Comparison of the fdLine and ULM Frequency Dependent EMTP Line Models with a Reference Laplace Solution

Arash Tavighi, José Ramon Martí, José Alberto Gutiérrez Robles

Abstract--**This paper presents time-domain simulation results obtained with the frequency-dependence fdLine and ULM line models in the EMTP for a variety of asymmetrical line configurations, including single-circuit, double-circuit and parallel lines. The results are compared with a reference frequency-domain solution based on the Laplace transform. The comparisons include open and short circuit conditions during the transient and steady-state periods. As observed from the simulations, both fdLine and ULM follow very closely the results obtained with the Laplace solution. This is in contrast with the traditional belief that fdLine does not give accurate results for asymmetrical line configurations.**

Keywords: **Frequency-dependent line models, EMTP, fdLine, Universal line model (ULM), Laplace transform solution, Asymmetrical overhead transmission lines.**

I. Introduction

THERE are two main approaches to solving for transient conditions in frequency-dependent transmission lines: Time-domain and Frequency-domain.

The frequency-domain solution describes the transmission line one frequency at the time over a frequency range, and transforms the frequency response into a time response using time-frequency transformations such as the Discrete Laplace Transform. The frequency-domain solution makes no approximations regarding diagonalizing transformation matrices and is, therefore, a very good reference when comparing time-domain frequency-dependent line models [1].

On the other hand, the solution of a complete system, in which a large variety of conditions are simulated, is more easily formulated in the time-domain. In direct time-domain simulations, every element of an electrical network is described by a set of differential equations. In order to obtain the time-domain models of these elements, the corresponding differential equations are solved (integrated) between discrete time intervals (e.g. trapezoidal rule in the EMTP). Transmission lines, however, are one of the most difficult elements to be modelled in time-domain transient simulations, due to the complexity of the frequency dependence functions involved. As a result, a series of factors such as time-step, numerical stability and accuracy become major considerations. In the pioneering work of [2], an efficient frequency-dependent line model was formulated by fitting the characteristic admittance and propagation wave functions with a low-order rational function approximation. This model was extended in [3] for high-order accurate approximations of the line functions while guaranteeing numerical stability by applying minimum-phase shift constraints.

The fdLine model of [3] has been widely used in EMTP programs (e.g. [4]-[7]) and is well-known for its efficiency and reliability. The rational function approximations in fdLine are based on Bode's Asymptotic Fitting (BAF) of the magnitude of the line functions using only real negative poles and zeros. This guarantees minimum-phase-shit functions and, therefore, causal and stable solutions. The original version of fdLine was implemented in 1981-1983 in the Bonneville Power Administration (BPA) EMTP [7]. This original BPA version was improved in the DCG/EPRI version developed in 1984-1986 [8]. Some of the specific improvements included: higher dynamics in the very low frequency region that solved encountered problems for very short lines, improved BAF algorithms that improved the overall accuracy of the fitting, and automatic error checking for short- and open-circuit conditions to optimize the single-frequency diagonalizing matrices used in fdLine. Even though fdLine has been widely used, there has been the belief that because it uses a single real transformation matrix to convert between modal and phase quantities, the model may not be accurate enough for strongly asymmetrical line configurations.

The Universal Line Model (ULM) was introduced in 1999 [9] to improve the accuracy of fdLine by using complex poles and zeros in the rational-function approximations and by not limiting the transformation matrices to a single frequency but fitting these matrices over the frequency range of the model using rational function approximations of the idempotent coefficients. By introducing these more detailed modelling functions, ULM relaxes the fitting requirements by allowing non-minimum phase rational functions. As a result both the real and imaginary parts of the line propagation and characteristic admittance functions need to be fitted. The frequency-dependent transformation matrices are also fitted in the form of idempotents coefficients [10].

As a result of the additional complexity of the model, ULM requires additional computational time which is an important consideration in real time simulators. In addition, the numerical stability of the model is harder to guarantee and in

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many cases special considerations need to be made to guarantee passivity. Even though fdLine is simple, does not require passivity considerations, and has been implemented in real time simulators the concern has remained of whether the model is accurate enough under strongly asymmetrical line configurations. Many of the arguments for this conclusion have been based on using a single transformation matrix versus fitting the transformation matrix functions. The results presented in this paper seem to indicate that fdLine has a similar accuracy to ULM when compared to a reference Laplace solution under asymmetrical overhead line configurations.

In some regions of the modelled transients, ULM gave slightly better results than fdLine (even though both models were very accurate), while in some other regions fdLine gave slightly better results than ULM (even though both models were very accurate). Six different line configurations were tested in this study. The MicroTran software of [4] (v3.25) was used for the simulations using fdLine, while the PSCAD software of [5] (v4.5.2.0) was used for the simulations running the ULM model. MATLAB was used for the Laplace solution. Even though particular software packages were used to run these tests, the cases are described with enough detail so that they can be run using other EMTP software implementations.

II. Case Studies

Six line configurations (Figs. 1 to 6) were tested under asymmetrical conditions as follows:

- **Test 1:** Single-phase line.
- **Test 2:** 3-phase single-circuit horizontal line [11].
- **Test 3:** 3-phase single-circuit vertical line [12].
- **Test 4:** 3-phase single-circuit delta line [12].
- **Test 5:** 3-phase double-circuit one-tower delta line [11].
- **Test 6:** 3-phase double-circuit two-tower horizontal line [13].

To compare the result with the Laplace solution, an M-file was written in MATLAB which directly maps the frequency response of the test cases to the time-domain [1]. To ensure the accuracy of the results obtained with the Laplace solution, a time simulation window with 2^{20} samples was chosen.

In the curve-fitting process of all the simulations, the maximum number of poles was set to 35, the frequency range considered was from 10^{-2} to 10^{8} *Hz*, the constant/real transformation matrix for fdLine ($\boldsymbol{Q_{real}}$) was set at 1 *kHz*, and the diagonal matrix of external insulator conductance ($\boldsymbol{G_{ext}}$) was chosen as a typical value of 3×10^{-8} (*S/km*) [14].

In the test cases, the transmission lines are connected to a balanced three-phase cosine source (Fig. 7) and the peak value of phase-a is applied at t = 0. The equivalent source impedance corresponds to the impedance of the generator plus its step-up transformer. The conductors at the receiving-end of the line are either shorted or open (connected to the ground with a resistance of 10^{-6} Ω or 10^{6} Ω, respectively.). The time steps of the EMTP simulation are indicated in the captions showing the line geometry. To guarantee the accuracy of the time-domain simulation, these time steps are taken to be 20 times smaller than the travelling time at the speed of light.

Figs. 1 to 6 show the line geometries and the terminal conditions of the conductors (open or shorted). Voltages and currents during these tests are plotted in Figs. 8 to 39.

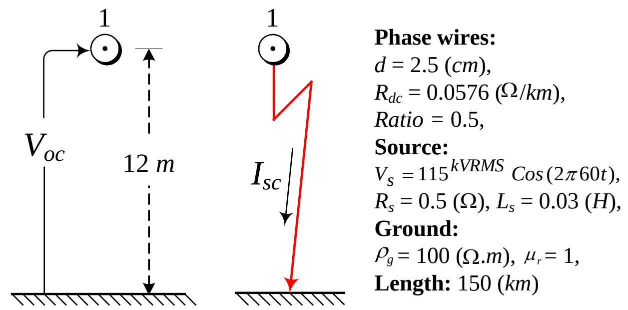

Fig. 1. Single-phase transmission line (open and shorted). (Δt = 5 μs)

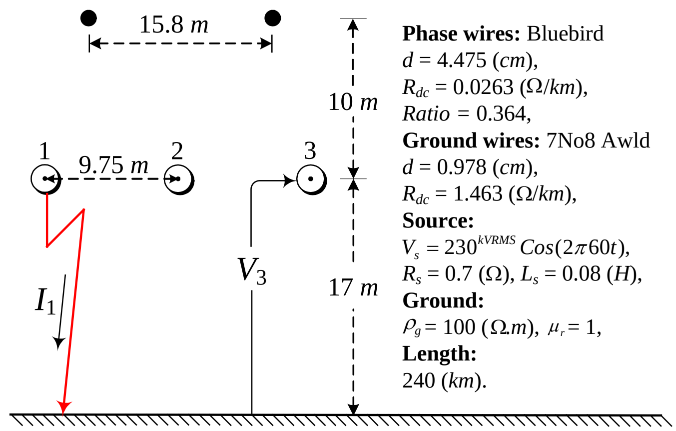

Fig. 2. Three-phase single-circuit horizontal transmission line. (Δt = 40 μs)

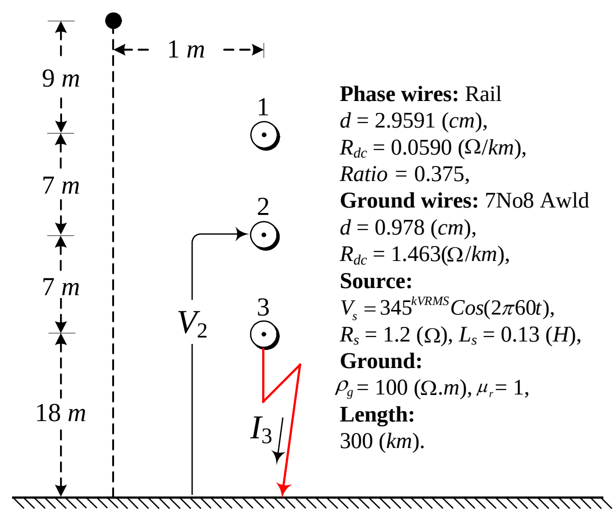

Fig. 3. Three-phase single-circuit vertical transmission line. (Δt = 50 μs)

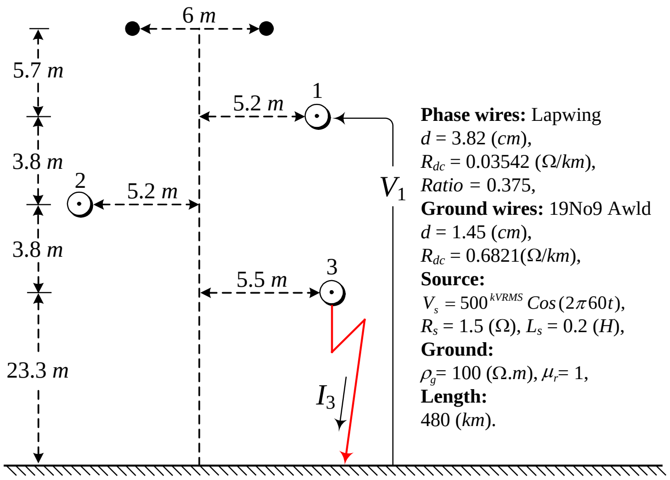

Fig. 4: Three-phase single-circuit delta transmission line. (Δt = 80 μs)

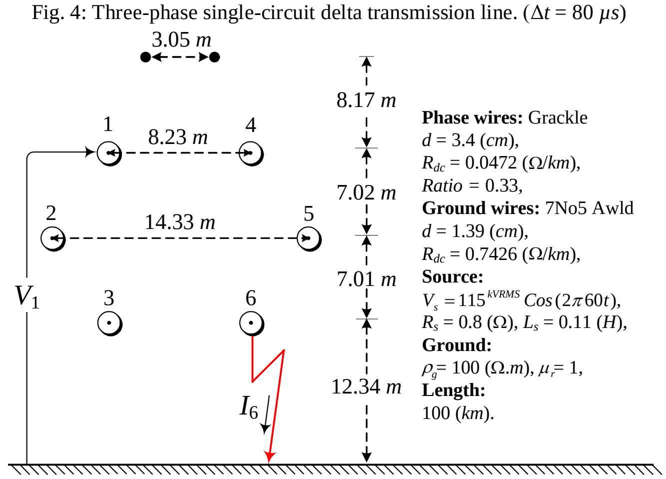

Fig. 5: Three-phase double-circuit one-tower delta transmission line. (Δt = 16 μs)

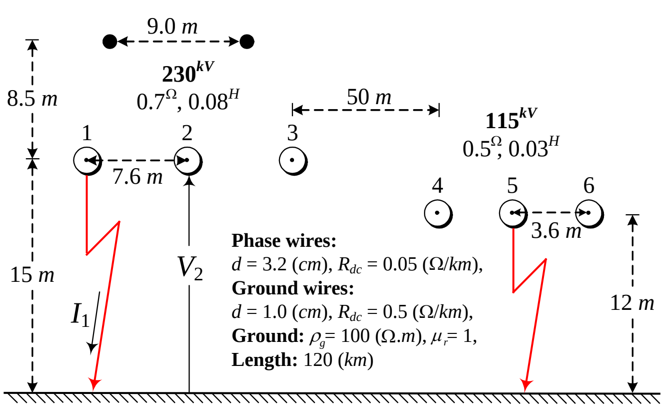

Fig. 6: Three-phase double-circuit two-tower horizontal transmission line. (Δt = 20 μs)

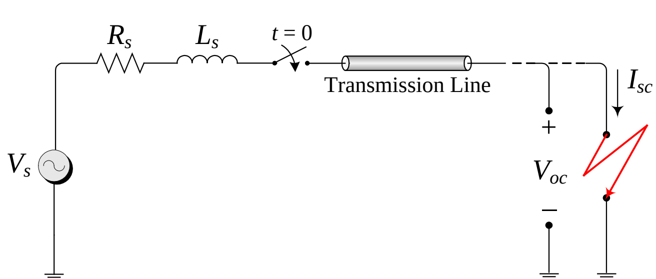

Fig. 7. One-line diagram of the equivalent circuit for the tests.

III. Comparison of Simulation Results

The simulation results of the line energization tests, obtained with the Laplace, fdLine, and ULM solutions are shown using "Black", "Green", and "Red" color codes, respectively.

Due to the closeness of the results presented in each plot, the differences are best viewed using the glass magnifier in a pdf viewer. The simulations are shown in two different time spans, a longer time span to show the evolution into steady-state, and a shorter time span to focus on the initial transient region.

Due to space limitations, only the phases with the worst results are shown. The simulations were run at the Δt's shown in the captions to the line configuration, Figs. 1 to 6.

Figs. 8 to 39 show the simulation results for open-circuit voltages and short-circuit currents for the following cases:

A) A single-phase line (Figs 8 and 9),

B) Three-phase single-circuit lines (Figs. 10 to 27),

C) Three-phase double-circuit lines in the same tower (Figs. 28 to 33),

D) Three-phase double-circuit lines in separate towers (Figs. 34 to 39),

E) Steady-State Solutions (Table 1).

A) Single-Phase Line

The results for the single-phase test are shown in Figs. 8 and 9. These results show, in the absence of transformation matrices needed in the multiphase cases, fdLine and ULM match the Laplace solution perfectly, thus, verifying the general validity of the models.

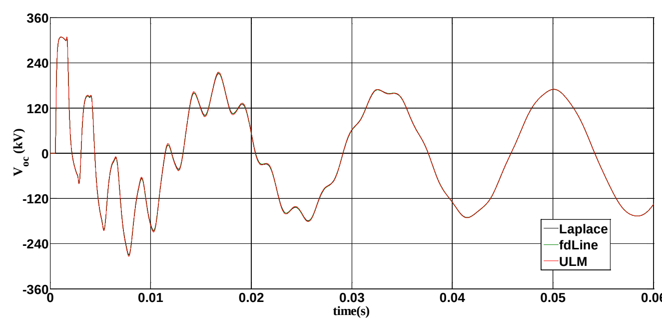

Fig. 8. Voltage at receiving-end of open single-phase line.

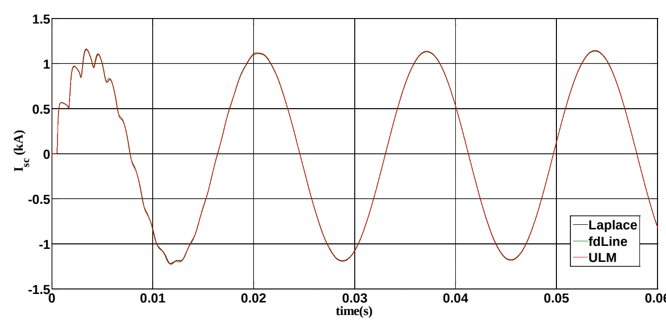

Fig. 9. Current at receiving-end of shorted single-phase line.

B) Three-Phase Single-Circuit Lines

The results for the Single-Circuit line tests are shown in Figs. 10 to 27. For the *horizontal* line of Fig. 2, the open-circuit voltages at the receiving-end of the line are shown in

Figs. 10 to 12. These results show that the solutions given by fdLine and ULM are perfectly matched, and they follow the Laplace solution very well (except in the jagged points of the curves in which the Laplace solution deviates slightly). Also for the short-circuit currents, Figs. 13 to 15 show that the results of fdLine and ULM for the *horizontal* line are quite in agreement with the Laplace solution.

For the *vertical* line of Fig. 3, the open-circuit voltages at the receiving-end of the line are shown in Figs. 16 to 18. These results show that fdLine follows the Laplace solution closer than ULM. The deviation of the open-circuit voltages at the peak points and the slight phase shift drift of ULM can be more clearly observed in Figs. 17 and 18. For the short-circuit currents, Figs. 19 to 21 show that ULM is closer to the Laplace solution than fdLine. Fig. 20 shows that the peak value of short-circuit current of fdLine is about 5% below the peak value of the Laplace solution.

For the *delta* line of Fig. 4, the open-circuit voltages at the receiving-end of the line are shown in Figs. 22 to 24. These results show that fdLine and ULM follow the Laplace solution well, but not as well as for the *horizontal* and *vertical* lines tests. Fig. 22 shows that the peak values of fdLine get closer to the Laplace solution than the ULM values as time advances. However, there is no perfect match between the three solutions at the peak values of the few initial cycles. For the short-circuit currents, Figs. 25 to 27 show that the short circuit currents of fdLine and ULM match Laplace solution very well.

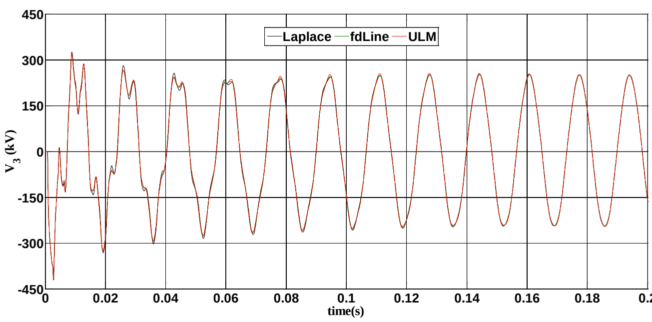

Fig. 10. Voltage at open conductor 3 of horizontal line.

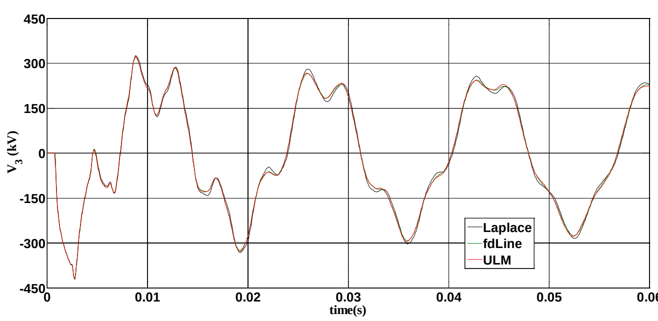

Fig. 11. Detail of voltage at open conductor 3 of horizontal line.

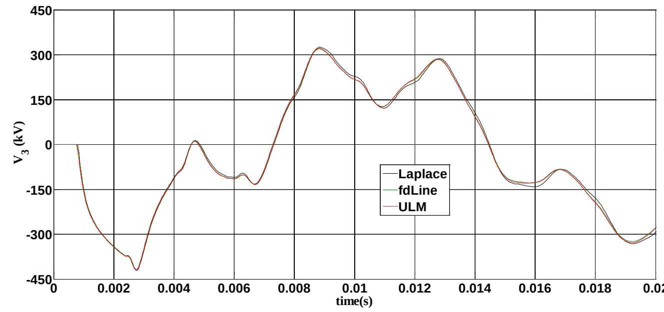

Fig. 12. Further detail of voltage at open conductor 3 of horizontal line.

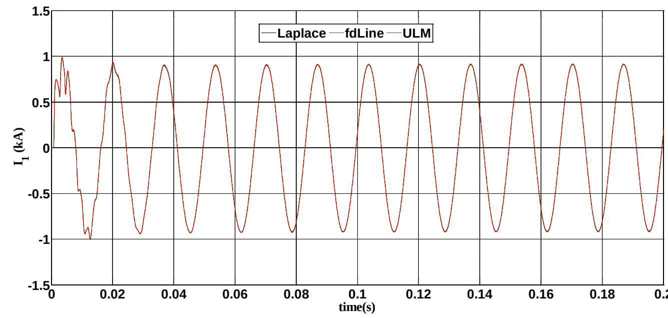

Fig. 13. Current at shorted conductor 1 of horizontal line.

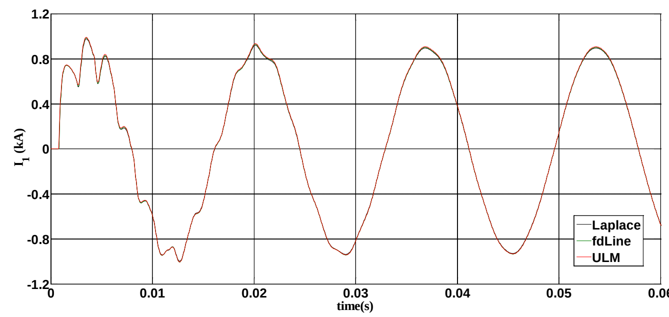

Fig. 14. Detail of current at shorted conductor 1 of horizontal line.

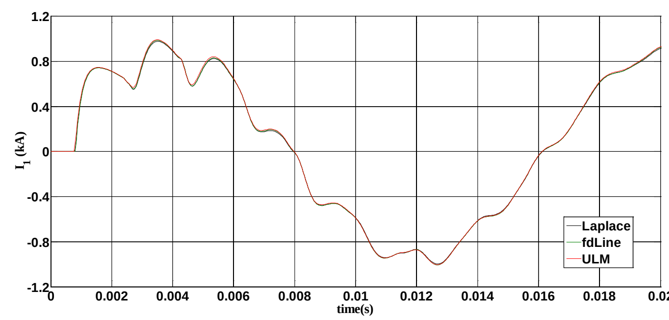

Fig. 15. Further detail of current at shorted conductor 1 of horizontal line.

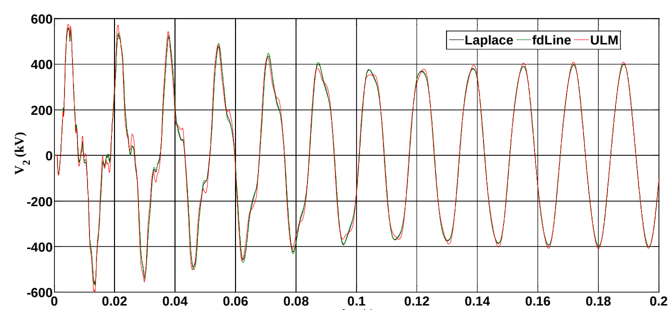

Fig. 16. Voltage at open conductor 2 of vertical line.

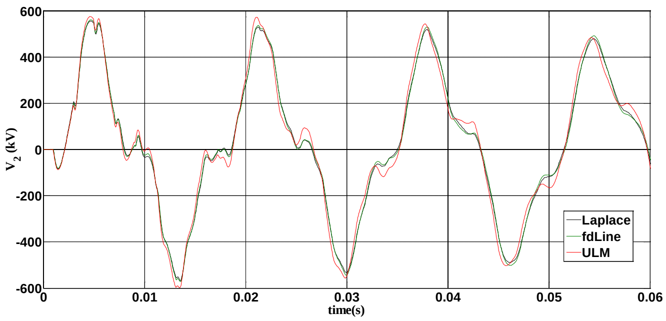

Fig. 17. Detail of voltage at open conductor 2 of vertical line.

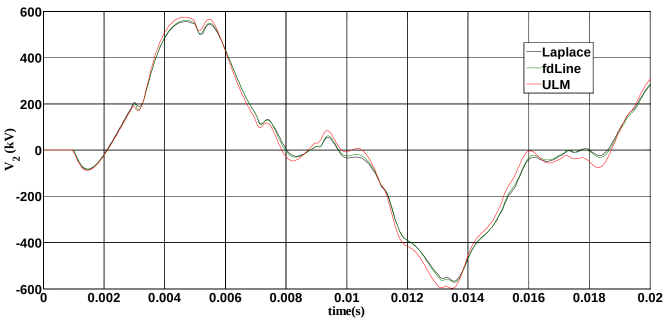

Fig. 18. Further detail of voltage at open conductor 2 of vertical line.

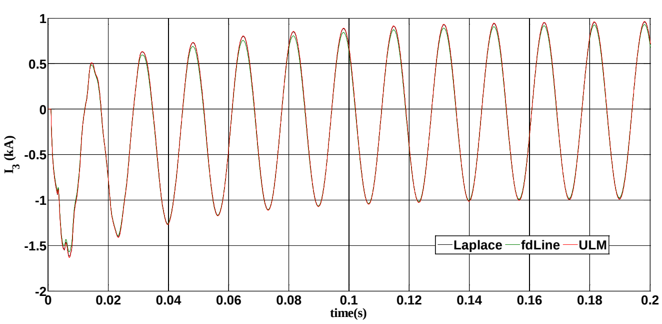

Fig. 19. Current at shorted conductor 3 of vertical line.

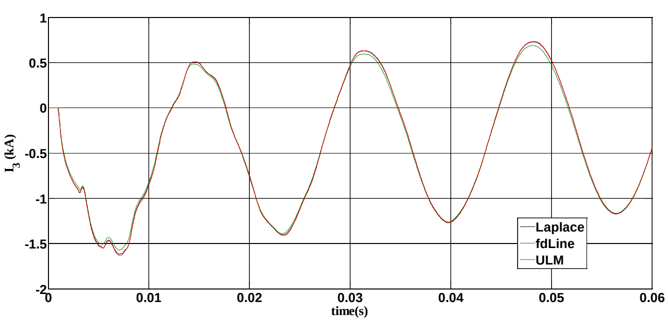

Fig. 20. Detail of current at shorted conductor 3 of vertical line.

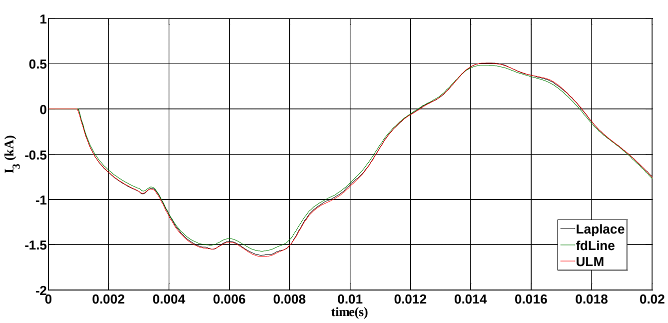

Fig. 21. Further detail of current at shorted conductor 3 of vertical line.

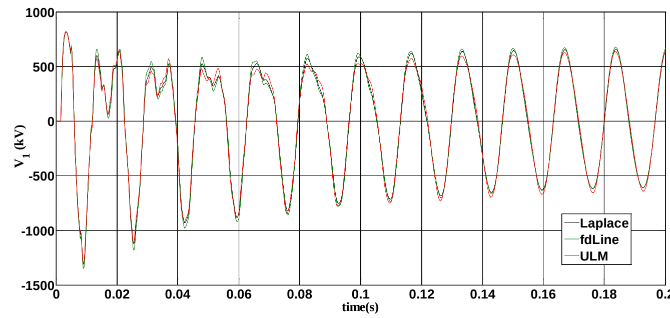

Fig. 22. Voltage at open conductor 1 of delta line.

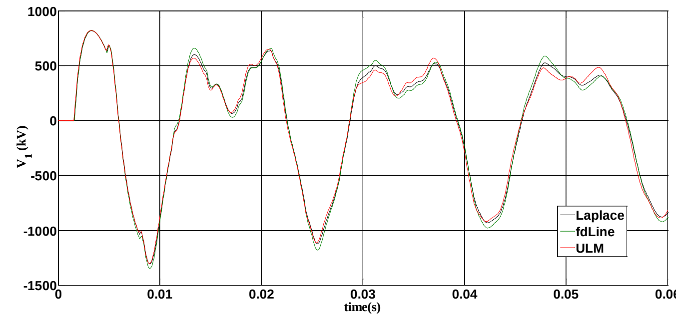

Fig. 23. Detail of voltage at open conductor 1 of delta line.

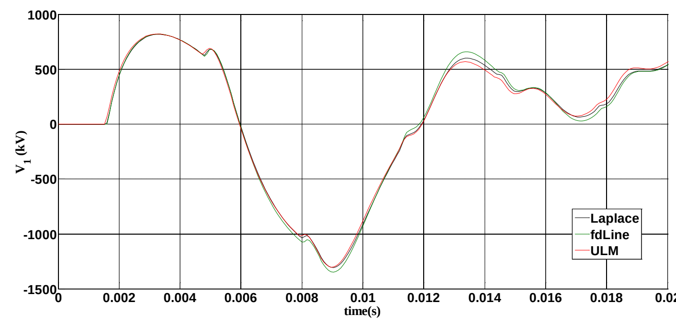

Fig. 24. Further detail of voltage at open conductor 1 of delta line.

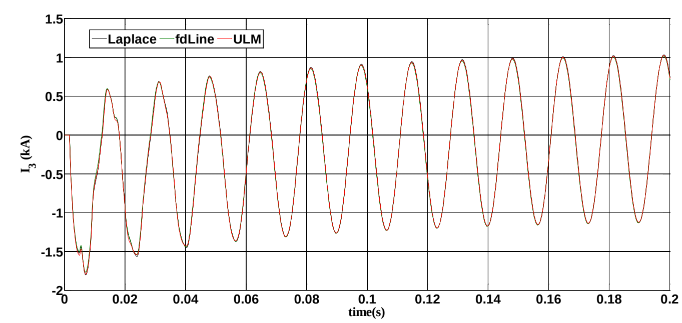

Fig. 25. Current at shorted conductor 3 of delta line.

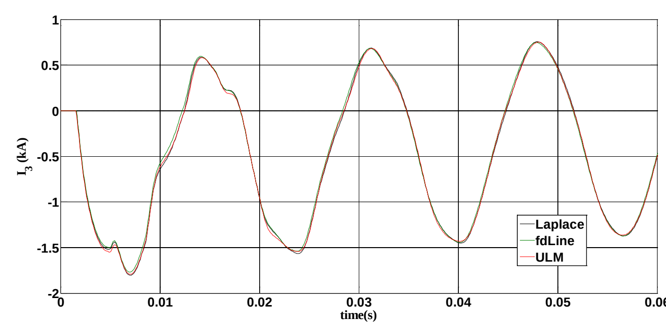

Fig. 26. Detail of current at shorted conductor 3 of delta line.

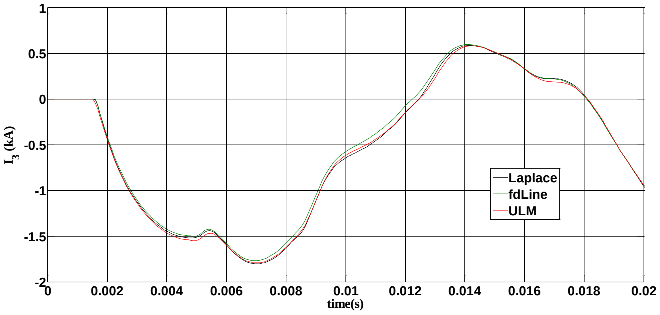

Fig. 27. Further detail of current at shorted conductor 3 of delta line.

C) Three-Phase Double-Circuit Line in the Same Tower

The fdLine and ULM models performed very accurately for the single-phase line and the three-phase single-circuit lines. As expected, however, larger differences between time-domain (fdLine and ULM) and frequency-domain (Laplace) simulations were observed for the double-circuit same-tower and double-circuit separate-tower tests. The results of these tests are shown in Figs. 28 to 33.

For the double-circuit *delta* lines of Fig. 5, the open-circuit voltages at the receiving-end of the line are shown in Figs. 28 to 30. As seen in Fig. 29, the fdLine and ULM solutions are very close to each other and follow the Laplace solution quite well in the initial few cycles and in the steady state solutions. However, as observed in Fig. 28, time-domain solutions deviate more from the Laplace solution in the time span between 0.02 to 0.1 seconds.

For the short-circuit currents, Figs. 31 to 33 show that both fdLine and ULM match the Laplace solution perfectly.

The results of these tests show that, contrary to common belief, fdLine gives very accurate results for the double-circuit line. These results are very similar to the results obtained with ULM.

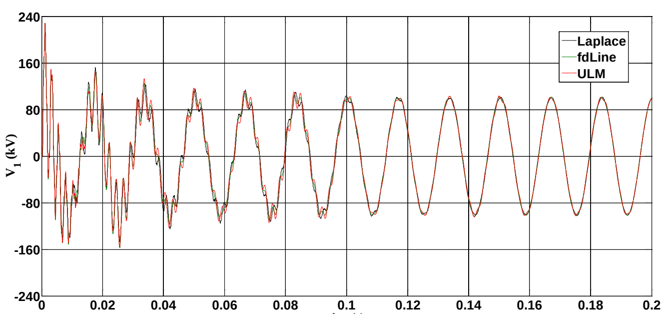

Fig. 28. Voltage at open conductor 1 of double-circuit one-tower delta line.

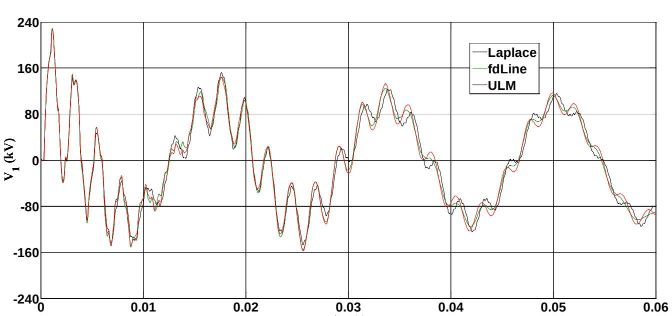

Fig. 29. Detail of voltage at open conductor 1 of double-circuit one-tower delta line.

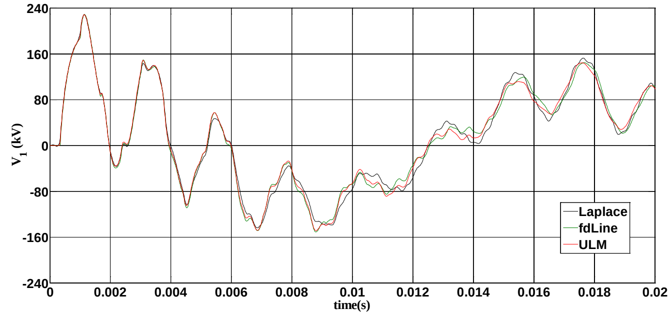

Fig. 30. Further detail of voltage at open conductor 1 of double-circuit one-tower delta line.

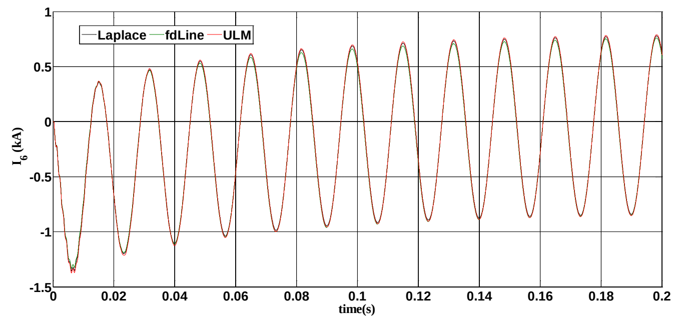

Fig. 31. Current at shorted conductor 6 of double-circuit one-tower delta line.

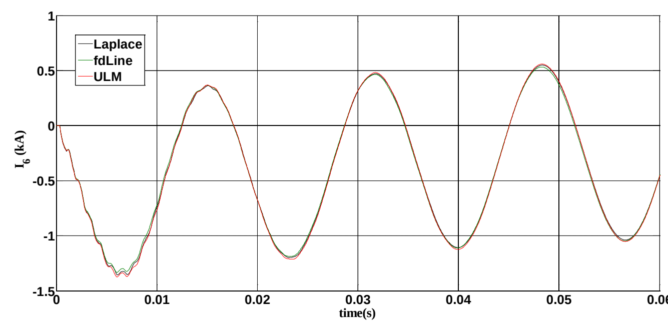

Fig. 32. Detail of current at shorted conductor 6 of double-circuit one-tower delta line.

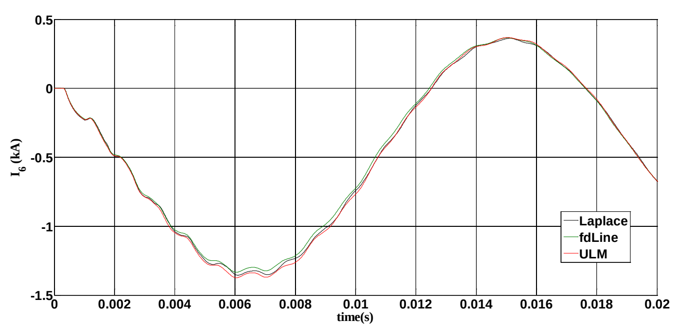

Fig. 33. Further detail of current at shorted conductor 6 of double-circuit one-tower delta line.

D) Three-Phase Double-Circuit Line in Separate Towers

Figs. 34 to 39 present the results obtained for the two parallel horizontal lines of Fig. 6 mounted in two separate towers.

For the open-circuit voltages, Figs. 34 to 36 show that the solutions given by fdLine and ULM are closer to the Laplace solution than they were for the double-circuit in the same tower case. As observed in Figs. 35 and 36, fdLine is slightly closer to the Laplace solution than ULM, particularly at the peak points.

For the short-circuit currents, Figs. 37 to 39 show that the results are very similar for both fdLine and ULM. However, both results present a slight vertical shift with respect to the Laplace solution.

It can be noted that for circuits in separate towers, there is less coupling between circuits than when they are are in the same tower. This might explain the slight edge of fdLine versus ULM in this case. Since ULM uses a more complex model for frequency dependent coupling (frequency dependent transformation matrices), it might be the case that, for numerical reasons, a simpler model, like fdLine that uses a constant transformation matrix has less numerical difficulties and provides better results for weakly coupled cases. This does not mean that fdLine only works well for weakly coupled cases, since it also gave very good results, comparable with those of ULM, for the strongly coupled cases when both circuits were in the same tower.

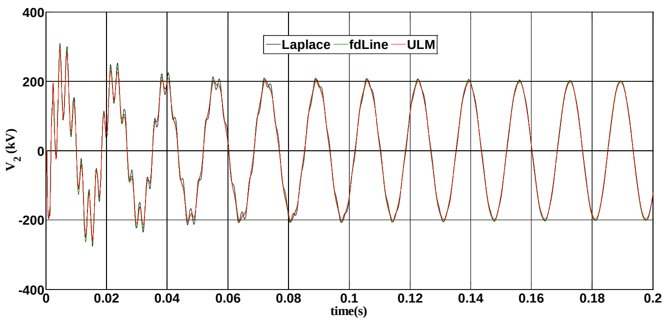

Fig. 34. Voltage at open conductor 2 of two parallel horizontal lines.

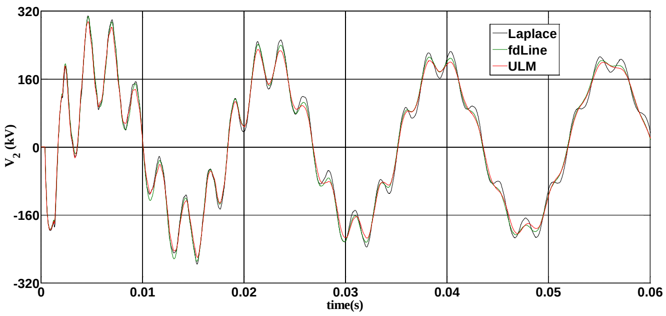

Fig. 35. Detail of voltage at open conductor 2 of two parallel horizontal lines.

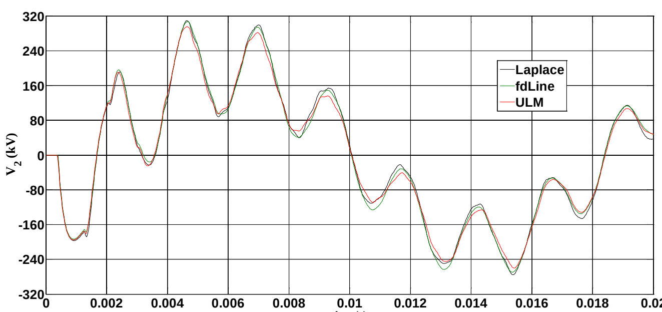

Fig. 36. Further detail of voltage at open conductor 2 of two parallel horizontal lines

E) Steady-State Solutions

Table I below shows the steady state solutions obtained running Laplace, fdLine, and ULM in the different test cases after the initial transient settles.

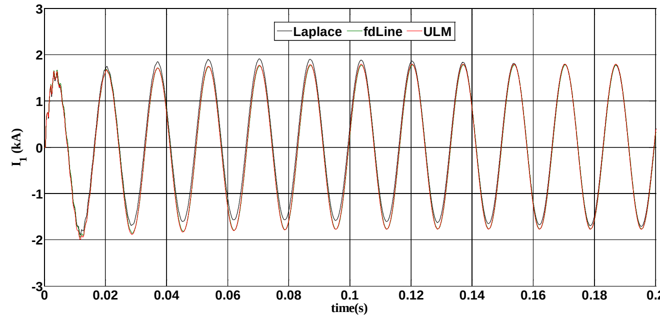

Fig. 37. Current at shorted conductor 1 of two parallel horizontal lines.

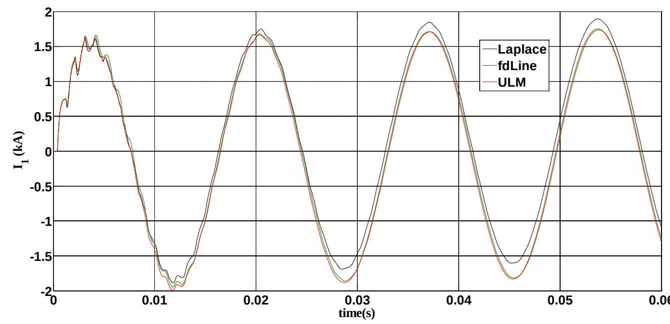

Fig. 38. Detail of current at shorted conductor 1 of two parallel horizontal lines.

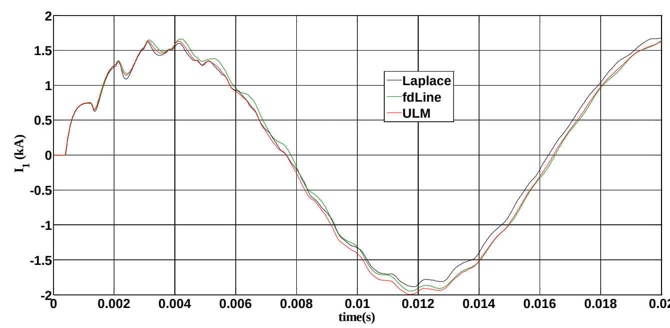

Fig. 39. Further detail of current at shorted conductor 1 of two parallel horizontal lines.

As it can be observed in Table I, the maximum error for the steady-state voltages was 1.16% for fdLine and 1.25% for ULM. The maximum error for the steady-state currents was 1.96% for fdLine and 2.59% for ULM. Considering the complexity of frequency-dependent line modelling, these are very good results for both fdLine and ULM.

TABLE I
COMPARISON OF STEADY STATE VALUES FOR THE DIFFERENT SOLUTIONS

#	*kV/kA*	Laplace	fdLine		ULM	
1	V_{oc}	167.81	167.9	0.05%	167.9	0.05%
	I_{sc}	1.155	1.159	0.36%	1.163	0.69%
2	V_3	247.7	246.9	0.32%	248.0	0.12%
	I_1	0.912	0.916	0.44%	0.919	0.77%
3	V_2	397.5	396.9	0.15%	399.4	0.48%
	I_3	0.970	0.951	1.95%	0.976	0.62%
4	V_1	631.5	638.8	1.16%	633.6	0.33%
	I_3	1.078	1.071	0.65%	1.072	0.56%
5	V_1	100.6	101.1	0.50%	100.5	0.10%
	I_6	0.809	0.803	0.74%	0.820	1.36%
6	V_2	200.4	202.2	0.90%	197.9	1.25%
	I_1	1.738	1.772	1.96%	1.783	2.59%

IV. CONCLUSIONS

In this paper, six different line configurations were simulated under asymmetrical short-circuit conditions using two well-known time-domain frequency-dependent line models: fdLine and ULM. The results were assessed using a frequency-domain Laplace solution. The Laplace solution is taken as reference because it does not use any approximation with regards to the modelling of frequency-dependence.

Three cases of three-phase lines were considered: single-circuit lines, double-circuit lines in the same tower, and double-circuit lines in separate towers. Open-circuit voltages and short-circuit currents were compared. For all cases, the fdLine model gave similar results to the ULM model and both models gave good results when compared to the reference Laplace solution. These results indicate that, contrary to traditional belief, a constant transformation matrix model like fdLine is capable of representing multi-circuit asymmetrical line configurations. In view of these results, we believe that more research is needed to understand the role of frequency-dependent transformation matrices in overhead transmission line modelling.

Even though, fdLine and ULM gave very accurate results compared to the Laplace solutions, there are still some discrepancies between time-domain and frequency-domain simulations that, in our opinion, are not yet clearly understood. The largest errors in the tests were observed in the voltages for the three-phase single-circuit delta line of Fig. 4 and in the three-phase double-circuit one-tower delta line of Fig. 5. The largest errors for the currents were observed for the three-phase double-circuit two-tower horizontal lines of Fig. 6. It was also observed that the open-circuit voltages of fdLine were closer to the Laplace solution than ULM in the tests of the single-circuit vertical line and double-circuit separate-tower horizontal lines. Alternatively, the short-circuit currents of ULM were closer to the Laplace solution than those of fdLine in the single-circuit vertical line test.

Overall, the waveforms of the short-circuit currents of both fdLine and ULM followed those of the Laplace solution closer than the waveforms of the open-circuit voltages, for the same line configuration.

Analysis of the steady-state period confirmed the very good performance of both fdLine and ULM. The maximum error was 1.25% (ULM) in the open-circuit voltages, and 2.59% (ULM) in the short-circuit currents for the test of two parallel horizontal lines in two separate towers.

Overall, fdLine and ULM performed extremely well in simulating asymmetrical overhead line configurations, with maximum errors that are very small compared to the uncertainties in the calculation of the line parameters, the uncertainties in the ground resistivity, and the errors inherent to formulating closed-form solutions for transcendental line functions.

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