.31	1.21	1.00
10	2.97	1.96	0.91	0.79
100	2.59	1.43	0.40	0.38
1000	3.62	1.86	0.44	0.34

TABLE III
RELATIVE DIFFERENCE BETWEEN Z_0 COMPUTED WITH THE PROPOSED FORMULA AND WITH THE BASE SOLUTION, h = 1 m

Resistivity Ω·m	*h/r*			
	40	100	1000	10000
	Relative difference (%)			
1	0.22	0.50	0.78	0.47
10	0.60	1.50	1.69	1.22
100	0.47	1.05	1.66	1.31
1000	6.77	3.82	1.63	1.16

The next section analyzes the effect of using the proposed formula on the accuracy of transient computations for a test case.

V. TEST CASE

The proposed formula is tested on a simple setup corresponding to the lightning protection system of a building (LPS), taken from [7]. The structure setup is shown in Fig. 5. Horizontal conductors were modeled as single-phase transmission lines with frequency dependent parameters, while vertical conductors were modeled in 3 different ways:

- Applying the same frequency domain model used as base solution in section IV.
- Computing the characteristic impedance from Hara's formula and introducing such value into the Bergeron model included in PSCAD.
- Computing the characteristic impedance from the proposed formula and introducing such value into the Bergeron model included in PSCAD.

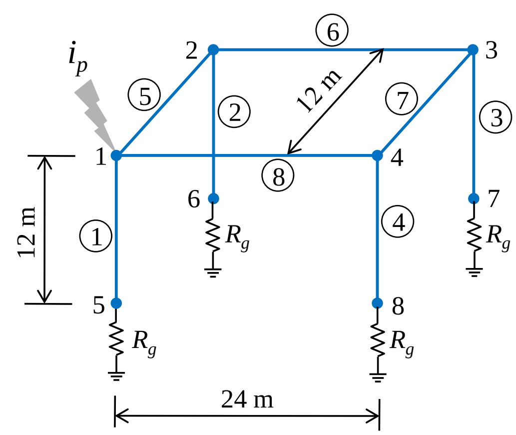

Fig. 5. LPS structure obtained from [6].

The transient current flowing along each branch of the structure was computed. The structure is excited by a lightning current waveform (i_p in Fig. 5) given by the following expression [15]:

$$i(t) = \sum_{i=1}^{n} t^{\delta_i} A_i\, e^{-\alpha_i t} \tag{14}$$

with n = 4. The remaining values used in (14) are listed in [15].

The maximum current values at each conductor of the structure are computed and compared to the experimental results reported in [7]. This is shown in Table IV by means of relative differences. Branch numbering can be identified (enclosed by circles) in Fig. 5.

Also, Table V shows a comparison of maximum voltage values at nodes 1 to 4 of the structure. These values were not measured in ref. [7], therefore in this case the FD model is used as base solution to compute relative differences.

TABLE IV
RELATIVE DIFFERENCE BETWEEN MEASURED AND COMPUTED VALUES OF MAXIMUM BRANCH CURRENTS

Branch	max[$i(t)$] Meas. [kA]	FD model [%]	Hara [%]	Proposed formula [%]
1	45.60	9.67	17.36	13.21
2	23.25	1.27	1.81	0.07
3	12.71	4.75	4.76	5.19
4	16.50	6.12	12.37	8.99
5	31.50	3.24	8.62	5.88
6	8.00	4.16	4.30	4.74
7	4.75	3.25	5.82	6.00
8	21.00	4.68	11.63	8.24
Mean		**4.64**	**8.33**	**6.54**

TABLE V
RELATIVE DIFFERENCE BETWEEN COMPUTED VALUES OF MAXIMUM NODE VOLTAGES

Node	max[$v(t)$] FD model [kV]	Hara [%]	Proposed formula [%]
1	903.01	12.88	3.19
2	444.06	19.14	4.31
3	230.22	25.59	4.09
4	300.01	20.79	3.04
Mean		**19.60**	**3.66**

From Tables IV and V, it can be noticed that the proposed formula increases the accuracy of the results in comparison to Hara's formula. According to such tables, the node voltages are by far more sensitive to the accuracy in the computation of characteristic impedance than the branch currents. From Table IV, there is a slight increase of accuracy in the computation of branch currents when using the proposed formula instead of Hara's formula (average relative difference decreases less than 2%). However, Table V shows that there is a very important increase of accuracy in the computation of node voltages when using the proposed formula instead of Hara's formula. The average relative difference in this case decreases almost 16%, reaching a value of 3.66%. Thus, for this case, using the proposed formula could have a large impact in obtaining an adequate protection of the building against lightning.

VI. CONCLUSIONS

The computation of characteristic impedance of vertical conductors for lightning transient analysis has been analyzed in this paper.

An initial parametric comparison of existing formulas (using FEM as base solution) showed that the formulas from Ametani, Hara and Jordan provide the best results. However, the results from Hara's expression are in closer agreement to FEM as the height/radius ratio increases. Therefore, this expression is used as base equation for the formula proposed in this paper, with modifications to account for the effects of ground resistivity and non-uniformity of electrical parameters. The results from a test case consisting of the lightning protection system of a building prove that the proposed formula yields a very important accuracy increase in the computation of transient overvoltages (average relative difference of 3.66% against 19.6% obtained with Hara's formula)

VII. REFERENCES

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