

# FDTD Analysis of Distribution Line Voltages Induced by Inclined Lightning Channel

Masashi Natsui, Akihiro Ametani, Jean Mahseredjian, Shozo Sekioka, Kazuo Yamamoto

**Abstract**--This paper investigates lightning induced voltages on a distribution line, when the lightning channel is not vertical but angled to the line and the earth representation is based on the finite-difference time-domain (FDTD) method. The effect of the current flowing into the earth, after the lightning channel touches the earth, is also investigated. The induced voltages are quite dependent on the angle of the channel to the line as can be easily estimated. It is found that the lightning angle with the line causes significant increase of the voltage and the effect of the earth resistivity on the induced voltage has been made clear when considering the inclined channel. Since the induced voltage is, in theory, proportional to the frequency of the inducing current, i.e. inversely proportional to rise time  $T_f$  when the inducing and induced circuits are parallel. When the lightning channel is angled, the proportional relation is less clear than that in the vertical channel. This fact has to be taken into account when discussing the effect of  $T_f$ . The current flowing in the earth is also affected by the lightning inclination and the voltage drop along the earth surface due to the current becomes larger than that in the vertical lightning case.

**Keywords:** lightning induced voltage, distribution line, angled channel, earth current, FDTD.

## I. INTRODUCTION

**P**OWER quality issues are becoming more and more significant, especially in the field of distribution systems, because a number of electronic devices, which are very sensitive to overvoltages, are installed in the distribution systems including home appliances [1]-[8]. Lightning to ground nearby the distribution systems produces induced lightning surges as is well-known [9]-[14]. An accurate evaluation of lightning induced voltages is, therefore, essential to protect these devices. In the induced surge calculations on a distribution line, the lightning channels is, in most cases, assumed to be straight and vertical to the ground surface and the distribution line, because existing analysis techniques are based on a transmission line (TL) theory which cannot handle non-uniform conductors in a three-dimensional space [15]-[19]. Also, a perfectly conducting ground is quite often assumed in a calculation due to the theoretical premise. In reality, a lightning channel is not straight nor vertical, and its return stroke current flows into a ground which is not perfectly conducting. The assumptions could result in incorrect

lightning induced surges. Therefore, these assumptions require a further careful investigation for the design of effective and adequate lightning protection methodologies.

In this paper, the effect of inclined lightning channel on a lightning induced voltage on a distribution line is investigated considering the earth resistivity by using a 3D finite-difference time-domain (FDTD) method [20]-[22]. The distribution line with finite length is placed above a lossy ground with an orthogonal three-dimensional space. The induced voltage is calculated at the position of the line closest to the base of the lightning channel.

The FDTD modeling of each element involved in a model circuit are described in Chapter II. FDTD computed results under various conditions of a lightning channel together with the return stroke velocity of the channel are shown, and the effect of the inclined lightning channel on the induced voltage is investigated in Chapter III. The contribution of the earth surface current to the induced voltage is discussed in Chapter IV. The remarks found in the investigations are summarized in Chapter V.

## II. MODEL CIRCUIT

In FDTD lightning study, lightning is generally modeled as a combination of a thin wire and a current source on the bottom of the lightning channel. In this paper, the lightning channel is represented by traveling-current-source model [22] to adjust the velocity and to obtain a smooth electromagnetic field.

Fig. 1 illustrates the model. The current source array is aligned vertically and horizontally along the arbitrary inclined lightning path and the velocity is adjusted by the delay time of the excitation for each current source. The height of the lightning channel is set to 500 m, and two directions of inclined channel are investigated: angle  $\theta$  on yz plane for the inclination toward the line, and angle  $\varphi$  on xz plane for the inclination along the line. The angle  $\theta$  and  $\varphi$  vary from  $-45^\circ$  to  $+90^\circ$  and  $0^\circ$  to  $+90^\circ$ , respectively. The angle of  $+90^\circ$  is used for the confirmation of consistency. Also, two cases of return stroke velocity,  $\beta = 1.00$  and  $0.33$ , and two values of the wave front duration,  $T_f = 1.0$  and  $0.1 \mu\text{s}$  are adopted in this study. Note that  $\beta$  is the ratio of the return stroke velocity to the light speed. A single overhead line with a height of 10 m is placed at 50 m away from the lightning base, which represents a 20 kV distribution line. In general, when the fast-rise return stroke current and low earth resistivity are assumed, the induced voltage becomes proportional to the line height and almost inversely proportional to the distance from the lightning base [9], [23].

Both ends of the line are connected to Liao's absorbing boundary of the working space of  $1060 \text{ m} \times 1060 \text{ m} \times 600 \text{ m}$ ,

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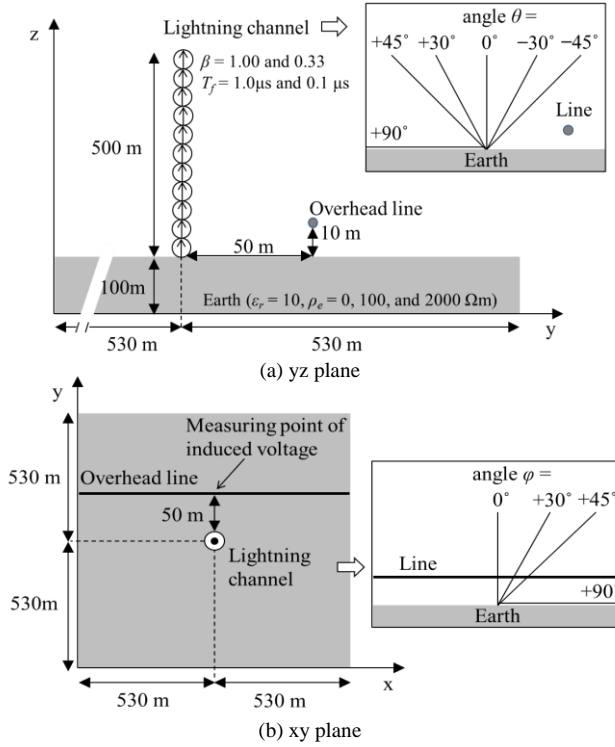


Fig. 1. FDTD experimental configuration: Lightning base is located at the center of the space and the distance between the base and a 10 m-height overhead line is 50 m. Inclination angle  $\theta$  and  $\varphi$  vary from  $-45^\circ$  to  $+90^\circ$  and  $0^\circ$  to  $+90^\circ$ , respectively.

which is composed of cubic cells of the several sizes between  $1\text{ m} \times 1\text{ m} \times 1\text{ m}$  and  $8\text{ m} \times 8\text{ m} \times 8\text{ m}$ . The minimum grid size is adopted in the area including the lightning channel base and the measuring point ( $140\text{ m} \times 140\text{ m} \times 140\text{ m}$ ). The measuring point of the induced voltage on the line is placed at the closest point from the lightning base. The depth of earth is taken as 100 m, and the earth resistivity  $\rho_e$  is set to  $0\ \Omega\text{m}$  (perfectly conducting earth: PCE),  $100\ \Omega\text{m}$ , and  $2000\ \Omega\text{m}$ . A normalized value of current 1A with  $T_f = 1.0$  and  $0.1\ \mu\text{s}$  shown in Fig. 2, is adopted for each current source as a lightning return stroke current.

### III. LIGHTNING INCLINATION

#### A. Effect of angle $\theta$

Fig. 3 shows induced voltage waveforms on a distribution line for various values of  $\theta$ . In this case (a) is for the perfectly conducting earth, (b) is for earth resistivity  $\rho_e = 100\ \Omega\text{m}$ , and (c) for  $\rho_e = 2000\ \Omega\text{m}$ . It is clear that for  $\theta = -45^\circ$ , i.e. the channel lightning channel being the nearest to the line, induces the highest voltage to the line. As  $\theta$  increases, i.e. the channel becomes further to the line, the induced voltage tends to decrease except the case of (c)  $\rho_e = 2000\ \Omega\text{m}$  as shown in Fig. 4. When  $\rho_e = 2000\ \Omega\text{m}$ , the induced voltages in the cases of  $\theta = -30^\circ$  and  $0^\circ$  are higher than that in the case of  $\theta = -45^\circ$ .

It is observed in Fig. 4 that the induced voltage becomes higher as the earth resistivity increases as is well known [24]–[25]. Also, it should be noted that the rise time of the induced voltage becomes larger as  $\rho_e$  increases, and thus the induced voltage sustains much longer in the case of  $\rho_e = 2000\ \Omega\text{m}$  than

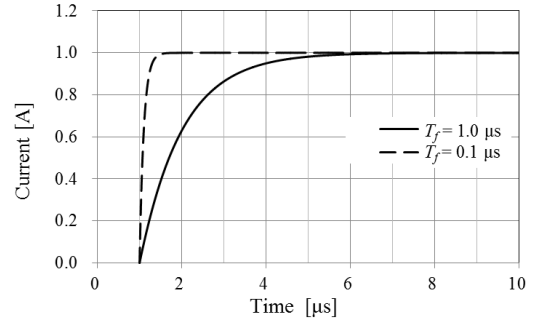


Fig. 2. Return stroke current waveforms.

that in the case of  $\rho_e = 100\ \Omega\text{m}$ . When  $\theta = +90^\circ$ , the lightning channel is placed above the earth surface by the height of 1 m, i.e. the channel is almost perpendicular to the distribution line, and thus the induced voltage is the smallest. For  $\rho_e = 0\ \Omega\text{m}$ , the voltage is almost zero, and increases to 5.5V for  $\rho_e = 2000\ \Omega\text{m}$ . Considering the fact that the channel is perpendicular to the line for  $\theta = +90^\circ$  is considered, the induced voltage may be caused by the current flowing on the earth surface after the lightning hits the earth. The characteristic of the current will be discussed in Chapter IV as the earth surface current.

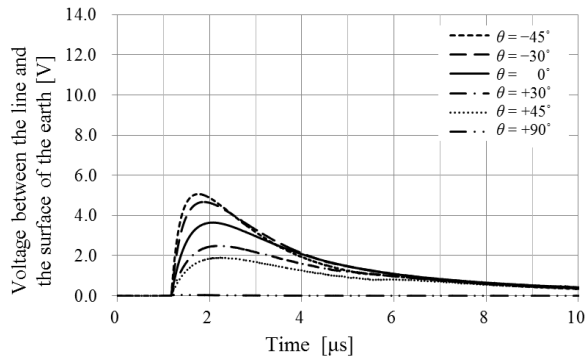
#### B. Effect of angle $\varphi$

Fig. 5 shows induced voltage waveforms with various angles  $\varphi$  for  $\rho_e = 0, 100$  and  $2000\ \Omega\text{m}$ . Fig. 6 shows the maximum voltage as a function of  $\varphi$ . The induced voltages become higher as  $\rho_e$  increases. The highest voltage is observed when  $\varphi = +45^\circ$  and the voltage decreases as  $\varphi$  becomes smaller except  $\varphi = +90^\circ$ . When  $\varphi = +90^\circ$ , the induced voltage becomes almost zero for  $\rho_e = 0\ \Omega\text{m}$ , nearly the same as that of  $\varphi = +45^\circ$  for  $\rho_e = 100\ \Omega\text{m}$ , and much higher for  $\rho_e = 2000\ \Omega\text{m}$ . The phenomena are estimated to be caused by the image current in the earth. When  $\varphi = +90^\circ$ , the lightning channel is also placed 1 m above the earth surface. Therefore, the image current becomes large under the channel for  $\rho_e = 0\ \Omega\text{m}$ , and it cancels almost all the electromagnetic field generated from the lightning channel. Therefore, nearly no voltage is induced on the line. On the other hand, the effect of the image current becomes less for large  $\rho_e$ , and the electromagnetic field can reach the line and induces the voltage.

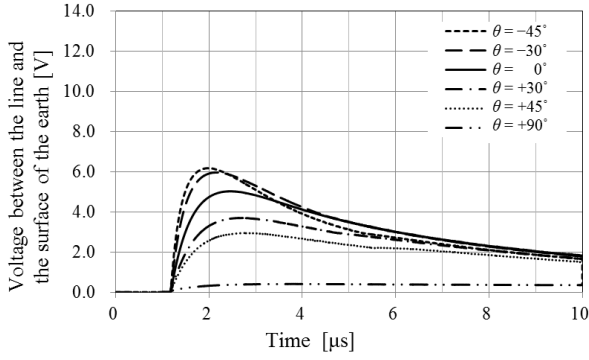
#### C. Effect of rise time $T_f$

Fig. 7 shows a comparison of the results for  $T_f = 0.1\ \mu\text{s}$  and  $1.0\ \mu\text{s}$  at angle  $\theta = -45^\circ$ . The first peaks of the induced voltage become larger and sharper due to the faster rise time of the lightning current.

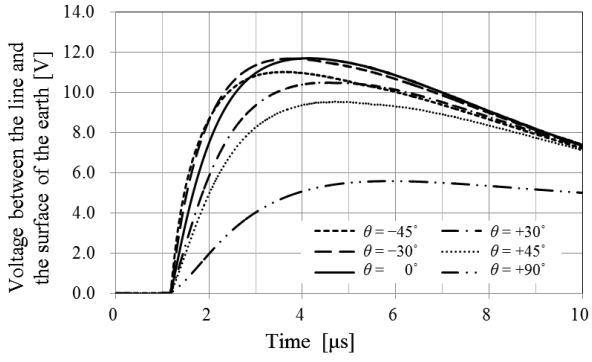
The relation between the peak voltage and angle  $\theta$  for  $T_f = 0.1\ \mu\text{s}$ , shown in Fig. 8 (a), is also similar to the result for  $T_f = 1.0\ \mu\text{s}$ . The peak voltage increases as angle  $\theta$  increases. The increase ratio of the voltage according to angle  $\theta$  is larger than those for  $T_f = 1.0\ \mu\text{s}$ , indicating that the induced voltage is more sensitive to the angle in shorter  $T_f$ . Note that the induced voltage is generally proportional to the frequency of the inducing current, i.e. inversely proportional to  $T_f$  when the inducing and induced circuits are parallel. However, the induced voltage, which is caused by the superposition of the electromagnetic field, does not simply increase.



(a)  $\rho_e = 0 \Omega\text{m}$

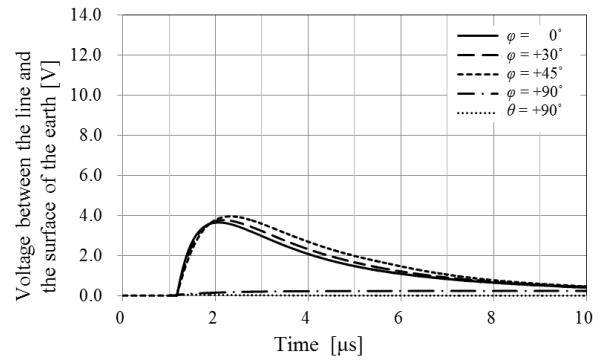


(b)  $\rho_e = 100 \Omega\text{m}$

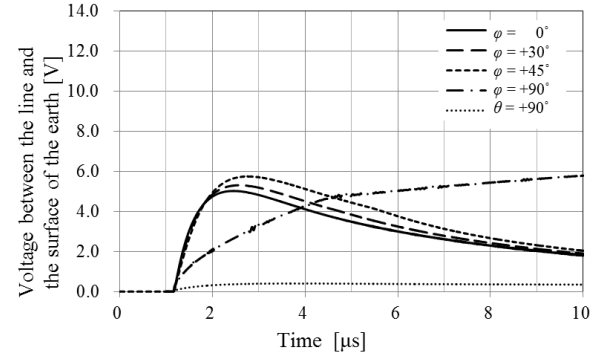


(c)  $\rho_e = 2000 \Omega\text{m}$

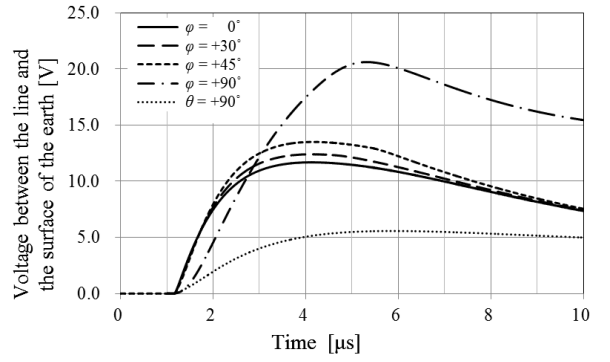
Fig. 3. Induced voltages on the overhead line with the inclined lightning of angle  $\theta$ :  $\beta = 1.00$  and  $T_f = 1.0 \mu\text{s}$ .



(a)  $\rho_e = 0 \Omega\text{m}$



(b)  $\rho_e = 100 \Omega\text{m}$



(c)  $\rho_e = 2000 \Omega\text{m}$

Fig. 5. Induced voltages on the overhead line with the inclined lightning of angle  $\phi$ :  $\beta = 1.00$  and  $T_f = 1.0 \mu\text{s}$ .

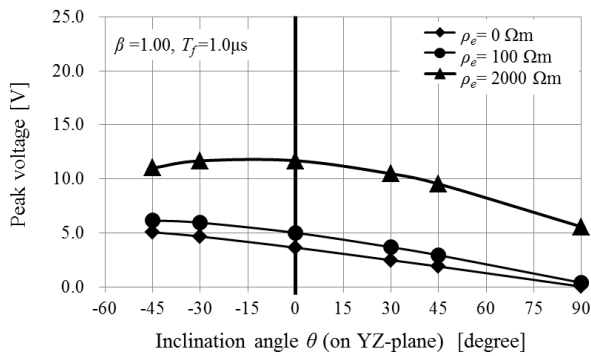


Fig. 4. Effect of the inclination angle  $\theta$  on the peak induced voltages:  $\beta = 1.00$  and  $T_f = 1.0 \mu\text{s}$ .

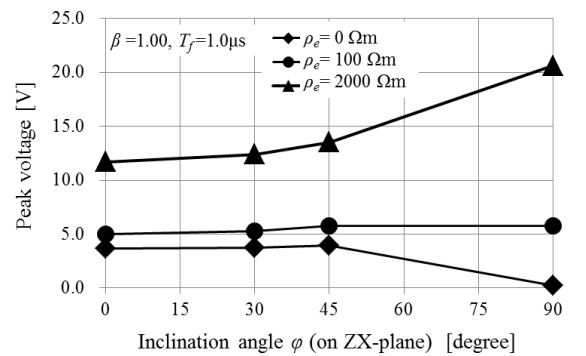


Fig. 6. Effect of the inclination angle  $\phi$  on the peak induced voltages:  $\beta = 1.00$  and  $T_f = 1.0 \mu\text{s}$ .

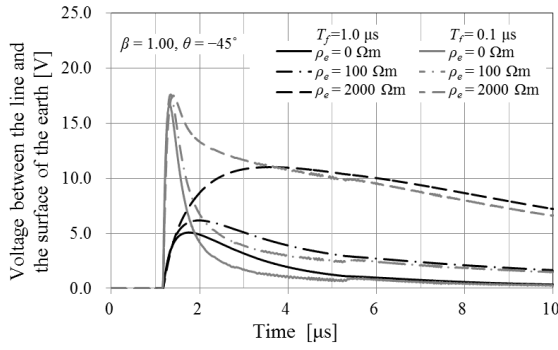
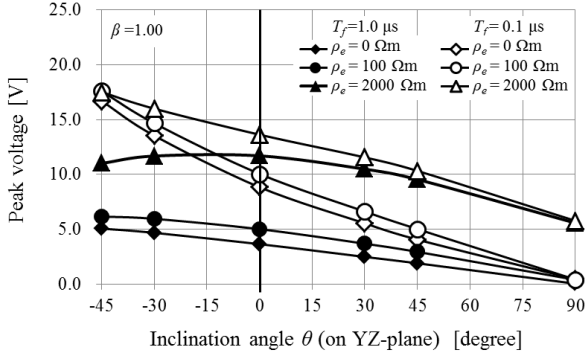
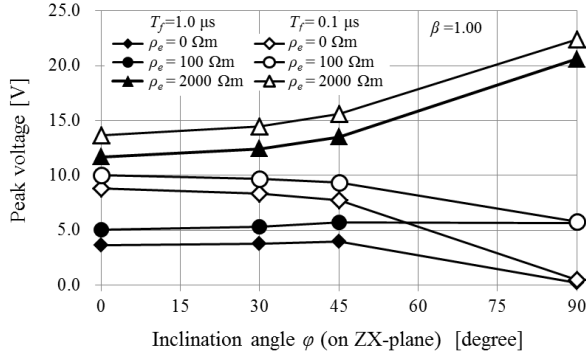


Fig. 7. Comparisons of the induced voltage waveforms between  $T_f=1.0\mu s$  and  $0.1\mu s$ :  $\beta=1.00$  and  $\theta=-45^\circ$ .

Fig. 8 (b) shows the effect of the angle  $\varphi$  on the induced



(a) Peak voltage versus angle  $\theta$



(b) Peak voltage versus angle  $\varphi$

Fig. 8. Comparison of the effect of the lightning inclination on the peak voltages between  $T_f=1.0 \mu s$  and  $0.1 \mu s$ :  $\beta=1.00$ .

voltage for  $T_f = 1.0$  and  $0.1 \mu s$ . The peak voltages also become higher for  $T_f = 0.1 \mu s$  than  $1.0 \mu s$ .

Considering the difference of the earth resistivity for the waveforms in Fig. 7, the induced voltages at  $\theta = -45^\circ$  with  $T_f = 0.1 \mu s$  show almost the same amplitude of the peak. However, the decay is different from each other, and similar to the result of  $T_f = 1.0 \mu s$ . These mean that the waveform of the induced voltage can be composed of the superposition of two components: in the first one, at least with  $T_f = 0.1 \mu s$ , the voltage is mainly induced by the lightning current without a significant effect of the earth condition, and in the second one,

the voltage is result from an interaction between the current and the earth.

#### D. Effect of return stroke velocity $\beta$

Fig. 9 illustrates the peak induced voltages for return stroke velocity  $\beta = 0.33$ . Both results of  $T_f = 1.0$  and  $0.1 \mu s$  shows almost the same manner to the result of  $\beta = 1.00$  shown in Fig. 8. The peak values show minor difference from the result of  $\beta = 1.00$ .

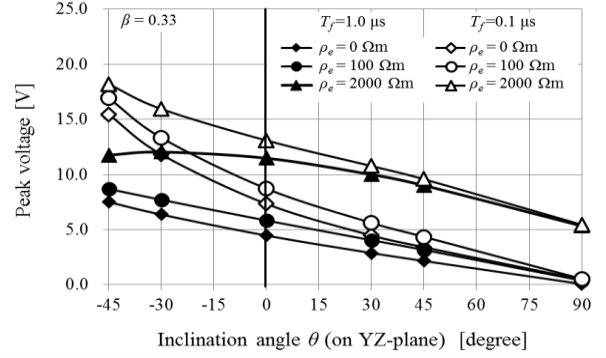


Fig. 9. Effect of the lightning inclination angle  $\theta$  on the peak voltages between  $T_f = 1.0 \mu s$  and  $0.1 \mu s$ :  $\beta = 0.33$ .

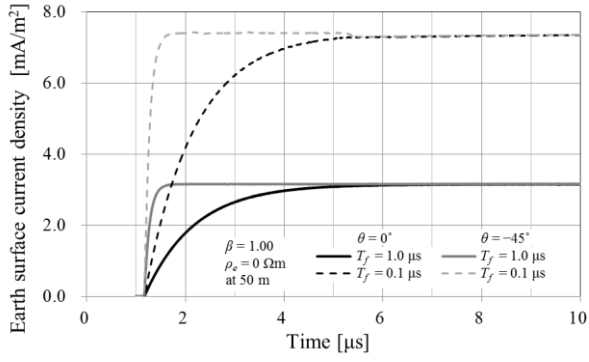
#### IV. EARTH SURFACE CURRENT

The distribution and the effect of the earth current are investigated by using the same models in the previous study. Fig. 10 shows the earth surface current density just under the overhead line (at the distances of 50 m from the lightning channel base) when the lightning of  $\beta = 1.00$  strikes the earth for: (a)  $\rho_e = 0 \Omega m$ , (b)  $\rho_e = 100 \Omega m$ , and (c)  $\rho_e = 2000 \Omega m$ . Note that the direction of the current flow from the overhead line toward the lightning base is considered as positive. It is observed from the figure that the earth surface current becomes larger when  $\theta$  decreases. On the other hand, at  $t = 10 \mu s$ , the current of  $\rho_e = 0 \Omega m$  at  $\theta = -45^\circ$  keeps higher value while the current of  $\rho_e = 100$  and  $2000 \Omega m$  become the same value at  $\theta = 0^\circ$ . This means that the resistivity of  $\rho_e = 100$  and  $2000 \Omega m$  are more dominant for the distribution of the current than inductive characteristic of the current path even if the lightning is inclined.

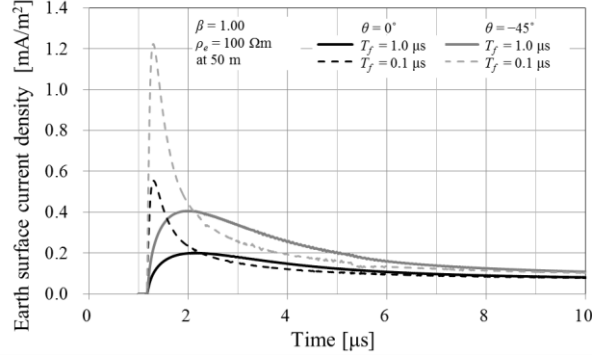
Fig. 11 shows the peak values of the earth surface current density. Note that solid black line and gray line indicate the currents which are assumed to be distributed evenly in the circular and hemispherical surface, respectively, i.e. the values are:

$$I_{\text{circular dist.}} = \frac{1}{2\pi r} [A/m^2] \cdot (1), \quad I_{\text{hemispherical dist.}} = \frac{1}{2\pi r^2} [A/m^2] \cdot (2)$$

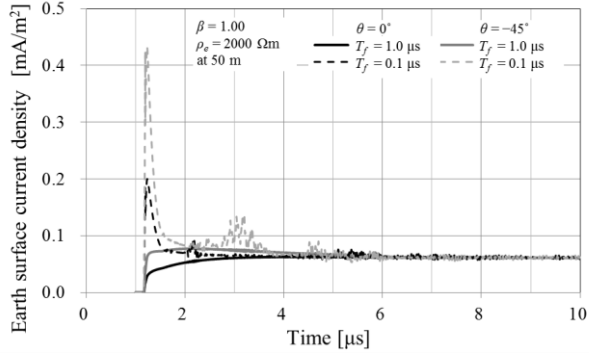
The result for  $\rho_e = 0 \Omega m$  at  $\theta = 0^\circ$  in Fig. 11 corresponds to the black line. This means that, as expected, the earth current flows only on the earth surface and scatters evenly for each direction. On the other hand, the result for  $\rho_e = 2000 \Omega m$  with  $T_f = 1.0 \mu s$  in Fig. 11 (a) matches the gray line, indicating that the current is distributed evenly on hemispherical surface due to the earth resistivity as assumed above.



(a)  $\rho_e = 0 \Omega\text{m}$



(b)  $\rho_e = 100 \Omega\text{m}$

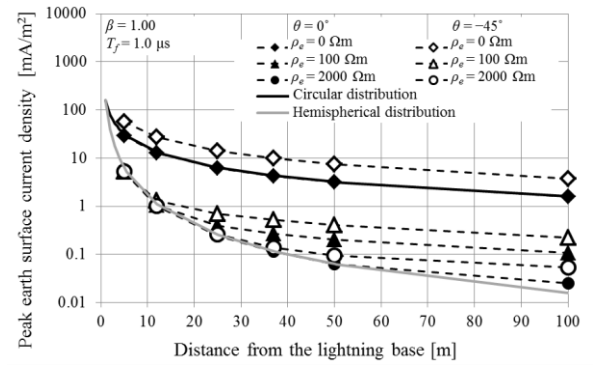


(c)  $\rho_e = 2000 \Omega\text{m}$

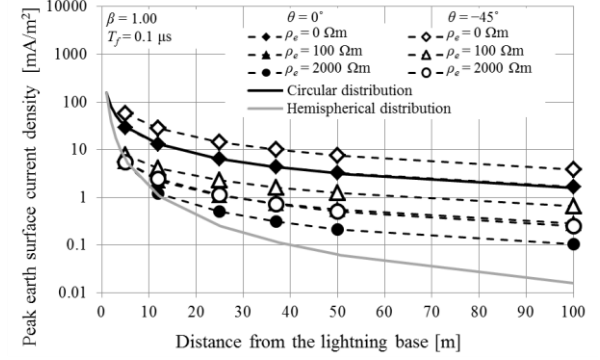
Fig. 10. The earth surface current density waveform in the earth of  $\rho_e = 0, 100$ , and  $2000 \Omega\text{m}$  for the vertical lightning:  $\beta = 1.00$ .

The effect of the inclined lightning on the earth surface current is also shown in Fig. 11. The current at  $\theta = -45^\circ$  becomes larger than that of  $\theta = 0^\circ$ .

Fig. 12 shows the example of the horizontal differential voltage between the earth surface under the measuring point of the overhead line and the surface of 50 m behind in y direction. Note that in  $\rho_e = 0 \Omega\text{m}$ , there is no differential voltage between the points. The ground potential significantly decreases due to the voltage drop of the surface current, and the effect of angle  $\theta$  is clearly observed. The peak value of the differential voltage is illustrated in Fig. 13. As expected, the voltage decreases noticeably in the case of higher resistivity  $\rho_e = 2000 \Omega\text{m}$  and shorter  $T_j = 0.1 \mu\text{s}$ . The earth voltage decrease occurs when lightning strikes the lossy earth.



(a)  $T_j = 1.0 \mu\text{s}$



(b)  $T_j = 0.1 \mu\text{s}$

Fig. 11. The peak values of the earth surface current density caused by the lightning inclined at  $\theta = 0^\circ$  and  $-45^\circ$ :  $\beta = 1.00$ .

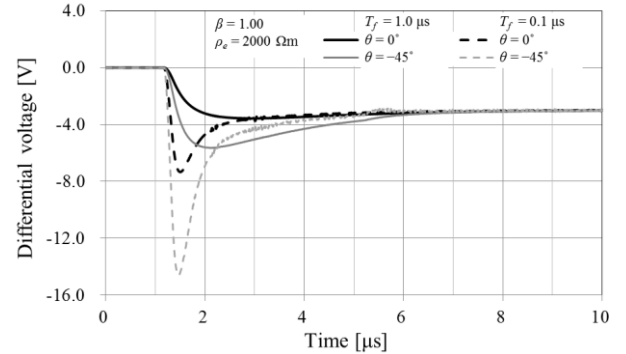


Fig. 12. Horizontal differential voltage between the earth surface under the measuring point of the overhead line and the surface of 50 m behind in y direction:  $\beta = 1.00$ ,  $\rho_e = 2000 \Omega$ .

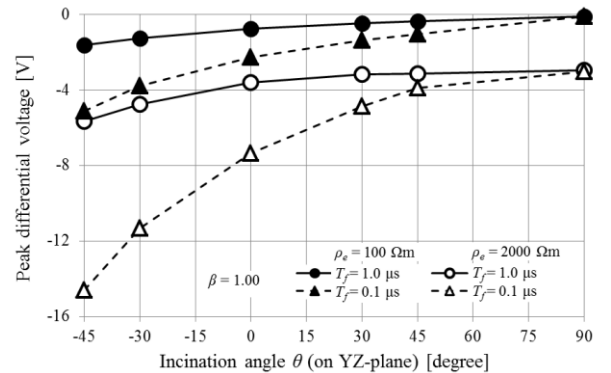


Fig. 13. Peak differential voltage drop values between the earth surface under the measuring point of the overhead line and the surface of 50 m behind in y direction:  $\beta = 1.00$ .

## V. CONCLUSION

Lightning induced voltages on a distribution line have been investigated by FDTD computations when the lightning channel is angled and lossy earth is assumed. It is found that when the lightning is inclined toward the line, it significantly increases the induced voltage and the increase ratio is larger in short rise time  $T_f$ . However, the inclination along the line causes only a minor increment of the voltage. This phenomenon is the same when the resistivity is relatively low. For lightning with a slower return-stroke velocity, the effects of angle,  $T_f$  and the earth resistivity on the voltage are almost the same as those for  $\beta = 1.00$ . The inclined lightning also affects the earth surface current and horizontal electric field distribution, which transiently induces larger earth potential drop in horizontal direction.

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