Three-Phase Compact Distribution Line Transient Analysis Considering Different Line Models

A. De Conti, A. C. Silva

***Abstract*--** **This paper discusses the simulation of electromagnetic transients on compact distribution lines taking as reference a full phase-domain model based on the FDTD technique. It is shown that the insulating layer covering the phase conductors must be included for an accurate transient simulation involving such type of line. It is also shown that Marti's model available in popular electromagnetic transient simulators can be used in such type of simulation as long as extra care is taken in the fitting of the model in frequency domain and some loss of accuracy is admitted.**

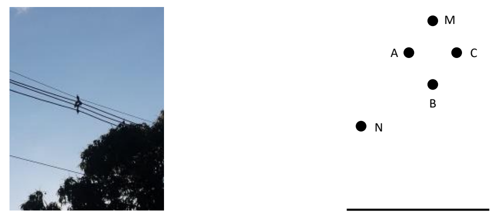
***Keywords*: transients, compact distribution lines, transmission line modeling.**

I. Introduction

In many countries, electric utilities have been using phase conductors covered with an insulating layer, typically a polymeric material such as XLPE and HDPE, to improve the overall performance of medium-voltage distribution lines. In Japan, for example, for many decades such strategy has been deployed to reduce the number of faults due to the undesired contact of conductors to trees and improve the performance of distribution lines against voltages induced by nearby lightning [1]. Insulated phase conductors also allow a significant reduction in clearances without compromising the overall insulation performance. This has motivated the proposition of compact configurations as an alternative to conventional structures based on the use of bare wires.

One of these compact configurations, also known as spacer-cable system, is shown in Fig. 1. It is often used in medium-voltage distribution lines from 15-kV to 34.5-kV classes [2]. It contains a polymeric spacer that sustains three phase conductors covered with an insulating layer. The spacer is supported by a messenger cable that is periodically grounded. At every grounding point, the messenger is connected to a neutral conductor located few meters below.

Despite the increasing use of compact distribution lines in many countries, studies dedicated to investigate the performance of such lines to electromagnetic transients are relatively scarce. This has been motivating a number of investigations in recent years, including the determination of the impulse withstand voltage of 15-kV class compact distribution line structures [3, 4], the calculation of lightning-induced voltages due to nearby lightning strikes [5, 6], and the assessment of the influence of the insulating layer covering the phase conductors on the propagation characteristics of compact distribution lines [7].



(a) (b)

Fig. 1. Three-phase compact distribution line.

One particularity of three-phase compact distribution lines is the fact that it is an inhomogeneous system comprising three insulated cables (the phase conductors) and two bare wires (messenger cable and neutral). Direct phase-domain models such as the universal line model (ULM) [8] and the solution of telegrapher's equations based on the finite-difference time-domain (FDTD) method [9] are expected to have no difficulty in the accurate simulation of electromagnetic transients in such kind of system. However, it is not clear if the modal-domain transmission line model with real and constant transformation matrix proposed by Marti [10], which is available in most electromagnetic transient simulators, can be successfully used in this type of simulation. The main purpose of this paper is to investigate this point.

This paper is organized as follows. Section II describes the main characteristics of the three-phase compact distribution line considered in this study. Section III presents details of the transmission line model used as reference in this investigation, which is based on the FDTD method, and of the modal-domain model proposed by Marti. Results and analyses are presented in Section IV, followed by conclusions in Section V.

II. Simulated System

The geometry considered in the simulations is illustrated in Fig. 1. It is a typical three-phase distribution line structure of 15-kV class [2]-[4]. The vertical and horizontal coordinates of the conductors are given in Table I. The phase conductors are covered with a 3-mm XLPE insulating layer, while the neutral and messenger are bare. Further details are given in Table II.

This work was supported in part by Brazilian agencies FAPEMIG (TEC-PPM-00280-17), CNPq (431948/2016-0 and 304117/2016-1), and CAPES (Finance Code 001).

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Paper submitted to the International Conference on Power Systems Transients (IPST2019) in Perpignan, France June 17-20, 2019.

TABLE I
COORDINATES OF THE SYSTEM OF CONDUCTORS SHOWN IN FIG. 1(B)

Cable	Label	Coordinates (m)	
		Horizontal	Vertical
Phase A	A	-0.095	8.83
Phase B	B	0	8.67
Phase C	C	0.095	8.83
Messenger	M	0	9.00
Neutral	N	-0.354	7.00

TABLE II
CABLE DETAILS

Cable	Core radius (mm)	External radius (mm)	ε_r (insulating layer)	DC Resistance (Ω/km)
A	4.10	7.10	2.3	0.822
B	4.10	7.10	2.3	0.822
C	4.10	7.10	2.3	0.822
M	4.75	4.75	-	4.5239
N	3.72	3.72	-	1.0949

III. TRANSMISSION LINE MODELS

A. FDTD Model

Telegrapher's equations describing voltages and currents along an overhead line with N conductors positioned over a finitely conducting ground can be written in time domain as

$$-\frac{\partial \boldsymbol{v}(x,t)}{\partial x} = \boldsymbol{L_e}\frac{\partial \boldsymbol{i}(x,t)}{\partial t} + \boldsymbol{\varsigma}(t) * \frac{\partial \boldsymbol{i}(x,t)}{\partial t} \quad (1a)$$

$$-\frac{\partial \boldsymbol{i}(x,t)}{\partial x} = \boldsymbol{G}\boldsymbol{v}(x,t) + \boldsymbol{C}\frac{\partial \boldsymbol{v}(x,t)}{\partial t} \quad (1b)$$

where $\boldsymbol{v}(x,t)$ and $\boldsymbol{i}(x,t)$ are N x 1 vectors describing voltages and currents at coordinate x of the line, at time t, $\boldsymbol{L_e}$, $\boldsymbol{C}$, and $\boldsymbol{G}$ are frequency-independent matrices of order N x N representing the external inductance, capacitance, and conductance per unit length, '*' represents the convolution integral, and $\boldsymbol{\varsigma}(t)$ is the transient impedance given by

$$\boldsymbol{\varsigma}(t) = \mathcal{L}^{-1}\left\{\frac{\boldsymbol{Z_i} + \boldsymbol{Z_g}}{s}\right\}. \quad (2)$$

In (2), $\boldsymbol{Z_i}$ is an N x N diagonal matrix containing the internal impedance of the cables, $\boldsymbol{Z_g}$ is a full N x N matrix containing the ground-return impedance, s is the Laplace variable, and $\mathcal{L}^{-1}\{\cdot\}$ is the inverse Laplace transform operator.

Equation (1) can be solved using the 1st order FDTD scheme presented in [9]. It consists in transforming partial derivatives into difference equations by subdividing the line in N_S segments with length Δx considering a time step Δt. The resulting set of equations can be used to calculate voltages and currents at any point of the line, at any time t. In this paper, the transient impedance matrix $\boldsymbol{\varsigma}(t)$ was fitted in frequency domain using the vector fitting technique considering a proper function and a single set of real poles for all its elements [11]. A total of 11 poles were required without need to enforce model passivity. The convolution integral appearing in (1a) was solved recursively as described in [9].

In [7] it was demonstrated that the compact line configuration shown in Fig. 1 is not affected by the proximity effect. For this reason, the per-unit-length parameters of the line can be calculated using approximate equations that are valid for widely-spaced cables. For calculating $\boldsymbol{C}$ including the presence of an insulating layer on the phase conductors, the formulation presented in [12], which is available in the Cable Constants (CC) routine in the Alternative Transients Program (ATP), was considered. The external inductance $\boldsymbol{L_e}$ was calculated assuming a system of bare wires because the insulating layer covering the phase cables presents no magnetic properties [9]. The internal impedance was calculated using the well-known formulation based on Bessel's equations [9]. For a direct comparison with results obtained with ATP, the ground-return impedance was calculated using Carson's equations. A conductance of 18.64 nS/km was assumed for $\boldsymbol{G}$, which is diagonal.

As long as the Courant condition $\Delta t \leq \Delta x / \upsilon$ is satisfied, where υ is the maximum wave phase velocity within the model (essentially the speed of light, for an overhead line with bare conductors), the FDTD method leads to an accurate solution of (1) directly in phase domain. For this reason, the FDTD model is considered the reference model in this paper.

B. Marti's Model

Marti's model is a frequency-dependent transmission line model that solves telegrapher's equations in modal domain assuming a constant and real transformation matrix calculated at a frequency determined by the user [10]. This model is implemented in most electromagnetic transient simulators and can be successfully used in transient studies as long as the transformation matrix necessary for decoupling telegrapher's equations into modes does not present a significant variation with frequency. It relies on the fitting of the characteristic impedance $Z_c(\omega)$ and of the propagation function $A(\omega)$ of each mode as a sum of rational functions, which are then used for calculating the recursive convolutions that are necessary for efficient time-domain simulations.

In this paper, two possibilities are considered for Marti's model. One of them is a Matlab code implemented by the authors that is able to deal with complex poles obtained from the fitting of $Z_c(\omega)$ and $A(\omega)$ using the vector fitting technique [11]. In this implementation, a dedicated set of poles is considered for each mode. The second possibility is the use of the model implemented in ATP, which requires the fitting of $Z_c(\omega)$ and $A(\omega)$ with real poles. For this second possibility, the line parameters are calculated either with the CC routine available in ATP or in Matlab, from which a punch file compatible with ATP is generated [13]. This latter approach was proposed in [13] for including frequency-dependent soil parameters in transient simulations in ATP.

IV. RESULTS AND ANALYSES

To investigate the transient response of the compact distribution line illustrated in Fig. 1 and compare the performance of different transmission line models, a 1-km long line is considered. An ideal voltage source was assumed to excite the sending end of phase A with a step voltage of 1-V amplitude. Phases B and C were both grounded at the sending end with 500 Ω resistances. The messenger and the neutral were also grounded at the sending end, but with 10 Ω resistances. The receiving end of the line was left open.

A. Influence of the insulating layer

This section illustrates the influence of the 3-mm XLPE insulating layer covering the phase conductors on the voltages that propagate along the line. In the simulations, a 1000-Ωm soil resistivity was assumed and two conditions were considered in the FDTD model: (a) a homogeneous system in which the insulating layer covering the phase cables was neglected and (b) an inhomogeneous system in which insulated phase cables were considered. The voltages calculated at the receiving end of the line are shown in Fig. 2.

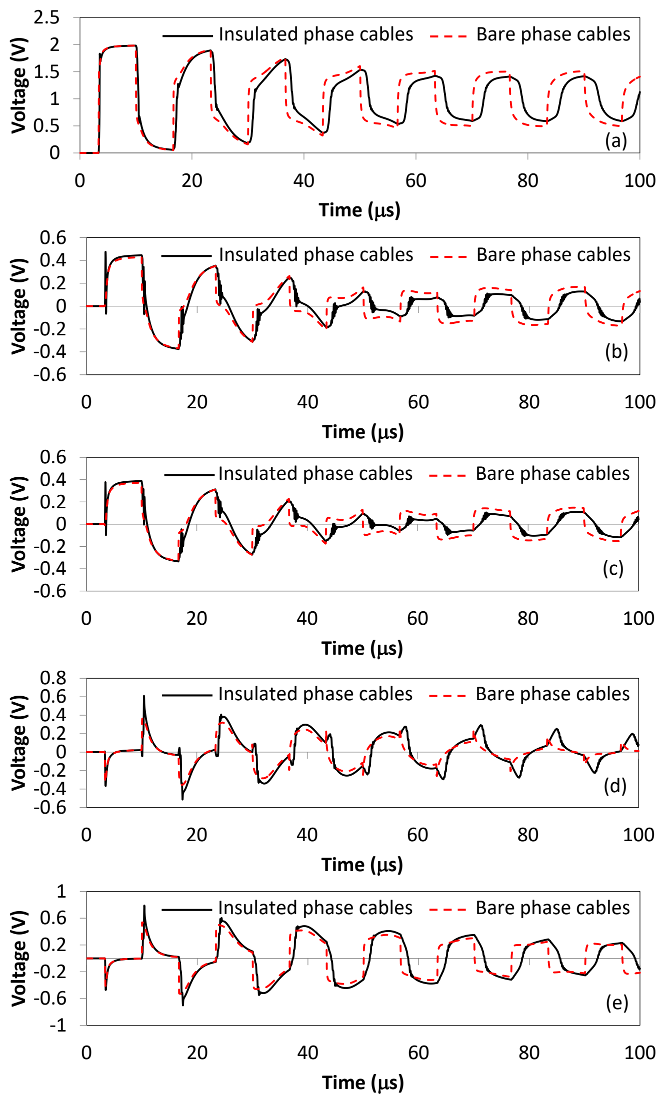


Fig. 2. Voltages at the receiving end of the line calculated for a 1000-Ωm soil with the FDTD method: (a) phase A; (b) phase B; (c) phase C; (d) messenger; (e) neutral.

It is seen in Fig. 2 that the initial peak voltage on phase A is not significantly affected by the presence of the insulating layer covering the phase conductors. On phases B and C, however, a high-magnitude spike is observed at early times. This spike is the result of the combination of voltage waves travelling at different propagation speeds, some approaching the speed of light at high frequencies whilst others are reduced by the influence of the cable insulation. The amplitudes of the negative voltages that are initially induced on both messenger and neutral remain essentially unaltered, but the peak voltages of the following oscillations are underestimated if the cable insulation is neglected. Also, significant deviations are observed on the tail of the voltages calculated on all conductors. It can thus be concluded that assuming bare cables for simulating compact distribution lines might not be sufficiently accurate in a rigorous transient study. Although not shown, similar results were obtained for a soil resistivity of 100 Ωm.

B. Transformation matrices

The analysis of the previous section was performed with an FDTD solution of telegrapher's equations which, although accurate, is not computationally efficient. It also presents difficulties for the simulation of complex distribution systems involving laterals, transformers, surge arresters and multiple grounding points, in which case it is preferable to resort to electromagnetic transient simulators.

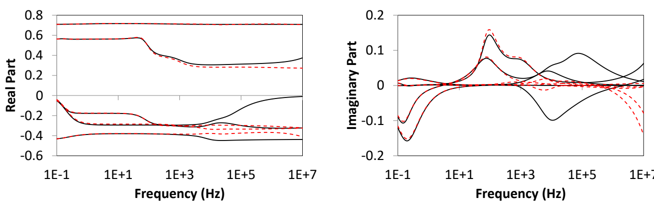
One of the transmission line models that are readily available in most electromagnetic transient simulators is Marti's model discussed in Section III-B. Since the main drawback of this model is the consideration of a real and constant transformation matrix for decoupling telegrapher's equations into modes, the first step before using Marti's model in the simulation of transients in compact distribution lines is to identify how this matrix varies with frequency.

Fig. 3 illustrates the real and imaginary parts of the columns of matrix $\boldsymbol{T_I}(\omega)$ obtained from the diagonalization of the product $\boldsymbol{YZ}$, where $\boldsymbol{Y}$ is the shunt admittance and $\boldsymbol{Z}$ is the longitudinal impedance of the line, both per unit length, using the Newton-Raphson routine proposed in [14]. The eigenvectors were rotated at each frequency to minimize their imaginary parts. The simulations considered either insulated or bare phase cables. As seen in Fig. 3, the elements of $\boldsymbol{T_I}(\omega)$ present a greater variation with frequency if the insulating layer covering the phase conductors is considered, especially at higher frequencies. In addition, the imaginary parts of the eigenvectors are closer to zero, at least at higher frequencies, if a system of bare cables is considered. This suggests that Marti's model is likely to lead to more accurate results in the simulation of the compact line as a system of bare cables, which is confirmed in the next section.

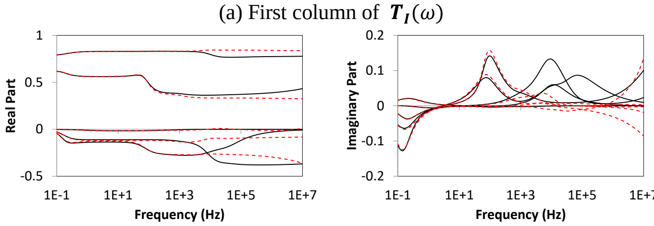
C. Simulations with Marti's model

1) Fitting with Complex poles

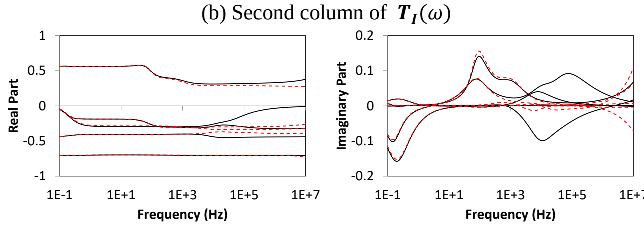
In this section, the simulations presented in Section IV-A are repeated considering an independent implementation of Marti's model in Matlab, in which complex conjugate poles are allowed in the fitting of $Z_c(\omega)$ and $A(\omega)$ with the vector fitting technique from 0.1 Hz to 10 MHz. Two frequencies were assumed in the computation of the transformation matrices, namely 100 kHz and 1 MHz. The former was chosen because the imaginary parts of the eigenvectors associated with the system with bare cables are naturally close to zero at this frequency. The latter was chosen to illustrate a condition in which the imaginary parts of the eigenvectors are comparatively larger. For the system with insulated phase cables, the chosen frequencies are convenient to illustrate the effect of the variation of the transformation matrix with frequency on the simulated results. In the simulations, only the real parts of the transformation matrices were retained. Overall, the fitting of $A(\omega)$ required from 7 to 13 poles, depending of the considered mode, while the fitting of $Z_c(\omega)$ required about 30 poles for all modes. No passivity enforcement was necessary. The results are presented in Figs. 4 and 5, which correspond, respectively, to a homogeneous system of bare conductors and to an inhomogeneous system with insulated phase conductors. For comparison purposes, the results obtained with the FDTD method were included as reference in the figures.



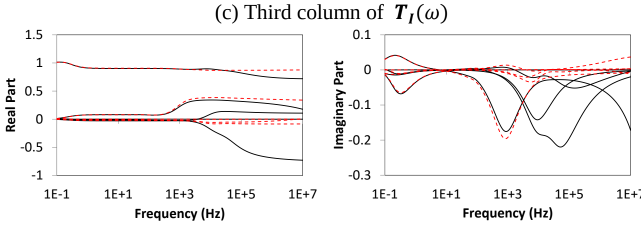
(a) First column of $\boldsymbol{T}_I(\omega)$



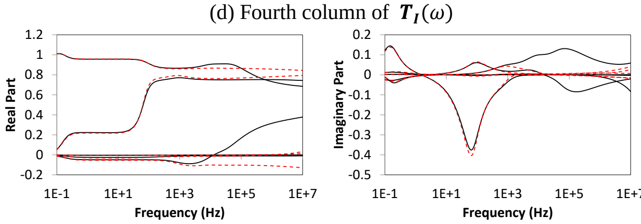
(b) Second column of $\boldsymbol{T}_I(\omega)$



(c) Third column of $\boldsymbol{T}_I(\omega)$



(d) Fourth column of $\boldsymbol{T}_I(\omega)$



(e) Fifth column of $\boldsymbol{T}_I(\omega)$

Fig. 3. Elements of the transformation matrix. Solid lines: insulated phase cables; dashed lines: bare phase cables.

It is seen in Fig. 4 that voltages calculated with Marti's model present an excellent agreement with those obtained with the FDTD method, regardless of the frequency assumed for the calculation of the transformation matrix, if a system of bare conductors is considered. This result is in line with the analysis presented in the previous section that indicates that for a homogeneous system the transformation matrix is closer to purely real and does not vary significantly with frequency.

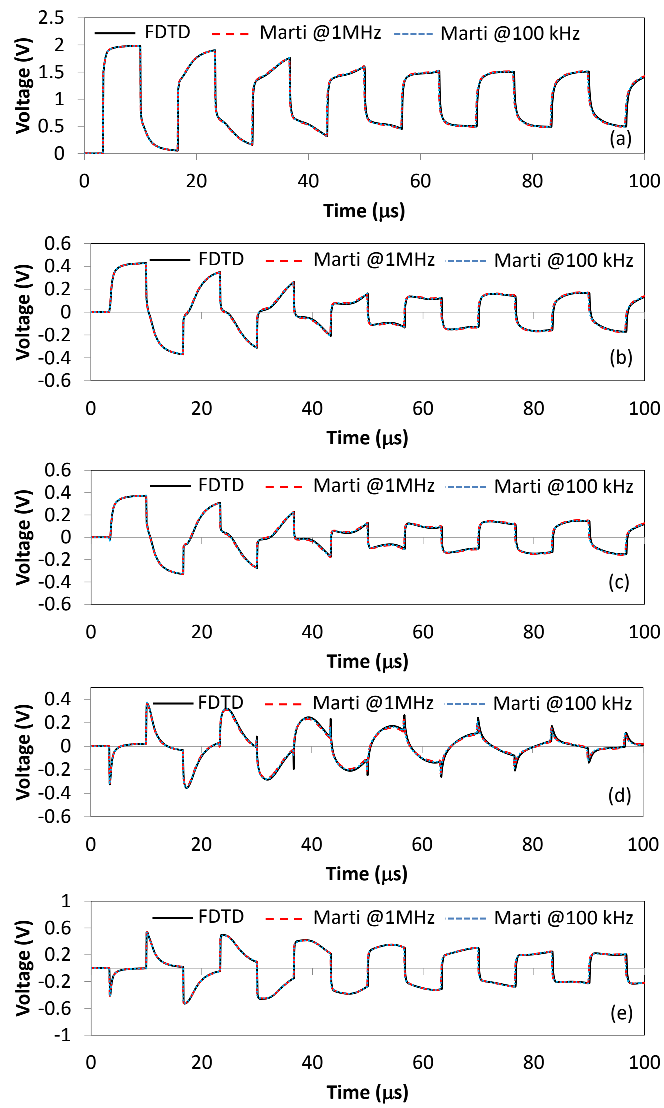


Fig. 4. Voltages at the receiving end of the line for bare phase cables and 1000-Ωm soil: (a) phase A; (b) phase B; (c) phase C; (d) messenger; (e) neutral. Tested model: Marti's model with complex poles in the fitting of $Z_c(\omega)$ and $A(\omega)$, simulated in Matlab. Reference model: FDTD.

In the case of an inhomogeneous system with insulated phase cables, shown in Fig. 5, voltages calculated at the receiving end of the line with Marti's model also present a good agreement with the FDTD method, even though small deviations are perceptible both in the high-frequency behavior and tail of the calculated voltages. The results obtained with the transformation matrices calculated at 100 kHz and 1 MHz are similar despite the greater variation of $\boldsymbol{T}_I$ with frequency when insulated phase cables are considered. Additional analysis indicated that selecting the frequency of 10 kHz for calculating the real transformation matrix necessary in Marti's model would also lead to results comparable to those illustrated in Fig. 5. This suggests that, as long as some loss of accuracy is admitted, Marti's model can be used in the calculation of transients in compact distribution lines even if insulated phase cables are considered.

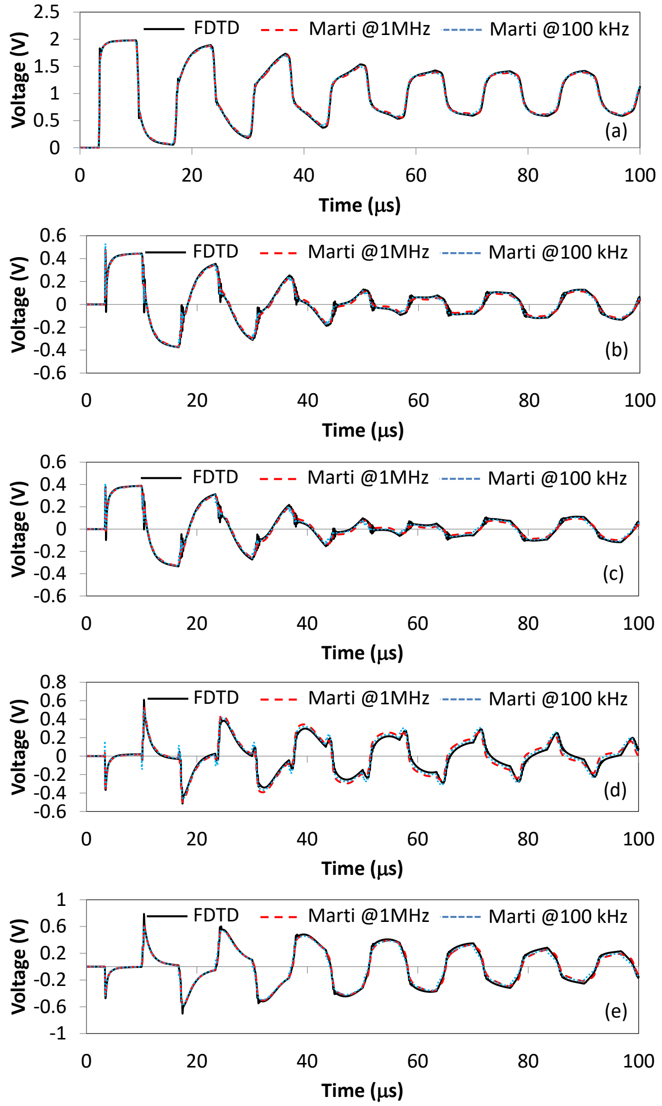


Fig. 5. Voltages at the receiving end of the line for insulated phase cables and 1000-Ωm soil: (a) phase A; (b) phase B; (c) phase C; (d) messenger; (e) neutral. Tested model: Marti's model with complex poles in the fitting of $Z_c(\omega)$ and $A(\omega)$, simulated in Matlab. Reference model: FDTD.

2) Fitting with Real poles

The performance of Marti's model for simulating transients on compact distribution lines is further investigated in this section with the assumption that the rational fitting of $Z_c(\omega)$ and $A(\omega)$ is performed strictly with real poles. This analysis is important because the fitting with real poles is less flexible than the fitting with complex poles, sometimes leading to larger fitting errors. Two approaches were considered. In the first case, $Z_c(\omega)$ and $A(\omega)$ were calculated from 0.1 Hz to 10 MHz considering 10 points per decade in Matlab and then fitted with the vector fitting technique. A similar number of poles as required for the fitting of $Z_c(\omega)$ and $A(\omega)$ considering complex poles was needed. Then, the obtained (real) poles and residues were written in the form of a punch file that was simulated in ATP using Marti's model, following the procedure proposed in [13]. In the second approach, $Z_c(\omega)$ and $A(\omega)$ were calculated also from 0.1 Hz to 10 MHz with 10 points per decade using the option "single core cable" with cables in air in the CC routine available in ATP, and fitted with the asymptotic technique available in this platform for use in the Marti model. The fitting of $A(\omega)$ required 6 to 13 poles, while the fitting of $Z_c(\omega)$ required 13 to 49 poles depending on the considered mode. In both cases, the transformation matrix was calculated at 100 kHz.

To compare the performance of the models, the transient analysis presented in the previous sections was repeated for both the homogeneous and inhomogeneous cases. The results obtained for the system of bare cables considering strictly real poles were identical to those shown in Fig. 4. For this reason, they are not shown here. The results obtained assuming insulated phase cables are shown in Fig. 6.

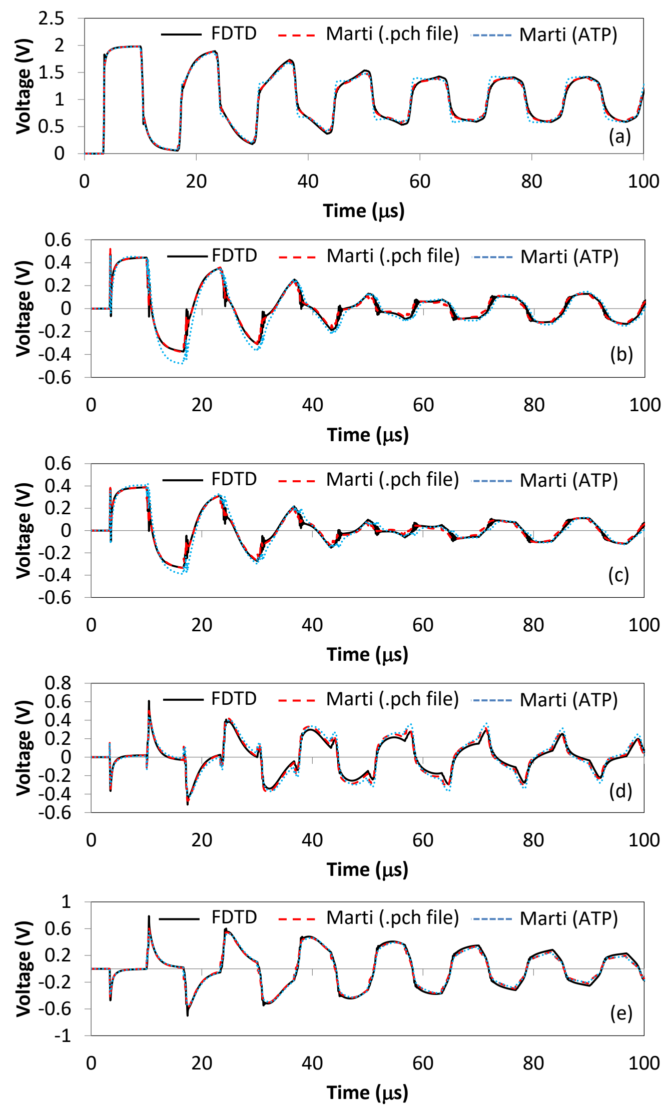


Fig. 6. Voltages at the receiving end of the line calculated for a 1000-Ωm soil assuming insulated phase cables and real poles in the fitting of $Z_c(\omega)$ and $A(\omega)$ for simulation with Marti's model in ATP: (a) phase A; (b) phase B; (c) phase C; (d) messenger; (e) neutral.

Despite minor differences observed mainly at steep voltage changes, it is seen in Fig. 6 that the results obtained in ATP considering the punch file generated in Matlab (labeled as “Marti .pch”) present a good agreement with the results obtained with the FDTD method. This means that, in principle, the use of real poles in the fitting of $Z_c(\omega)$ and $A(\omega)$ does not necessarily compromise the model performance. In fact, the results obtained with real poles are very close to those obtained admitting the use of complex poles in the fitting of $Z_c(\omega)$ and $A(\omega)$. On the other hand, the results directly obtained in ATP (parameter calculation in CC routine, asymptotic fitting of $Z_c(\omega)$ and $A(\omega)$, and simulation using Marti’s model), labeled in Fig. 6 as “Marti (ATP)”, present a larger deviation from the reference model. Such deviation is more perceptible in the first negative peak of the voltages induced on phases B and C, which are 26% and 16% larger than those calculated with the FDTD method, respectively.

In order to understand the reasons for such discrepancy, a detailed investigation was performed. First, $\boldsymbol{Z}$ and $\boldsymbol{Y}$ calculated in ATP were compared with their counterparts calculated in Matlab at several frequencies from 0.1 Hz to 10 MHz, and the differences were no larger than 1%. This means that the parameters are correctly and equivalently calculated in both platforms. Then, it was observed that the transformation matrix calculated in ATP is not purely real. This matrix was artificially modified by turning to zero all its imaginary terms, but simulations with this newly obtained matrix were seen to lead to results that are equivalent to those shown in Fig. 6. Finally, several attempts were made to improve the fitting of $Z_c(\omega)$ and $A(\omega)$ in ATP by adjusting the upper and lower frequency limits considered in the fitting, by increasing the maximum allowed number of poles, and by changing the number of points per decade considered in the parameter calculation. The model performance was seen to be very sensitive to the fitting parameters. In many cases, especially when increasing the number of points per decade, the fitting did not converge for some of the modes. Actually, the results shown in Fig. 6 can be considered the best among a large set of test results. This means that, although real poles can in principle be used in the fitting of $Z_c(\omega)$ and $A(\omega)$ for simulation of the compact line configuration shown in Fig. 1 in ATP, the model performance can be inaccurate if proper care is not taken in the fitting process. For this reason, for simulating the tested three-phase compact line in ATP it is preferable to perform the parameter calculation and then the fitting with the vector fitting technique in Matlab, and later to write the resulting model in the form of a punch file that is compatible with ATP, as proposed in [13].

V. Conclusions

The analysis presented in this paper shows that an accurate transient simulation involving compact distribution lines requires the consideration of the insulating layer surrounding the phase conductors. Marti’s model can be used for simulating this type of line, but some loss of accuracy is expected due to the use of a real and constant transformation matrix. Also, extra care must be taken in the model fitting. Good results were obtained when the fitting was performed with the vector fitting technique in Matlab, with the line model being exported to ATP via punch file. On the other hand, larger errors were observed in transient simulations with Marti’s model when the model parameters were directly fitted in ATP.

VI. References

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