Enhancing Phasor Estimation in Digital Protective Relays

A. K. X. S. Campos, W. L. A. Neves, D. Fernandes Jr.

***Abstract*-- The performance of protective relays phasor estimation algorithms depend on the signals of voltage and current fed to relays. During transients such as the ones produced by short-circuits, these voltage and current signals have undesirable components, such as harmonic components, interharmonics and DC decaying exponential component. Commonly used techniques apply cascaded low-pass, high-pass and Fourier Filters to filter out undesirable frequency components. In this work, it shown that the set of low-pass and high-pass filters could be merged into a suitably designed band-pass filter with better frequency response. The response of the band-pass filter cascaded with Fourier Filters are compared against classical filtering methods.**

***Keywords*: Protective Relays; Estimation Algorithms; Harmonic Components; Interharmonics Components; Decaying DC Component; Fourier Filters; Filtering Methods.**

I. Introduction

The protective relays use voltage and current phasors to make decisions during faults and maneuvers at electrical power systems. However, voltage and current signals present non-fundamental components: harmonic components, interharmonics and decaying DC component [1] – [3]. Simultaneous transients on voltage and current signals can cause even greater problems in protection devices [4].

Some alternatives have been proposed to filter out undesirable frequency components. These studies have focused on eliminating non-fundamental frequency components or mitigate the effect of these components in the protection relay performance [5] – [7].

Many studies have devoted special attention to removing DC exponential decaying in the current signal during transient phenomena [7] – [9]. Other works have been concerned to mitigate the transient components in the voltage signal provided by capacitive voltage transformers to protective relays [6].

An ideal filtering of non-fundamental frequency components should present unitary gain for the fundamental frequency of signal and zero gain for other frequency components. In practice, cascade of filters are used to approximate this characteristic.

Here, a suitably designed band-pass filter replaces the set of cascaded low-pass and high-pass filters. The response of the newly designed filter cascaded with Fourier Filters are compared against four techniques largely used in practice.

II. Digital Protective relay

Voltage transformers and current transformers are used to provide, respectively, voltage and current signals of the electric power system to protection relays.

The information of the voltage and current phasors are used by relays to perform arithmetic and logical operations, to enable the protection functions if needed and to send signals to other equipment belonging to the protection system, such as circuit breakers. A basic relay architecture and classical filtering algorithms are described next.

A. A Basic Relay Architecture

Digital protective relays comprise subsystems with specific functions. The basic architecture of digital relays with block diagram is shown in Fig. 1. Digital protective relay usually have built-in anti-aliasing analog filters, A/D Converter, a phasor estimation algorithm and a data processing unit.

The A/D converter is used to convert signals from analog to digital form at intervals defined by the sampling rate. The phasor estimation algorithms are used to obtain the fundamental component of signals, which is sent to the data processing unit whose function is to control the operation of the relay.

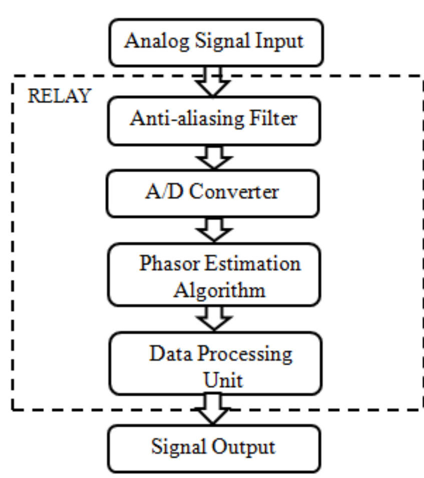

Fig. 1. Basic Relay Architecture Representation.

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B. Analog Filtering

The Nyquist sampling theorem postulates that a given signal of frequency f_{cs} can be completely reconstructed if the sampling rate of the scanning process is at least equal to $2.f_{cs}$, to avoid the overlapping spectra phenomenon called aliasing.

Sampling rate used in relays normally ranges from 480 Hz to 6000 Hz. Low-pass filters with cutoff frequency of half the sampling rate are generally used as anti-aliasing filters.

The low-pass filter has ideal unity gain for the desired frequency range, and zero gain from the cutoff frequency onwards. In practice, Butterworth and Chebyshev low-pass filters are used as anti-aliasing analog filters. Butterworth filters are the most used in digital relays due to their flat frequency response in their passband.

C. Phasor Estimation Algorithms

The phasor estimation algorithms calculate the phasor angles and magnitude within a sampling window. Some classical phasor estimation algorithms are briefly described here.

1) Discrete Fourier Transform (DFT) Algorithm

The basic approach used in a DFT algorithm is to extract the fundamental component of the measured waveform. The filtering process is performed by a pair of filters: Sine filter and Cosine filter.

There are two variations of this algorithm: the full cycle DFT (FDFT) and the half cycle DFT (HDFT), the first using samples from one cycle of the signal and the second using the samples from half cycle.

Assuming $x(t)$ to be a periodic signal, N the number of samples per cycle of that signal and Δt the sampling step used, the estimation of the fundamental frequency phasor, using the FDFT algorithm, is given by the correlation between its samples in a cycle with the samples in a cycle of sine and cosine reference signals. Then the calculated phasor components are given by:

$$X_x(k) = \frac{2}{N}\sum_{k=0}^{N-1} x(k)\cos\left(\frac{2\pi k}{N}\right), \quad (1)$$

$$X_y(k) = \frac{2}{N}\sum_{k=0}^{N-1} x(k)\sin\left(\frac{2\pi k}{N}\right), \quad (2)$$

The magnitude $|\hat{X}|$ and the phase $\angle X$ of the signal phasor are given by:

$$|\hat{X}| = \sqrt{(X_x)^2 + (X_y)^2}\ , \quad (3)$$

$$\angle X = \arctan\left(\frac{X_y}{X_x}\right). \quad (4)$$

The HDFT algorithm is similar to FDFT algorithm, but it uses only half the window size. Then the phasor components calculated using the HDFT algorithm are given by:

$$X_x(k) = \frac{4}{N}\sum_{k=0}^{N/2-1} x(k)\cos\left(\frac{2\pi k}{N}\right), \quad (5)$$

$$X_y(k) = \frac{4}{N}\sum_{k=0}^{N/2-1} x(k)\sin\left(\frac{2\pi k}{N}\right). \quad (6)$$

The magnitude of the frequency response of the FDFT algorithm and the HDFT algorithm are shown in Fig. 2 and Fig. 3, respectively.

The Fourier algorithms are very popular in filtering harmonic components of a signal by means of very simple calculations. However, the decaying DC component is not readily removed during the estimation process because of its aperiodic behavior and a relatively broad spectrum.

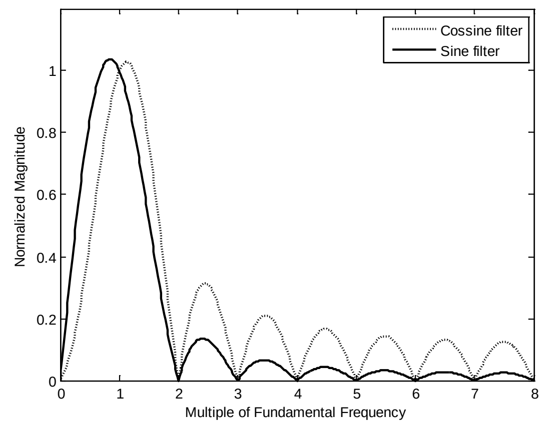

Fig. 2. Magnitude response of the FDFT algorithm.

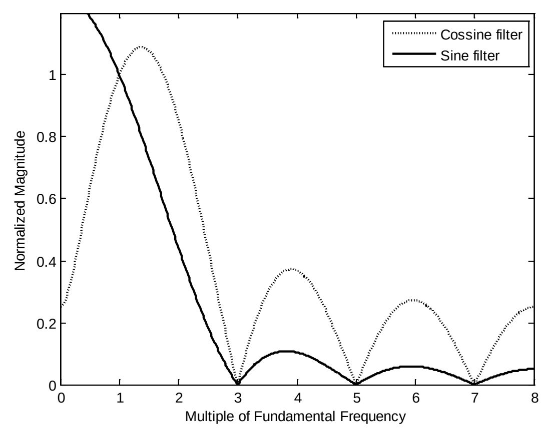

Fig. 3. Magnitude response of the HDFT algorithm.

2) Modified Cosine Filter Algorithm

In reference [10] the authors introduces the Cosine Filter Algorithm, which is based on orthogonal Fourier filter for the current data windows and delayed a quarter cycle.

For the cases shown in [10], the algorithm has good performance in filtering decaying DC component current, but produces a delay of one quarter cycle to estimate the phasor. This delay introduced by the Cosine Filter Algorithm motivated the development of a new version of this filter called Modified Cosine Filter [8], whose equations are shown below:

$$X_x(n) = \frac{2}{N}\sum_{k=0}^{N-1} x(k)\cos\left(\frac{2\pi k}{N}\right), \quad (7)$$

$$X_y(n) = \frac{X_x(n-1)\cos\left(\frac{2\pi}{N}\right) - X_x(n)}{sen\left(\frac{2\pi}{N}\right)}, \qquad (8)$$

where $X_x(n)$ e $X_y(n)$ are the nth sample of components X_x e X_y of the phasor.

3) GUO Algorithm

An algorithm to mitigate decaying DC component by means of a recursive estimation phasor is presented in [9]. This method is based on the following equations:

$$X_x(n) = X_{x\_prev}(n) \qquad (9)$$

$$X_y(n) = X_{y\_prev}(n) + \frac{2}{N}\operatorname{cotg}\left(\frac{\pi}{N}\right)(PS_2 - PS_1), \qquad (10)$$

where:

$$PS_1 = \sum_{k=1}^{N/2} x(2k-1) \qquad (11)$$

$$PS_2 = \sum_{k=1}^{N/2} x(2k) \qquad (12)$$

$$X_{x\_prev}(n) = \frac{2}{N}\sum_{k=0}^{N-1} x(k)\cos\left(\frac{2\pi k}{N}\right) \qquad (13)$$

$$X_{y\_prev}(n) = \frac{2}{N}\sum_{k=0}^{N-1} x(k)\,sen\left(\frac{2\pi k}{N}\right) \qquad (14)$$

III. Proposed Technique

A technique to filter out undesirable frequency components of the phasor signals is proposed here. The scheme of the proposed procedure is shown in the block diagram of Fig. 4.

The signal $x(t)$ is filtered by an optimized analog band-pass filter, followed by an A/D converter , and finally the samples are used for calculating phasor X of signal $x(t)$. DFT Algorithms are used in the calculation step of the components of the signal phasor.

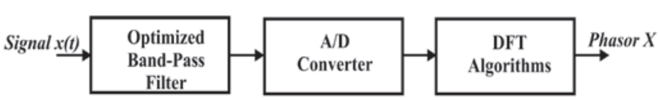

Fig. 4. Proposed procedure scheme.

The goal of the optimized band-pass filter is to prevent aliasing phenomenon and to reduce the non-fundamental frequency components.

A. Optimized Band-Pass Filter Parameters

The optimization process is to find the filter parameters to minimize the merit function, the difference between the desired (ideal) function and the approximated function. The merit function is given by:

$$F(a) = \sum_{i=1}^{N}\left(\frac{Y_i - H(f_i; a_1, \ldots, a_k)}{\sigma_i}\right)^2, \qquad (15)$$

where:

Y_i is the ith sample of the set of N data points of a guide function (samples of the functions that have the desired characteristics for the band-pass filter);

f_i is the ith value of variable data set (in this case, the frequency data points);

$H(f_i; a_1, \ldots, a_k)$ is the ith sample of the approximate functions with k parameters to be determined;

σ_i is the standard deviation of the data set;

a is the vector containing the parameters of the approximate function.

The approximate filter function H can be written as a rational function with the numerator and denominator polynomials in the s plane ($s = 2\pi fj$). The function H is written as:

$$H(s) = a_1 \prod_{n=1}^{m} \frac{s}{a_{2n}s^2 + a_{2n+1}s + 1}, \qquad (16)$$

where a_k are parameters of the approximate function and m is the number of rational function products.

Note the Y_i is a guide function only. Here, the magnitude of Y_i points were chosen to be a bell shaped Gaussian curve and Y_i phase was chosen to be linear.

The Levenberg-Marquardt Method is used in a computational routine to minimize and find the parameters of function H. This routine is coded in Matlab®.

The calculated parameters of H are shown in Table I, and the curves obtained for the magnitude and phase response of the optimized pass-band filter are shown in Fig. 5 and Fig. 6. The magnitude and phase are plotted as a function of multiples of the fundamental frequency of the signal.

The optimization process was performed for various rational function products, however single rational transfer function has shown to produce fairly good results.

TABLE I
PARAMETERS OBTAINED BY THE OPTIMIZATION

Parameters a_k		
$k = 1$	$k = 2$	$k = 3$
3.0857×10^{-3}	9.1239×10^{-6}	2.9837×10^{-3}

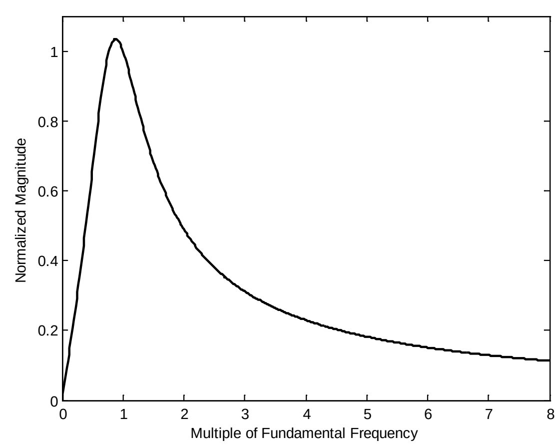

Fig. 5. Magnitude response of the Optimized Band-Pass Filter.

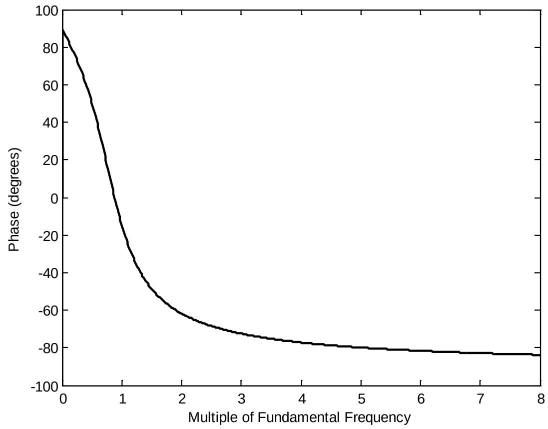

Fig. 6. Phase response of the Optimized Band-Pass Filter.

B. Optimized Band-Pass Filter with DFT Algorithms

To illustrate the performance of the frequency response of the combination of the Optimized Band-pass Filter (OBP), and the Fourier Algorithms (FDFT and HDFT), the frequency response of the transfer function OBP was multiplied by the transfer functions of FDFT Filters (Fig. 7) and HDFT Filters (Fig. 8).

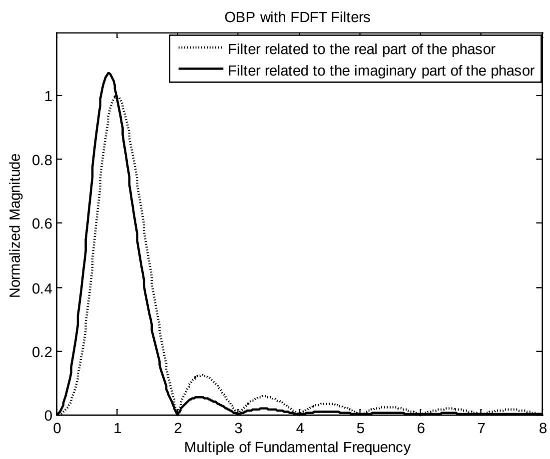

Fig. 7. Magnitude response of the combination: OPB filter with FDFT filters.

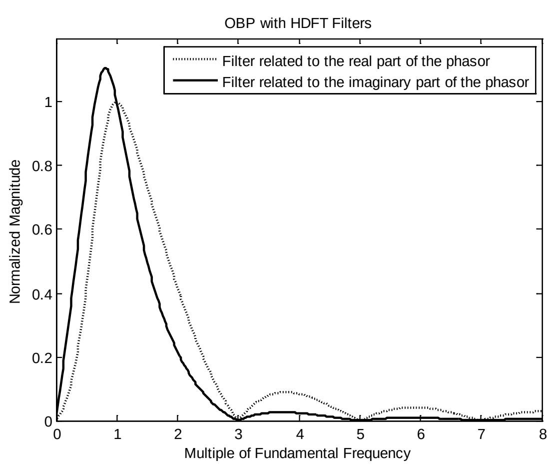

Fig. 8. Magnitude response of the combination: OPB filter with HDFT filters.

At a first glance, it can be seen from Fig. 7 and Fig. 8 that Fourier Filters perform better is preceded by the proposed band-pass filter. The filtering procedure is very selective in order to efficiently filter out the harmonic components and interharmonics.

IV. Evaluation Technique

In this section comparisons are made in terms of frequency and time response between the proposed technique and classical phasor estimation methods.

At this stage, Matlab® is used to implement filtering techniques and to obtain their frequency responses. The time-domain analytical signals coded on Matlab® are used for phasor estimation.

A. Frequency Response

The filtering scheme of the classical phasor estimation methods is shown in Fig. 9.

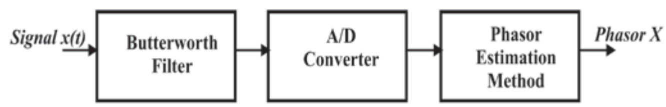

Fig. 9. The filtering scheme of phasor estimation methods.

Reference [7] uses the principles of electromechanical and static relays to devise a filtering algorithm to be used in digital relays to mitigate the decaying DC component, which was called Digital Mimic Filter (DMF).

The frequency response are plotted for several combination of filters: Combination 1 – Classical third order Butterworth (with cutoff frequency equal to 180 Hz) plus Digital Mimic Filter with a pre-defined time constant plus FDFT filters; Combination 2 – Classical third order Butterworth (with cutoff frequency equal to 180 Hz) plus mimic filter with a pre-defined time constant plus HDFT filters; Combination 3 - Classical third order Butterworth (with cutoff frequency equal to 180 Hz) plus the Modified Cosine Filter (MDF); and Combination 4 - Classical third order Butterworth (with cutoff frequency equal to 180 Hz) plus Guo Algorithm.

The magnitude frequency response for the Combinations 1, 2, 3 and 4 are shown in Fig. 10, Fig. 11, Fig. 12 and Fig. 13. The results obtained using the proposed technique (Fig. 7 and Fig. 8) are better than those obtained from the classical filtering methods (Figs. 10, 11, 12 and 13) in terms of filtering out interharmonics components.

It can be seen in Fig. 10, 11, 12 and 13 that the frequency response of Combinations 1, 2, 3 and 4 presents a higher gain in the high order frequency components. This can lead to errors in the phasor estimation when present these components.

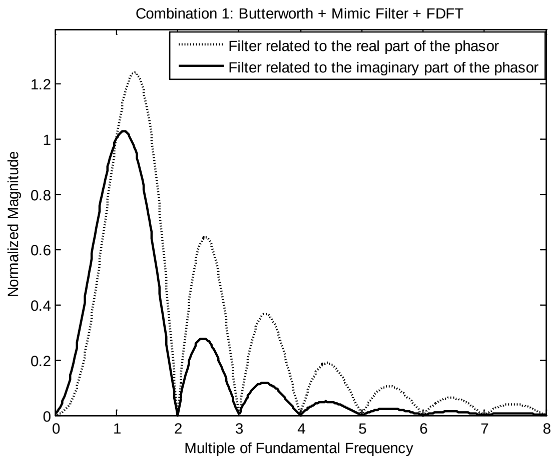

Fig. 10. Magnitude response of the Combination 1.

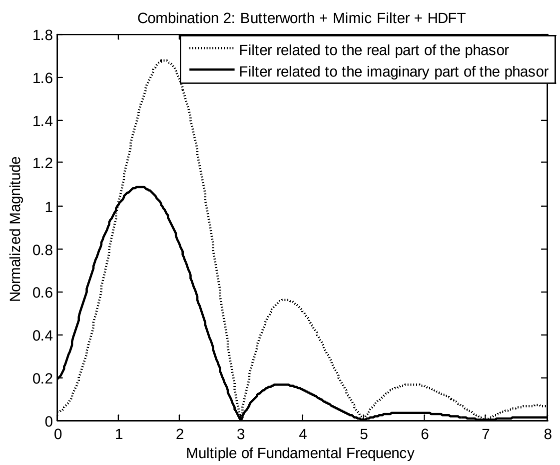

Fig. 11. Magnitude response of the Combination 2.

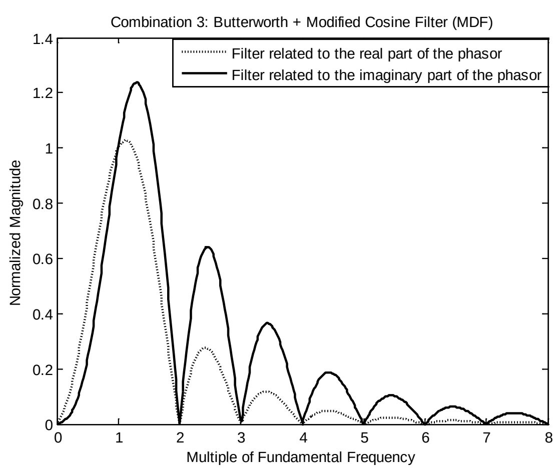

Fig. 12. Magnitude response of the Combination 3.

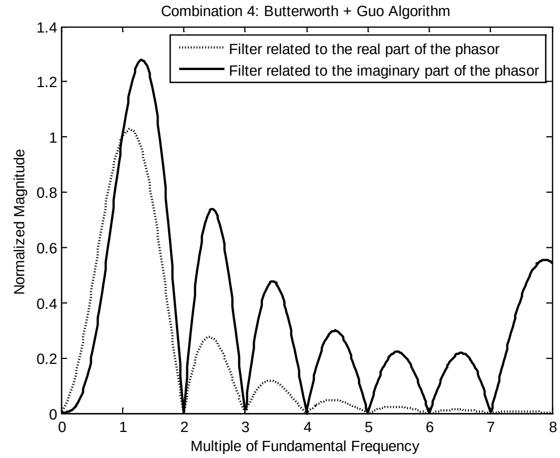

Fig. 13. Magnitude response of the Combination 4.

B. Time Response

Analytical signals containing the fundamental component and non-fundamental components as harmonic components, interharmonics and decaying DC component were employed for performing phasor estimation by the proposed technique and the classical methods.

Three analytical signals are used here for phasor estimation: a signal containing fundamental and harmonic components ($x_1(t)$), a signal containing the fundamental component and interharmonics ($x_2(t)$) and a signal containing fundamental component and DC decaying exponential component ($x_3(t)$):

$$x_1(t) = \cos(2\pi 60t) + \frac{1}{h}\sum_{h=2}^{8}\cos(2\pi 60ht)] \quad (17)$$

$$x_2(t) = \cos(2\pi 60t) + \frac{1}{(h/2)}\sum_{h=3,5,7}\cos(2\pi 60(h/2)t] \quad (18)$$

$$x_3(t) = e^{-t/\tau} + \cos(2\pi 60t), \quad (19)$$

where τ is one cycle of the fundamental frequency.

The phasor magnitude of the signal x_1 calculated using classical methods (Combinations 1, 2, 3, and 4) are shown in Fig. 14, and the magnitude calculated by the proposed technique is shown in Fig. 15.

From the results shown in Figs. 14 and Fig. 15 we can see that the performance of the OBP with HDFT Filters (Fig. 15) is better than the performance of the Combination 2 and Combination 4 (Fig. 14) in filtering out the non-fundamental components.

The phasor magnitude of the signal x_2 calculated from classical methods (Combinations 1, 2, 3, and 4) are shown in Fig. 16, and the magnitude calculated by the proposed method is shown in Fig. 17.

From the results shown in Figs. 16 and Fig. 17 we can see that the performance of the OBP with HDFT Filters and OBP with FDFT Filters (Fig. 17) are better than the performance of the Classical Methods (Fig. 16) in filtering out the interharmonics components.

In Fig. 18 is shown the phasor magnitude of the signal x_3 calculated from classical methods (Combinations 1, 2, 3, and 4), and the magnitude calculated by the proposed method is shown in Fig. 19. We can see that the proposed technique has a smoother response at the first instants of time.

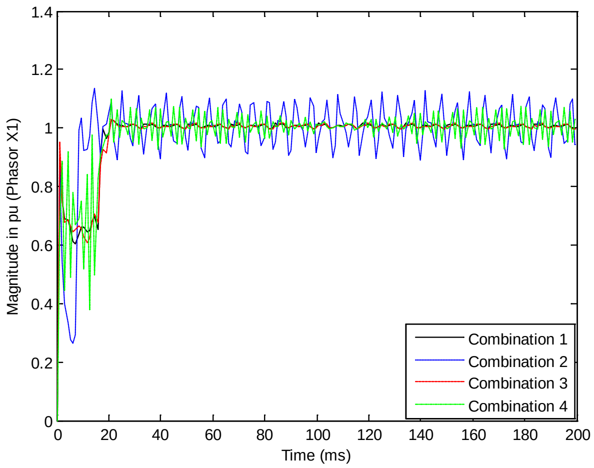

Fig. 14. Fundamental phasor magnitude of the signal x_1 (Classical methods).

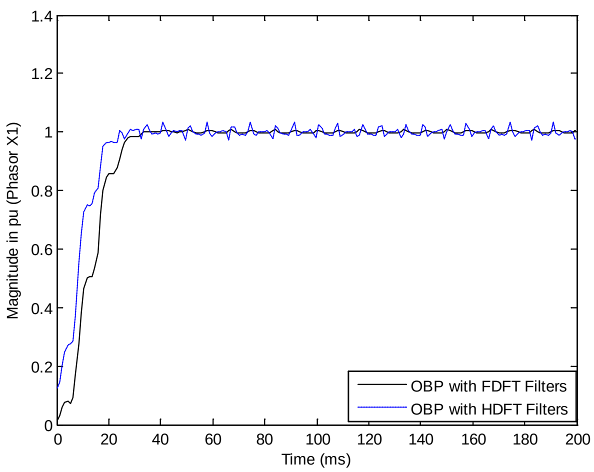

Fig. 15. Fundamental phasor magnitude of the signal x_1 (Proposed technique).

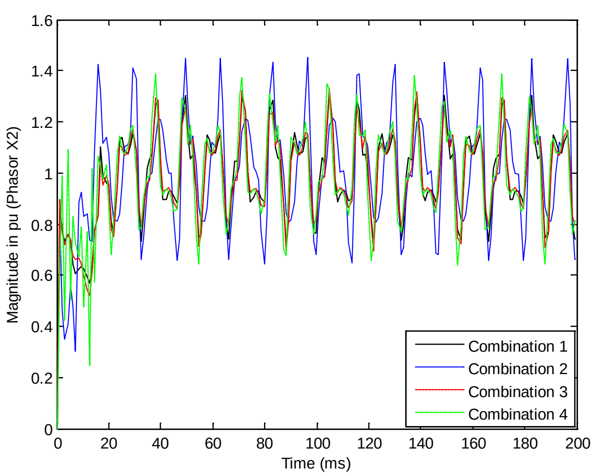

Fig. 16. Fundamental phasor magnitude of the signal x_2 (Classical methods).

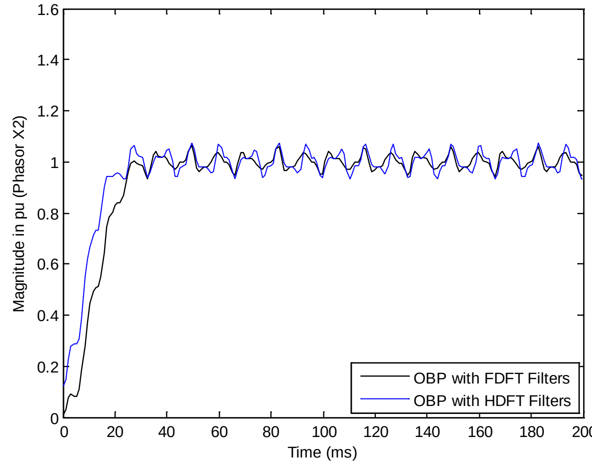

Fig. 17. Fundamental phasor magnitude of the signal x_2 (Proposed technique).

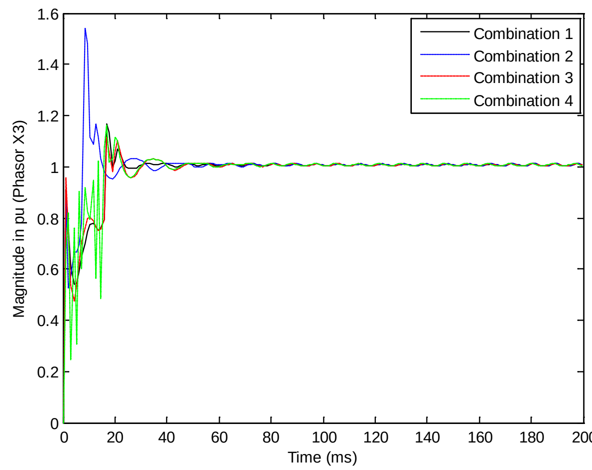

Fig. 18. Fundamental phasor magnitude of the signal x_3 (Classical methods).

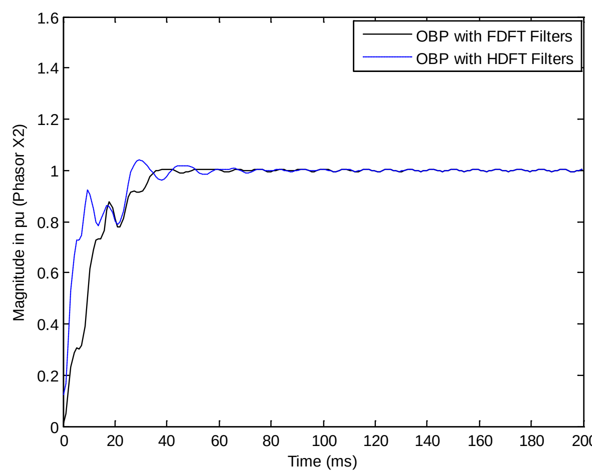

Fig. 19. Fundamental phasor magnitude of the signal x_3 (Proposed technique).

The work developed here is just an initial study regarding to the use of a band-pass filter in connection with Fourier filters with aim to improve phasor estimation.

It should be pointed out that narrower passband resulted in the higher the setting response time [11]. Further investigations are needed to determine the pass-band filter parameters that produce simultaneously good frequency response with suitable setting time.

V. ACKNOWLEDGEMENTS

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VI. CONCLUSIONS

In this paper a technique for calculating the fundamental component of the phasor signals from the power electrical system is presented. A pass-band filter obtained from an optimization process with the Fourier filters are used in this technique. As a result, the proposed technique is less sensitive to non-fundamental frequency components.

The performance of the proposed technique was compared with commonly used algorithms reported in the literature. The obtained results indicate that the proposed technique showed better frequency response when compared with results obtained from other filtering methods.

Although the proposed technique produces good frequency responses, a thorough investigation is required to determine the optimal band-pass filter that produce simultaneously good frequency response with suitable setting time.

VII. REFERENCES

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