Comparative Solutions of Numerical Oscillations in the Trapezoidal Method used by EMTP-based Programs

L. F. R. Ferreira, B. D. Bonatto, J. R. Cogo, N. C. de Jesus, H. W. Dommel and J. R. Martí

Abstract--This paper presents comparative solutions of different algorithms to the problem of numerical oscillations, which are caused by the trapezoidal integration rule, used by EMTP-based programs. These numerical oscillations emerge mostly from the abrupt variation of the voltage between the terminals of a capacitor, or from the abrupt current change in inductors, or even when non-linear elements, with piece-wise linearized models swap among linear regions. The importance of this review work stands on the exact understanding of the numerical oscillations problem and the evaluation of different methods proposed to solve this problem. Indeed, several approaches have been developed, but not all of them are really effective and only someones have been implemented in public or commercial software to eliminate numerical oscillations.

Keywords: EMTP, numerical oscillation, trapezoidal integration method, electromagnetic transients, computer modelling, simulation.

I. INTRODUCTION

THE Electromagnetics Transients Program (EMTP) is one of the most versatile and important tools for studies in the continuous-time domain related to electrical systems, such as transmission lines projects, insulation coordination and fault analysis. Back in the 60s, a series of papers, originated the EMTP [1]-[2], and since then various others researches contributed to improvements and development of new models. Nowadays, several EMTP-based programs are available for studies and research in academia and professional use in the electricity industry, as for instance: ATP [3], MicroTran [4], EMTP-RV [5], eMEGAsim and HYPERsim [6], RTDS [7]. Therefore, it is essential for electrical engineers and other users to fully comprehend both the potentialities and the limitations of the fundamental algorithm of the EMTP. The correct interpretation of physical phenomena in electrical systems must be clearly distinguished from numerical problems from the computational simulation scheme, therefore justifying this paper.

The strategy used in the EMTP algorithm to solve any circuit is to transform the continuous-time differential equations, which describes the behavior of the system under simulation, into discrete-time difference equations with solutions in time intervals of fixed time-step size Δt. The trapezoidal integration rule was preferred by its accuracy and stability [8]-[9] added to its simplicity in computational code, when compared to other existing integration rules.

However, it is intrinsic to the trapezoidal integration rule the possibility of introducing artificial numerical oscillation (non-existent in the analytic response) in the time domain simulation. These numerical oscillations may harmfully interfere the correct interpretation of the simulation response when the user is unaware of its causes and possible solutions. Specifically, some variables of the circuit oscillate around the correct value in a way that the average at each two points is the correct answer. These numerical oscillations are essentially caused by two situations [9]-[10]. The first occurs in the switch (between linearized regions) of non-linear elements, when used the piece-wise linearization, given the fact there is an abrupt change in the digital equivalent conductance of the element. The second situation is due to switching operation, causing a modification in the system's topology and, consequently, abruptly changing the inductor's current and/or the capacitor's voltage.

Several strategies were proposed and developed aiming to solve the problem of numerical oscillations in EMTP-based programs, but not all of them are really effective and only someones were implemented in the available software. This paper presents comparative solutions of different algorithms to the problem of numerical oscillations, which are caused by the trapezoidal integration rule, used by computer programs based on the EMTP.

One possibility is to average every two points of the output data, after completion of the simulation, technique also called output averaging.

Another strategy is to restart the simulation (re-solve) taking as initial conditions the state immediately preceding the beginning of the oscillation in the system.

The Alternative Transients Program (ATP) adopts the strategy of introducing a damping resistor ($R_P = K_P \cdot 2L/\Delta t$, where Δt is time step size) in parallel with inductances, or a resistor ($R_S = K_S \cdot \Delta t/2C$) in series with capacitances. The insertion of this artificial resistor is easy to implement in the

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program, and results in a gradual damping of the oscillations, but it also introduces a distortion in the response throughout the whole simulation.

The use of Backward Euler integration method along all simulation time, results in no occurrence of numerical oscillations, but this method has not a good precision as the trapezoidal, and also introduces large phase distortion [12]-[13].

The CDA - Critical Damping Adjustment technique proposed in [10]-[11] changes temporarily the time-integration method from trapezoidal to Backward Euler during two time steps sizes of $\Delta t/2$, when specific conditions that may cause numerical oscillations are detected. After two numerical solutions, solved with $\Delta t/2$, the program returns to the trapezoidal method for better precision in the remaining of the simulations. The CDA technique has been implemented, for example, in the software Microtran [11].

Another technique, equivalent to the CDA, was proposed by J. R. Cogo and H. W. Dommel. Instead of changing the method of integration, even temporarily with the CDA technique, this proposal is to use the average of the last two historical source values calculated for the trapezoidal method, as the historical source for the next iteration, during two subsequent half-steps, whenever there is occurrence of conditions for oscillations. This technique, for now called THTA – Trapezoidal History Term Averaging seems to be equivalent to the CDA technique. With this, it is guaranteed the advantages of accuracy by using the trapezoidal integration method, and eliminating numerical oscillations without need to change to the Backward Euler method as in the CDA strategy.

Therefore, this paper reviews the main causes of numerical oscillations in the trapezoidal method and provides a comparative analysis among some algorithms available to eliminate such problem in EMTP-based programs.

II. EMTP DIGITAL COMPONENTS MODELING

The component modeling approach adopted in the EMTP is to transform the continuous-time differential equations to discrete-time difference equations. This transformation is made using an integration rule, typically the Trapezoidal or Backward Euler methods.

A. Modeling using the Trapezoidal Integration Rule

In the case of an inductor, one can obtain its digital model starting from its basic equation [2], as follows:

$$v(t) = L\frac{di(t)}{dt} \rightarrow \int_{t-\Delta t}^{t} v(\tau)\, d\tau = L\int_{t-\Delta t}^{t} di(\tau) \qquad (1)$$

The first integral represents the area below the curve $v(t)$ from $t - \Delta t$ until t. Applying the trapezoidal rule to it, one may approximate to a trapezoid-like area:

$$\int_{t-\Delta t}^{t} v(\tau)\, d\tau = \text{Area} \cong \frac{[v(t-\Delta t)+v(t)]\,\Delta t}{2} \qquad (2)$$

The Thévenin equivalent discrete-time inductor model

recurrence formula is:

$$v_L(t) = \frac{2L}{\Delta t}.i_L(t) + \left[-\frac{2L}{\Delta t}.i_L(t-\Delta t) - v_L(t-\Delta t)\right] \qquad (3)$$

The EMTP algorithm uses the nodal method to solve the digital equivalent circuits [2]. It is necessary to convert (3) to obtain the "Norton discrete-time equivalent model" of an inductor, expressed in (4) and (5).

$$i_L(t) = \frac{\Delta t}{2L}.v_L(t) - i_{h_{Tr}}(t) \qquad (4)$$

$$i_{h_{Tr}}(t) = \left[-i_L(t-\Delta t) - \frac{\Delta t}{2L}.v_L(t-\Delta t)\right] \qquad (5)$$

Fig. 1 shows the graphical representation of the three models for an inductor. Analogous derivation can be done for a capacitor.
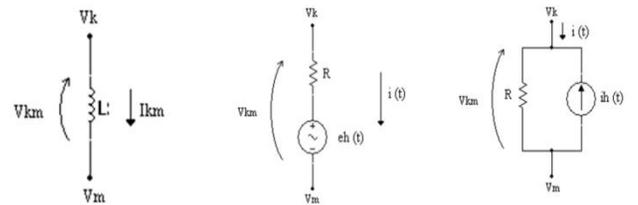
Fig. 1. Inductor and its "Thévenin and Norton discrete-time equivalent models".

Several techniques intend to minimize the numerical oscillations caused by the use of the trapezoidal rule under certain conditions, mentioned in the introduction. Here it is presented a comprehensive analysis of two techniques which completely eliminates the numerical oscillations in an effective way, and in the shortest time possible - the CDA and THTA.

B. Backward Euler's discrete model of an inductor

The CDA scheme uses the Backward Euler (BE) rule temporarily. Therefore, it requires a new approximation for the area below the curve $v(t)$. Equation (7) shows the "Thévenin discrete equivalent model" for an inductor with BE rule, while (8) and (9) shows the "Norton's discrete equivalent model".

$$\int_{t-\Delta t}^{t} v(\tau)\, d\tau = \text{Area} \cong v(t).\Delta t \qquad (6)$$

$$v(t) = \frac{L}{\Delta t}.i_L(t) + \left[-\frac{L}{\Delta t}.i_L(t-\Delta t)\right] \qquad (7)$$

$$i_L(t) = \frac{\Delta t}{L}.v(t) - i_{h_{BE}}(t) \qquad (8)$$

$$i_{h_{BE}}(t) = -i_L(t-\Delta t) \qquad (9)$$

C. Comparative results between the Trapezoidal and Backward Euler methods

It is now important to compare the two integration method when faced to a situation that causes a numerical oscillation.

Let the circuit in the Fig. 2 be the most didactic example. The step current of 1 [A] is applied in a 1 [H] inductor. The circuit is initially de-energized.
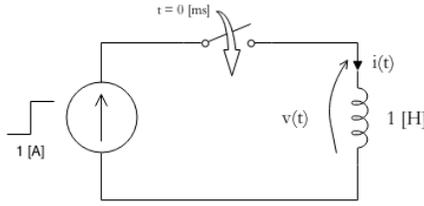
Fig. 2. Voltage and current at the inductor under analysis.

Analyzing the example, it is known from the electromagnetic transients theory that, facing an abrupt current change, the inductor generates an voltage impulse on its terminals and after that the voltage falls to zero. To make the analysis easier, the variable of interest v(t) is presented in (10)-(13).

	Trapezoidal Rule	
	$v_L(t) = \dfrac{2L}{\Delta t} \cdot i_L(t) + \dfrac{2L}{\Delta t} \cdot i_h(t)$	(10)
	$i_h(t) = \left[-i_L(t - \Delta t) - \dfrac{\Delta t}{2L} \cdot v_L(t - \Delta t) \right]$	(11)
	Backward Euler Rule	
	$v_L(t) = \dfrac{L}{\Delta t} \cdot i_L(t) + \dfrac{L}{\Delta t} \cdot i_h(t)$	(12)
	$i_h(t) = -i_L(t - \Delta t)$	(13)

Table I presents the results of five iterations following the EMTP fundamental algorithm with (10)-(13). A graphical representation is shown in Fig. 2.

One can fill the Table I using the trapezoidal rule following the steps:

a. At the time $t = 0$ all the variables are null, considering the circuit as initially de-energized.

b. At $t = 1\,\Delta t$, $I_{h_{Tr}}(\Delta t)$ is calculated using the recurrence formulae (11) and (10):

$$i_{h_{Tr}}(\Delta t) = \left[-i_L(0) - \frac{\Delta t}{2L} \cdot v_L(0) \right] = 0$$

$$v_{L_{Tr}}(\Delta t) = \frac{2L}{\Delta t} \cdot 1 + 0 = +\frac{2L}{\Delta t}$$

a. The next step, at $t = 2\Delta t$, $v_{L_{Tr}}(2\,\Delta t)$ becomes:

$$i_{h_{Tr}}(2\,\Delta t) = \left[-1 - \frac{\Delta t}{2L} \cdot \frac{2L}{\Delta t} \right] = -2$$

$$v_{L_{Tr}}(2\,\Delta t) = \frac{2L}{\Delta t} \cdot 1 + \frac{2L}{\Delta t} \cdot (-2) = -\frac{2L}{\Delta t}$$

b. Repeat until the end of the simulation.

TABLE I
CALCULATION OF V_L USING THE TRAPEZOIDAL AND BACKWARD EULER METHODS.

t	$i_L(t)$	$I_h(t)$ Tr	$v_L(t)$ Tr	$I_h(t)$ BE	$v_L(t)$ BE
0	0	0	0	0	0
1 Δt	1	0	$+2L/\Delta t$	0	$L/\Delta t$
2 Δt	1	−2	$-2L/\Delta t$	−1	0
3 Δt	1	0	$+2L/\Delta t$	−1	0
4 Δt	1	−2	$-2L/\Delta t$	−1	0
5 Δt	1	0	$+2L/\Delta t$	−1	0

To calculate the two remaining columns, with the results for the Backward Euler rules, use the recurrence formulae (13) and (12). The results in the Table I shows that the numerical oscillation exist only with the use of trapezoidal method. It also is sustained and alternates between $+2L/\Delta t$ and $-2L/\Delta t$, showing that the oscillation is not damped and will continue with these values until the end of the simulation or another change in the circuit happens. Reducing the time step size does not solve numerical oscillations. The results also show that numerical oscillations always assume values around the correct value, following that the average at each two points is the correct value. A capacitor presents similar behavior when confronted with a voltage step, but now the numerical oscillation occurs in current variable.
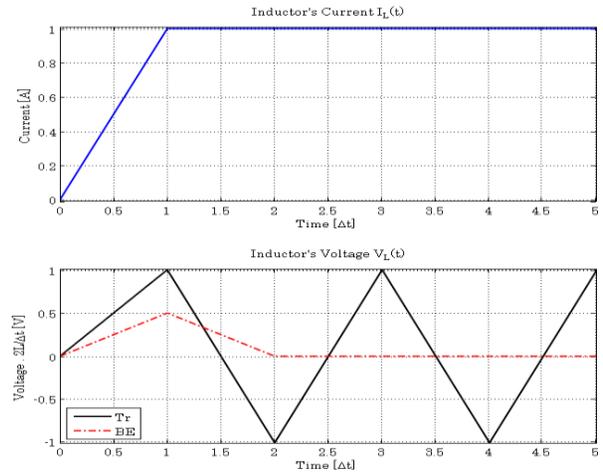
Fig. 3. Current across and Voltage at the terminals of the inductor.

Fig. 3 and the last two columns in Table I also show that the Backward Euler method is critically damped. It needs just one more iteration after the current step happens to bring the voltage back to 0 [V], the correct value for $t \geq 2\,\Delta t$. The comparison between (11) and (13) shows the exact origin of the numerical oscillations. The trapezoidal method requires the inductor´s voltage in the last step in order to calculate the historical current. If, at any given time, the conditions to start numerical oscillations are met, which in this case is a step-change in the current, the inductor produces a voltage impulse. And then, the term $-(\frac{\Delta t}{2L}) \cdot v_L(t - \Delta t)$ carries this impulse through all the rest of the simulation. Also, the negative sign makes the result oscillate around the correct value. Therefore, the problem of numerical oscillation is inherent from the trapezoidal method, and an efficient solution requires an attack at the root of the cause.

III. THE CDA AND THTA SCHEMES TO ELIMINATE NUMERICAL OSCILLATIONS

A. Critical Damping Adjustment - CDA

The CDA technique uses the Backward Euler rule to eliminate the numerical oscillation, taking advantage of its critically damped characteristic. In order to avoid changes in the conductance matrix, the CDA scheme runs for two integration steps of $\Delta t/2$, which brings the same equivalent conductance for all the elements in the circuit and no modification in the conductance matrix needs to be done, which does save computational effort [10]. The limitation for just two time step integration is due to fact that the Backward Euler introduces a phase shift in the simulation result at the long run, and also because the trapezoidal method is much more accurate for the same Δt. An important comparison study between the two methods is developed in [10].

B. Trapezoidal History Term Averaging - THTA

Professor Dommel, during a graduation course at The University of British Columbia, mentioned another possibility to eliminate numerical oscillations, while continue using the trapezoidal method, based on some ideas proposed by Professor João Roberto Cogo from UNIFEI. This method, freely denominated by Prof. Bonatto as "THTA-Trapezoidal History Term Averaging" consists in the determination of the average of the history source values, during two time integration steps of $\Delta t/2$, as in (14)-(15):

$$I_{THTA}(t) = \frac{I_{h_{Tr}}\left(t - \frac{\Delta t}{2}\right) + I_{h_{Tr}}(t)}{2} \qquad (14)$$

$$v_{L_{THTA}}(t) = \frac{2L}{\Delta t} \cdot i_L(t) + \frac{2L}{\Delta t} \cdot I_{THTA}(t) \qquad (15)$$

Equation (14) requires the calculation of the value of the history current source in time t. After that, the average is computed and this value is used as the new history source value. This technique seems to be equivalent to CDA, with the computational advantage of keep using the trapezoidal method during all the simulation. For this the program needs to store the last history vector in memory in case a numerical oscillation arises. Table II provides a step-by-step solution for the didactic circuit of Fig. 2 using this technique, and Fig. 4 shows its graphical representation.

To fill the table, use the following script:
a. Initially, all the variables are null, given the circuit is de-energized.
b. The current in the inductor is 1 [A] starting from the first solution until the end of the simulation.
c. Calculate the history source $I_{h_{Tr}}\left(\frac{\Delta t}{2}\right)$ using (11).
d. Calculate $I_{THTA}\left(\frac{\Delta t}{2}\right)$ and $v_L\left(\frac{\Delta t}{2}\right)$ using (14) and (15). This is the first step of the THTA.

$$I_{THTA}\left(\frac{\Delta t}{2}\right) = \frac{I_{h_{Tr}}(0) + I_{h_{Tr}}\left(\frac{\Delta t}{2}\right)}{2} = \frac{0 + 0}{2} = 0$$

$$v_{L_{THTA}}\left(\frac{\Delta t}{2}\right) = \frac{2L}{\Delta t} \cdot i_L\left(\frac{\Delta t}{2}\right) + \frac{2L}{\Delta t} \cdot i_{THTA}\left(\frac{\Delta t}{2}\right) = \frac{2L}{\Delta t}$$

e. Repeat the steps (c) and (d) for the second step of THTA.

$$i_{h_{Tr}}(\Delta t) = \left[-i_L\left(\frac{\Delta t}{2}\right) - \frac{\Delta t}{2L} \cdot v_L\left(\frac{\Delta t}{2}\right)\right] = -1 - \frac{\Delta t}{2L} \cdot \frac{2L}{\Delta t} = -2$$

$$I_{THTA}(\Delta t) = \frac{I_{h_{Tr}}\left(\frac{\Delta t}{2}\right) + I_{h_{Tr}}(\Delta t)}{2} = \frac{0 - 2}{2} = -1$$

$$v_{L_{THTA}}(\Delta t) = \frac{2L}{\Delta t} \cdot i_L(\Delta t) + \frac{2L}{\Delta t} \cdot i_{THTA}(\Delta t) = \frac{2L}{\Delta t} - 1 \cdot \frac{2L}{\Delta t} = 0$$

f. Continue with the EMTP solution method using the trapezoidal method.

TABLE II
COMPARATIVE ANALYSIS OF V_L USING THTA AND CDA.

t	$i_L(t)$	$I_h(t)$ Tr	$I_{THTA}(t)$	$v_L(t)$ THTA	$I_h(t)$ BE	$v_L(t)$ CDA
0	0	0	−	0	0	0
$\Delta t/2$	1	0	$\frac{0+0}{2} = 0$	$\frac{2L}{\Delta t}$	0	$\frac{2L}{\Delta t}$
1 Δt	1	−2	$\frac{0-2}{2} = -1$	0	−1	0
2 Δt	1	−1	−	0	−1	0
3 Δt	1	−1	−	0	−1	0
4 Δt	1	−1	−	0	−1	0
5 Δt	1	−1	−	0	−1	0

Fig. 4. Voltage and current at the inductor. Numerical oscillations are eliminated.

One can see, while observing the Table II and Fig. 4, that CDA and THTA are equivalent, considering that the area below the impulse voltage is equal to L. The MicroTran [4] package removes the peak voltage from the final answer considering this as an intermediate solution step, and possibly due that an extremely high frequency event would be quickly damped in a realistic power system [11].

For a practical verification of this technique, an educational computer program based in the EMTP algorithm was developed in the Matlab platform. The simulation results for some selected circuits obtained from this program are shown in the next section.

IV. SIMULATION RESULTS AND DISCUSSIONS

The first case presents the simulation of a simple RL circuit powered by a 10 [V] sinusoidal source, as show in Fig. 5. The switch will open after the current passes through zero and numerical oscillations will then arise in the inductor´s voltage. The second case shows another didactic example, where numerical oscillations will start two times, showing the need to effectively eliminate these oscillations to ensure the reliability of the results.

A. Test Circuit 1
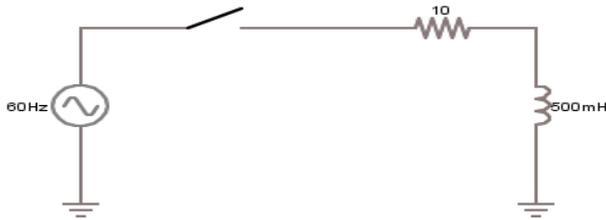
Fig. 5. Simple RL circuit and switch powered by a sinusoidal source.

The switch is scheduled to open at $t = 5$ [ms], but its effective opening happened at $t = 8,2$ [ms] as shown in Fig. 6. This is the moment when the current crosses zero and the switch model allows the interruption of the current [8]-[9].
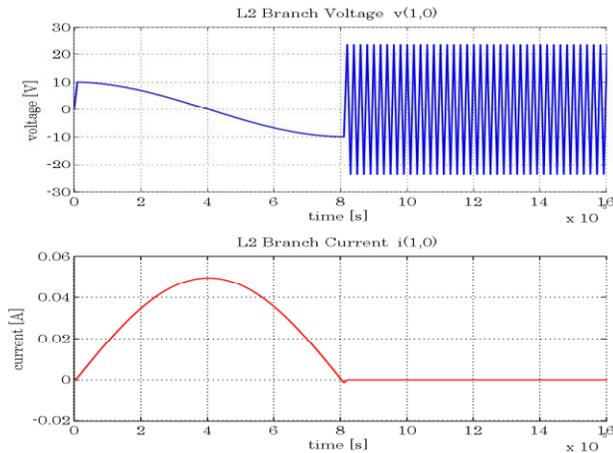
Fig. 6. Voltage and current in the inductor using the trapezoidal method.

The current interruption causes an abrupt change in the rate of change of the current. The inductor responds that with a voltage impulse on its terminals, trying to maintain the current flow. The trapezoidal method, based in (10)-(11), uses information of both current and voltage in the previous step to calculate the new current and voltage. The starting of the numerical oscillations happens exactly because of this dependency of the voltage history value in the previous step, which is an impulse, right after the opening of the switch. As

seen in (11), there is a variation in the voltage at each step because the value of the voltage impulse is added or subtracted from the correct value. In the present example, the expected voltage value is 0 [V], while the results obtained with the trapezoidal method are $+23.8$ and -23.8 [V]. The average of the two values is exactly 0 [V].

The next analyses are restricted to the voltages because this is the variable with possible numerical oscillation. In a similar analysis, the current across a capacitor may present the same characteristics.

The first alternative to remove the numerical oscillations is to use a moving average of two points, called "output averaging". The results using this technique are presented in Fig. 7.

Observe that the plot clearly shows the voltage impulse on the terminals of the inductor. The voltage impulse of 6,93 [V] is just representative of an interruption of the current. This technique is not recommended for studies of overvoltages caused by switching operation. Although is a simple technique requiring only the post-processing of the data output. Also, no modification in the core of the program is required.
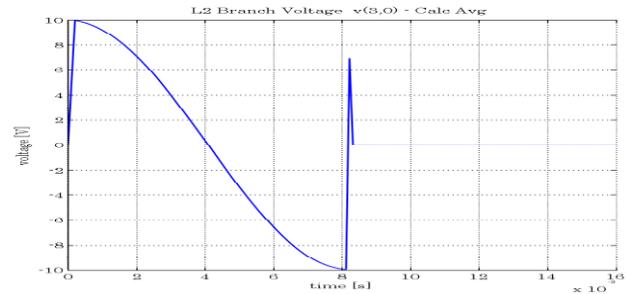
Fig. 7. Output averaging of the voltage.

The software ATP through its graphical interface ATPDraw adopts another strategy, consisting in the insertion of a fictitious resistor connected in parallel to the inductor (and in series with capacitors). This solution causes a damping in the numerical oscillation, which takes some time to disappear.

The speed of the damping may be adjusted by the variable K_P. The ATPDraw documentation recommends K_P values between 5 and 10. The value of the fictitious resistor is calculated by (16).

$$R_P = K_P \cdot 2L/\Delta t \tag{16}$$

For this example,

$$R_P = K_P \cdot \frac{2*500\cdot10^{-3}}{100\cdot10^{-6}} = K_P \cdot 10 \cdot 10^3 \tag{17}$$

Fig. 8 presents the ATP results for $K_P = 7.5$. Fig. 9 shows how the value of the K_P variable changes the damping time. The plots are for $K_P = 5$, 7.5 and 10, respectively.

Lower values of K_P eliminate the numerical oscillations faster, but it inserts an artificial element to the circuit and may lead to incorrect results values in certain studies.
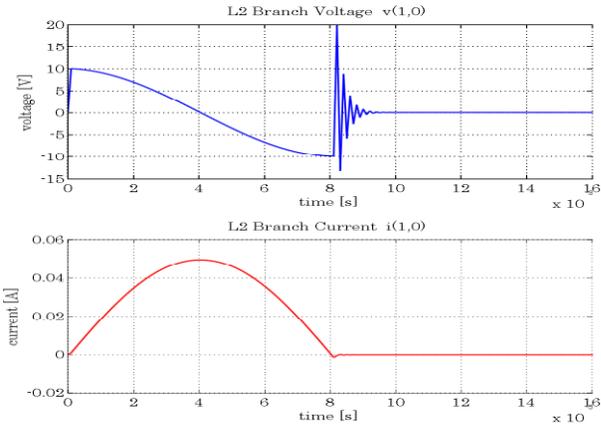
Fig. 8. Damping of numerical oscillations using $K_P = 7.5$.
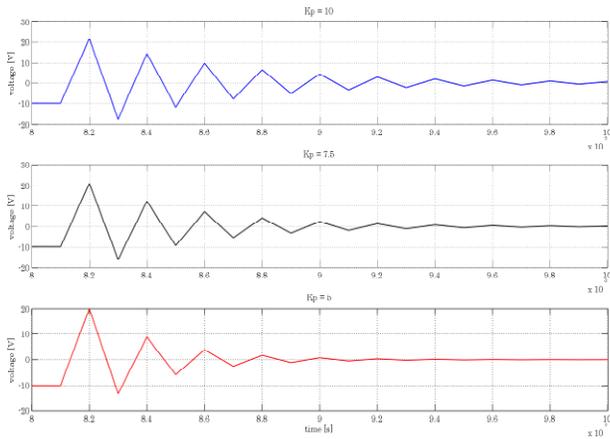
Fig. 9. Comparative damping of numerical oscillations using a parallel resistor.

The Backward Euler integration method offers the best path to remove numerical oscillations, for it is critically damped and totally immune to them. Fig. 10 presents the result for the same test circuit of Fig. 5 using this method.

One of the greatest drawbacks, however, is the phase distortion introduced in the result.

For equivalent errors to the trapezoidal method, it would be necessary to reduce Δt, rising the simulation time considerably in the simulation of complex power systems.
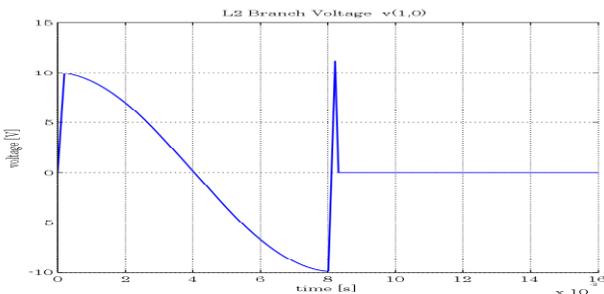
Fig. 10. Simulation using the Backward Euler integration method.

If the Backward Euler rule is used for two steps, the time enough for elimination of the oscillations, together with the Trapezoidal method, for the rest of the simulation, the best results are obtained. This is the case of the CDA scheme,

which is applied every time the conditions for numerical oscillations are met. Fig. 11 shows the result using the CDA technique.
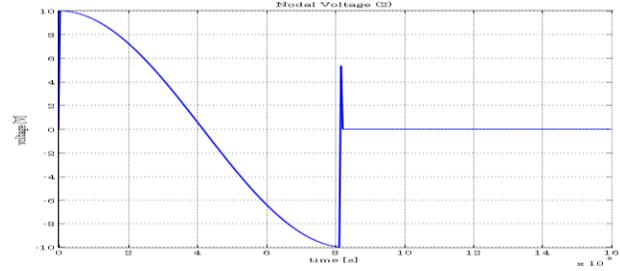
Fig. 11. Computational simulation using the CDA technique.

The voltage peak of $5.35\,[V]$ is the exact same if the simulation was conducted using the THTA scheme, shown in Fig 12.
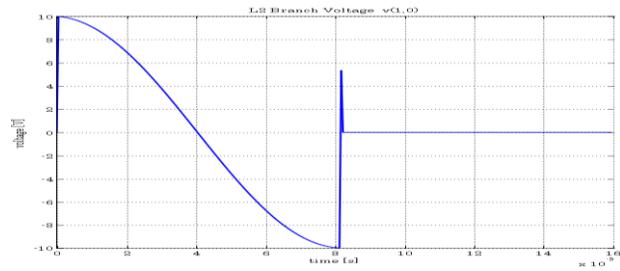
Fig. 12. Computational simulation using the THTA technique.

B. Test Circuit 2

The second circuit solved is one representative of a substation connected to a transmission line with a fault, illustrated in Fig. 13. The problem was adapted from [9]. The switch CH6 is closed at $t = 5[ms]$, simulating the transmission line energization. The switch CH2 is initially closed, acting as a ground fault. To clear the fault and create two situations for numerical oscillations, thus testing the robustness of the proposed THTA technique, both switches are set to open. The first situation is the closing of the switch CH6, putting a charged capacitor in parallel with an uncharged capacitor. The second situation will be the opening of SW2, interrupting current across the line inductor.
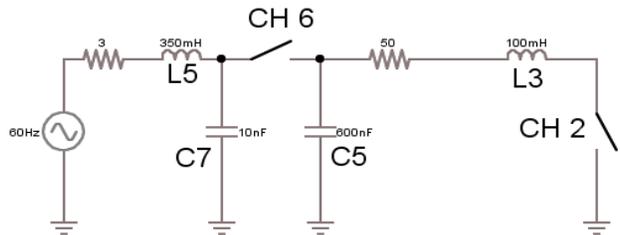
Fig. 13. Substation connected to a transmission line with a fault.

Fig. 14 shows the simulation with the trapezoidal method. The numerical oscillation begins at $t = 5\,[ms]$, instant when the switch CH6 closes and, as a result, the voltage on capacitor C7 was instantaneously applied to C5. Notice that the numerical oscillations in the capacitor current are sustained

and stop only by the opening of both switches, interrupting the supply and the short-circuit.
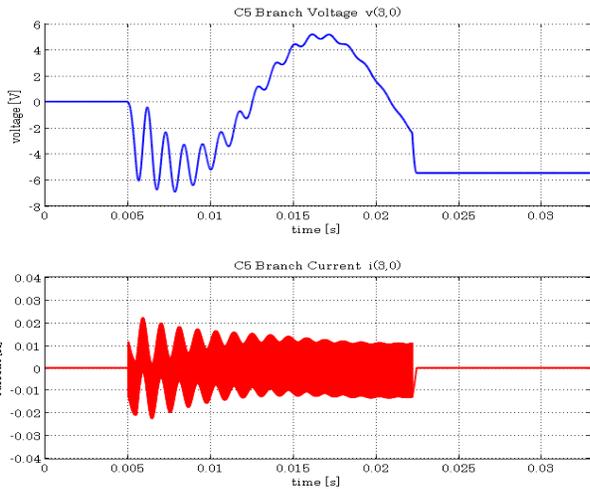
Fig. 14. Computational simulation using the trapezoidal rule.

Fig. 15 shows the "output averaging" of the current on the capacitor C5. Fig. 16 illustrates the use of an artificial resistor to damp the numerical oscillation. Fig. 17 and 18 shows the equivalence between the CDA and THTA techniques.

The second element to be analysed in this educational test case is the inductor, presenting numerical oscillations on its voltage signal after the opening of the switch CH2 at $t = 22.5\ [ms]$, as illustrated in Fig. 19. The values swings around $0\ [V]$, the expected value after the interruption of the current.

Fig. 20 illustrates the output averaging for the inductor's voltage. Fig. 20 shows the effect of an artificial resistor. It is important to know that each artificial resistor must be adjusted by the user via each one of its respective variables K_P and K_S.

Fig. 21 and Fig. 22 illustrate the equivalence for the CDA and THTA techniques, respectively.
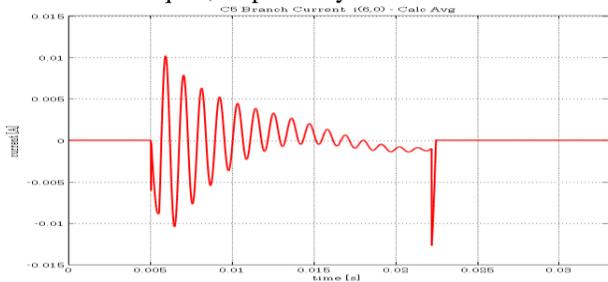
Fig. 15. Averaging of the C5 Capacitor's current.
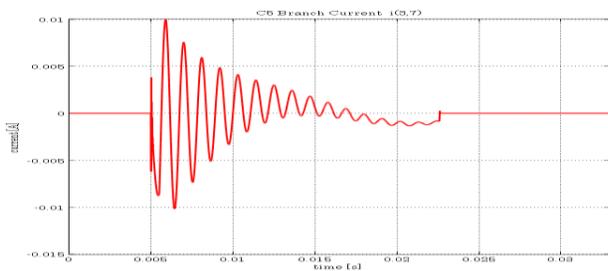
Fig. 16. Current at the C5 capacitor with a damping resistor.
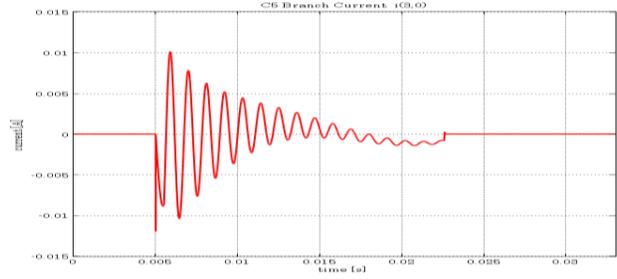
Fig. 17. Current at C5 capacitor using CDA.
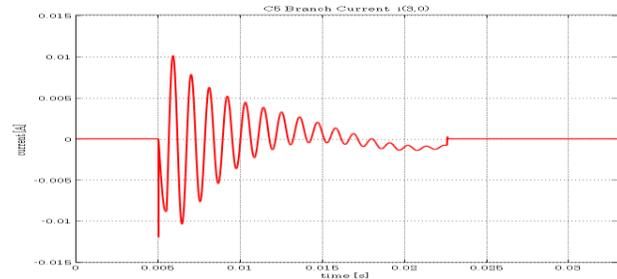
Fig. 18. Current at C5 capacitor using THAT.
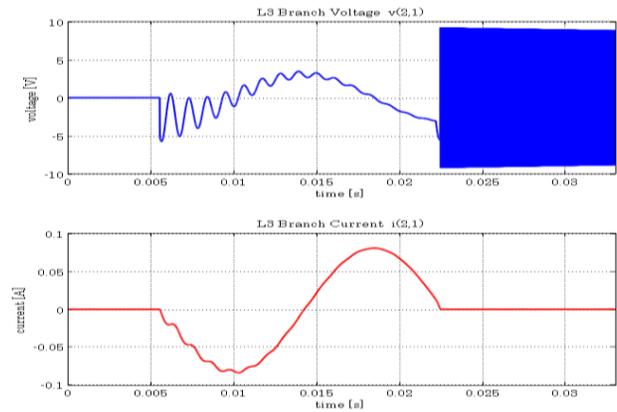
Fig. 19. Voltage and current at L3 inductor, when Trapezoidal method is used.
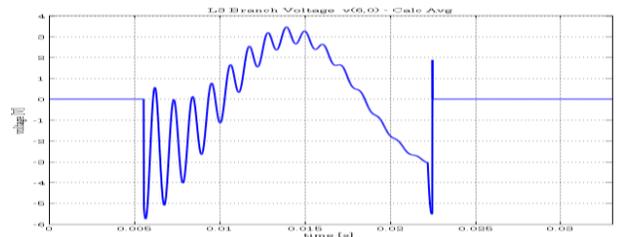
Fig. 20. Output averaged L3 inductor's voltage.
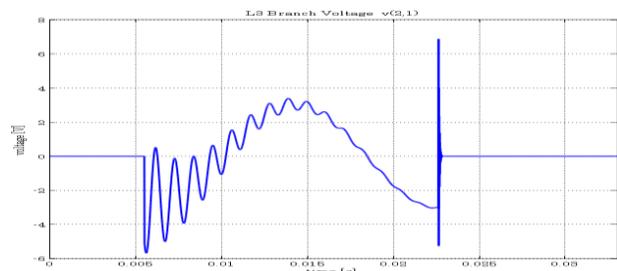
Fig. 21. Voltage at L3 inductor damped by a fictitious resistor.
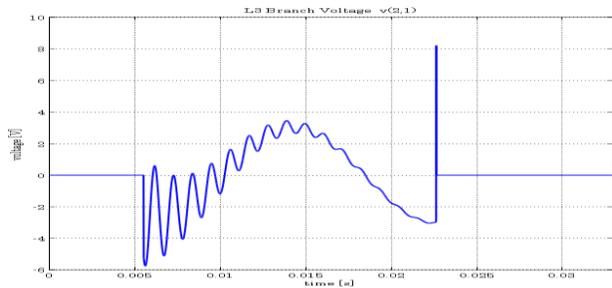
Fig. 22. Voltage at L3 inductor using CDA.
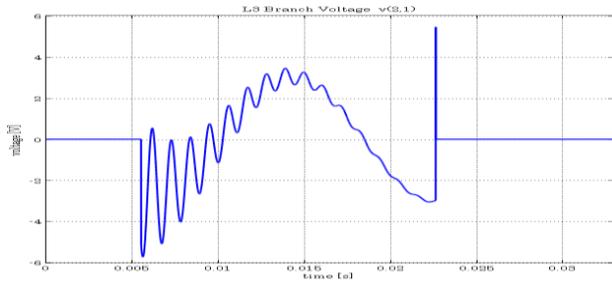
Fig. 23. Voltage at L3 inductor using THTA.

Fig. 24 shows the equivalence of CDA and THTA by comparing the values at each point. The unique mismatches found are the ones when the switches operate. A closer look at the numbers in Fig. 24 proves this to be truncation errors of the numerical framework, around the order of 10^{-19}.
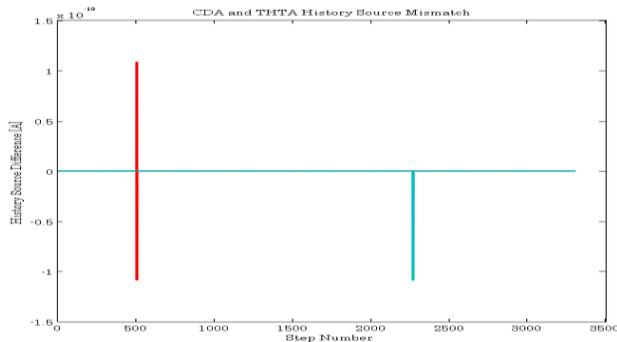
Fig. 24. "Mismatch" between CDA and THTA results.

Since the voltage impulse occurs in the intermediate step ($\Delta t/2$), and is probably quickly damped in a real power system, therefore, the voltage impulse may be skipped while plotting the final result. A final plot is shown in Fig. 25.
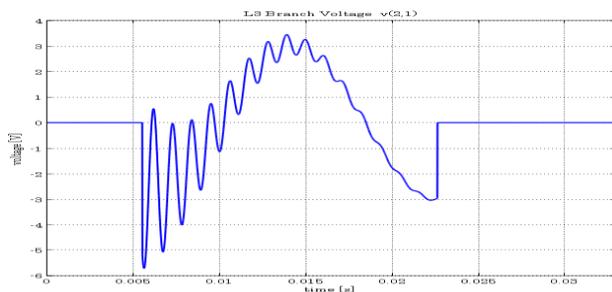
Fig. 25. Voltage at L3 inductor without impulse

V. CONCLUSIONS

This paper presented a clear review of numerical oscillations problems in EMTP-based programs, its causes and some techniques for effectively damping such oscillations. Among them, the CDA and the THTA techniques completely solves the problem by treating the root cause. These schemes require just half time step solutions to fully remove the numerical oscillations. They are the most elegant and effective, but as far as the authors now CDA technique was only implemented in the software MicroTran, the UBC's version of EMTP e probably a similar form of it in the EMTP-RV.

One important technical advantage of the THTA technique is that, unlike the CDA, it is not needed to derive and code all the components using the Backward Euler integration rule. It only needs the value of the history source of the previous solution step to produce a new value immune to numerical oscillations.

This paper initially focused on the didactic analysis of RLC circuits with the numerical oscillations started by operation of switches. Another important cause for numerical oscillations are the non-linear elements, based in the piece-wise linearization and switching of equivalent components. At each swap in the linearized region there is an abrupt change in the system, which may be abrupt enough to start new numerical oscillations. Therefore, further studies of the effectiveness of the THTA are possible, working with non-linear elements, and power electronics applications. The relevance of this work relies on its educational use by students and engineers and eventually the facilitation of implementation of CDA or THTA in other EMTP-based programs.

VI. REFERENCES

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