Wideband EMT-Compatible Model for Grounding Electrodes Buried in Frequency Dependent Soil

B. Salarieh, H. M. J. De Silva, B. Kordi

***Abstract*—The advent of sensitive electronic equipment, which requires high-quality electric power and, at the same time, is more susceptible to electromagnetic interference, poses new challenges for power system transient simulation techniques and algorithms. Power transmission line networks, due to their vast physical dimensions, are the most critical components of a complex power system from the reliability point of view in relation to direct lightning strike. The majority of outages in transmission lines are caused by back-flashovers and the tower-footing grounding impedance plays an important role in this context. To have an accurate estimation of the overvoltages in transmission lines, electromagnetic transient (EMT) compatible equivalent circuits of the grounding electrodes has been developed in this paper and the effect of frequency dependent effects has been studied using Finite Element Method (FEM). The objectives of this paper are to accurately estimate the overvoltages/surge in the transmission lines, and to develop a model to predict back-flashovers. The developed models will significantly improve the accuracy of EMT analysis of lightning discharge and its impact on the complex power systems.**

***Keywords*—Power system transient simulation, tower-footing grounding impedance, lightning strike, EMT-compatible, Finite Element Method**

I. Introduction

LIGHTNING is known to be a major cause of outages in power systems. It may result in damages to equipment, high repair costs, and loss of revenue to utility companies. The most vulnerable component of a power system to lightning is the transmission line system. Transmission lines expand over hundreds of kilometers and are exposed to lightning discharge. The outages due to a lightning strike are caused by three different mechanisms: flashover, midspan-flashover, and back-flashover. The first type occurs when lightning strikes directly on the phase conductors due to the absence of a shielding wire. When a lightning strike directly hits either the top of a transmission tower or the ground wires, surge waves are generated that flow through the tower structure that has been hit and consequently the voltage of the cross-arms raises instantly and a flashover can occur if the electric field exceeds the air dielectric strength This phenomenon is called back-flashover and is the main cause of outages for lines of voltages below 500 kV [1]. Different approaches can be taken to address the back-flashover problem such as improving the critical flashover voltage of insulators, reducing tower footing impedance, or installing surge arresters. Among all these methods, the reduction of tower footing impedance is the most effective way to lower the amount of over-voltages on insulator strings and lower the rate of back-flashovers [2].

The grounding system of transmission towers consists of metallic conductors that connect the tower to the electrodes, the grounding electrodes which are oriented vertically and/or horizontally to direct the major part of the lightning surge current to the earth, and the ground in which the electrodes are buried. In the lightning studies of the transmission towers, many authors adopted a constant potential approach and represented the tower footing impedance by a lumped resistance, which is a reasonable approximation at low-frequencies [2]–[5]. However, as the bandwidth of the injected current through the tower structure increases, capacitive and inductive behaviors of the grounding system play an important role in the accuracy of the transient analysis. Additionally, the electrical parameters of soil are strongly frequency dependent [6], [7], which are disregarded in the more common models of grounding systems [8], [9] and need to be considered in the accurate wideband modeling of the grounding electrodes [10].

Existing models in electromagnetic transient (EMT) simulators consider the whole transmission line and the grounding system as one single unit without considering the effect of the frequency-dependent parameters of the ground on the performance of the grounding system.

The objective of this paper is to develop a frequency-dependent time-domain macromodel for vertical and horizontal grounding electrode configurations that are compatible with EMT simulators. This model will enable the accurate estimation of the overvoltages/surge in the transmission line and prediction of back-flashovers. We have developed a simulation model for the system of electrodes and ground using finite element method (FEM) in a wide frequency range of 5 Hz to 5 MHz. Once the admittance of the grounding system is determined in the frequency domain, we will develop a circuit macromodel that has the same frequency response as the grounding system. The macromodel is compatible with any EMT simulator. We will demonstrate the performance of the model in the time domain.

II. Formulation of the Problem

Finite Element Method (FEM) is a differential equation based approach that requires the the whole problem space be divided in to a number of smaller regions [11]. In this paper

B. Salarieh is with the Department of Electrical & Computer Engineering, University of Manitoba, Winnipeg, MB, Canada (e-mail: salarieb@myumanitoba.ca).

H. M. J. De Silva is with Manitoba HVDC Research Centre, Manitoba Hydro International, Winnipeg, MB, Canada (e-mail: jeewantha@pscad.com).

B. Kordi is with the Department of Electrical & Computer Engineering, University of Manitoba, Winnipeg, MB, Canada (e-mail of corresponding author: behzad.kordi@umanitoba.ca)

Paper submitted to the International Conference on Power Systems Transients (IPST2019) in Perpignan, France June 17-20, 2019.

we solve the wave equation in the frequency domain that is represented by

$$\nabla \times (\frac{1}{\mu_r} \nabla \times \mathbf{E}) - k_0^2 \varepsilon_r \mathbf{E} = 0 \qquad (1)$$

where $\mathbf{E}$ is the electric field vector in the frequency domain, k_0 is the wave number, μ_r is the relative permeability, and ε_r is the relative permittivity of the medium [12]. An advantage of the FEM solution of (1) is that one can easily incorporate the inhomogeneity and/or frequency-dependence of the medium.

To analyze the performance of grounding electrodes subjected to lightning surges, the calculation region can be divided into two sections: one is the conductor domain including the electrode of radius r_1 buried in soil with relative permeability of μ_r, relative permittivity of ε_r, and conductivity of σ. The unbounded ground is represented by a hemisphere of radius r_0. The other is the non-conductive air region, which is represented by a cone of height h, a lower radius of r_0, and an upper radius equal to the electrode's diameter. A voltage source of amplitude 1 V is applied between the electrode's top surface and the upper face of the cone by means of defining a numerical port between them. The port is a rectangle with a width equal to the electrode's diameter and its length is equal to h. The solution of (1) using FEM needs suitable boundary conditions in order to represent the electromagnetic model in an accurate way. Perfect electric conductor (PEC) boundary condition is applied to all outer surfaces of the cone and the hemisphere. A 3D view of the proposed model is shown in Fig. 1. The sensitivity analysis described in Section III aims to justify the shape of the computational domain and to select the optimum value for this radius and other parameters affecting the accuracy of results.

III. Sensitivity Analysis and Convergence

In this section, first the size of the computational domain is justified and then the parameters influencing the accuracy of the proposed model are assessed and their optimum value is obtained. The variables considered here are the truncation radius r_0, the length of the port used to excite the electrodes h, and the frequency dependency of soil parameters.

A. Shape and Size of the Computational Domain

The outermost surface of the computational domain serves as the current closing path. There is no such truncation in reality, so the effect of size of the computational region should be minimized in the surge response of the electrode. The ground has been modeled by a hemisphere, because there should be a symmetry around the electrode in order to avoid forcing the current to go in a particular direction. The sensitivity analysis done to find the suitable hemisphere radius tries to minimize the effect of this imaginary ground around the electrode. The air region is modeled as a nonuniform coaxial cable. The current going through this coaxial cable can be considered as a wave which will be scattered cylindrically at high frequencies. The waves reflected back from the air's outer surface, propagate into the computational region and could cause an error. These errors show themselves as resonances in the simulated frequency-domain impedance of

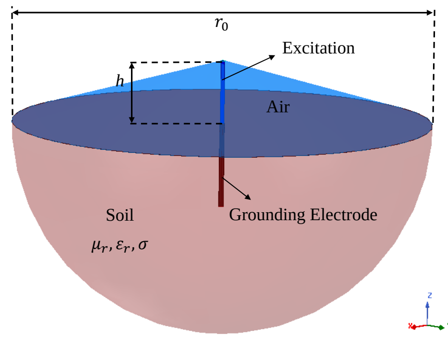


Fig. 1. Proposed FEM model representing the grounding electrode buried in soil (brown hemisphere); The upper-ground part is represented with the blue cone filled with air. The excitation is equivalent to a voltage source of 1V in amplitude (Note that the dimensions are not exact in the figure).

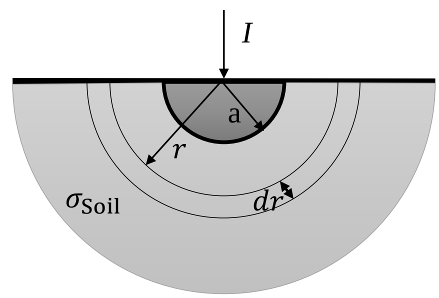


Fig. 2. Current injection in a perfect conducting hemispherical electrode of radius r_0 buried in homogeneous soil with conductivity of σ_{Soil}; a differential segment with a radial length of *dr* is specified.

the electrode. To minimize this error, the air region is modeled as a cone, which is equivalent to having a non-uniform axial cable. As the distance between the PEC wall and the source of scattered waves (central conductor) decreases, the resonances will occur at higher frequencies. This will cause these unwanted resonances to occur in frequencies outside the frequency range of ineterest.

B. Truncation Radius

Consider a PEC hemispherical electrode of radius a buried in homogeneous soil with a conductivity of σ_{Soil} as shown in Fig. 2. By integrating the resistance of differential shells, the DC resistance of such electrode/soil configuration is calculated as

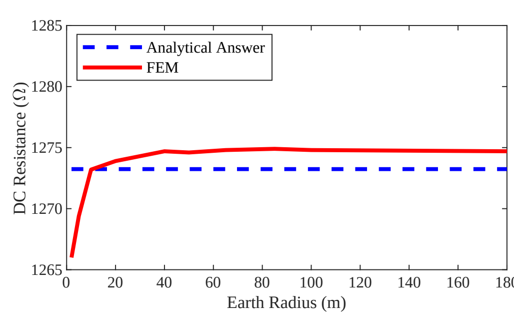


Fig. 3. DC resistance of a PEC hemispherical electrode buried in homogeneous soil as a function of the ground's radius obtained from FEM.

TABLE I
THE MAGNITUDE OF IMPEDANCE FOR A PORT WITH WIDTH OF 25 MM AND VARYING LENGTH.

Port length (mm)	Magnitude of Port Impedance (Ω)
10	0.05241
50	0.6276
100	1.656
1000	30.71
2000	70.09
5000	203.9

$$R_{\text{DC}} = \int_a^{\infty} \frac{dr}{2\pi r^2 \sigma_{\text{Soil}}} = \frac{1}{2\pi a \sigma_{\text{Soil}}} \ (\Omega). \tag{2}$$

The simulation of the grounding system shown in Fig. 2 considering the truncation radius as the variable will allow us to determine the effect of the truncation radius. The analytic DC resistance obtained from (2) is $1,273\ \Omega$ assuming $a = 12.5$ mm and $\sigma_{\text{Soil}} = 0.01$ S/m. Fig. 3 shows the simulated DC resistance as a function of ground's truncation radius. From this figure it can be concluded that considering a hemisphere of radius 60 m would be sufficient to model the resistance of the grounding electrode accurately. As the frequency increases, the skin depth of the soil will decrease that means if the truncation radius is large enough at $f = 0$ (DC), it is sufficiently large at any other frequency.

C. Port Length

The existence of the numerical port creates a parasitic inductance that can be calculated using [13]

$$L = 0.2 \times h \times (\ln \frac{2h}{w} + \frac{0.223w}{h} + 0.5) \text{ (nH)} \tag{3}$$

where h and w are the length and width of the port in millimeters, respectively. A radius of 12.5 mm for the electrode requires a port width of 25 mm. Table I shows the magnitude of port's impedance ($2\pi f L$) for a number of port lengths in the maximum frequency of concern in this paper that is 5 MHz.

In this table, it can be seen that the port length has to be less than 100 mm in order to have a negligible impedance in comparison with the grounding system. According to the results of Section IV, the minimum measured impedance in the whole frequency range has a value of around $8\,\Omega$, so a port impedance of $0.05\,\Omega$ at 5 MHz (0.63% of the grounding impedance) can be considered as negligible.

D. Frequency Dependence of Soil Parameters

Considering the electrical conductivity, permittivity, and magnetic permeability of soil, the first two parameters are strongly frequency dependent, whereas the last one is similar to the magnetic permeability of air in most cases. It has been found that the resistivity and permittivity of soil has a decreasing trend as the frequency increases [14]. Additionally, it was realized that the decrease of soil resistivity and permittivity, resulting from the frequency dependent effects, is responsible for significant decrease of the grounding impedance, and this effect in more pronounced for soils with high resistivity [15]. There have been many laboratory measurements done to determine the frequency dependence of soil parameters, *e.g.* [6], [7], [14]. An example of the frequency-dependence of soil electrical parameters is given by [16]

$$\sigma = \sigma_0 \left[1 + (1.2 \times 10^{-6} \times \sigma_0^{-0.73}) \times (f - 100)^{0.65}\right] \tag{4a}$$

$$\varepsilon_r = 7.6 \times 10^3 f^{-0.4} + 1.3. \tag{4b}$$

In (4a), σ_0 is the soil conductivity at 100 Hz, and ε_r in (4b) is the soil relative permittivity at frequency f. Although the expressions for determination of frequency dependence of soil electrical parameters are not causal, it has been shown that in the presence of a non-zero initial value in time-domain results, it remains relatively small, making it applicable for engineering applications [17]. Using these expressions, the impact of frequency dependent parameters of soil on the lightning response of vertical and horizontal arrangements of electrodes is accurately assessed in the following.

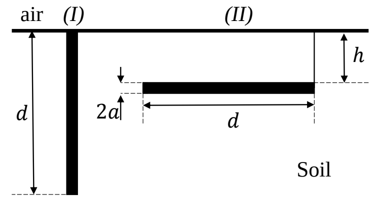


Fig. 4. Vertical (I) and horizontal (II) grounding electrodes buried in homogeneous soil with constant parameters.

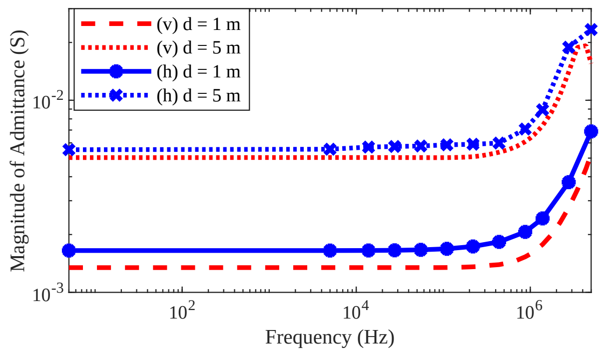
IV. SIMULATION RESULTS

In this section, the tower-footing grounding admittance for vertical and horizontal electrodes is determined and the effects of frequency dependence of soil parameters are investigated.

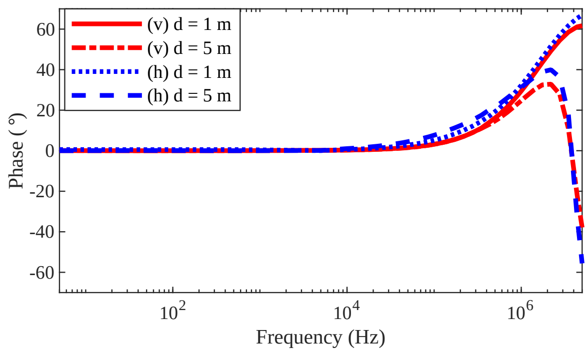
A. Admittance of Vertical and Horizontal Electrodes

In this part, the grounding admittance of vertical and horizontal electrodes are compared for different electrode lengths considering constant soil parameters. Figure 4 shows the typical arrangement of vertical and horizontal electrodes for transmission towers buried in a soil with constant conductivity and relative permittivity of σ_0 and ε_r, respectively. We have considered a burial depth of $h = 0.5$ m and an electrode radius of $a = 12.5$ mm. The grounding admittance is evaluated for two different electrode lengths of $d = 1$ and 5 m assuming constant soil conductivity $\sigma_0 = 0.001$ (S/m) and a relative permittivity of $\varepsilon_r = 10$. As it was mentioned earlier, the effect of frequency dependency of soil electrical parameters on the grounding impedance is more for highly-resistive soils. Additionally, the approximate soil resistivity for wet, moist, and dry soil is 10, 100, and 1000 Ωm [18], and the typical value assumed for soil permittivity is $10 - 20$, which for dry soil is less than or equal to 10 [19]. For this reason, we have considered the case of a dry soil in this paper.

Figure 5 shows the magnitude and phase of the tower-footing admittance for the vertical (red lines) and horizontal (blue lines) electrodes in different lengths. It is observed that the grounding electrodes behave like a lumped

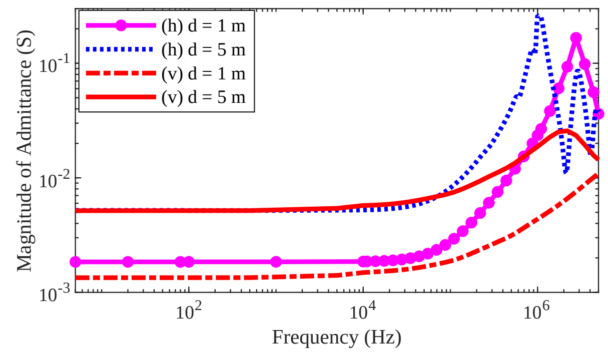


(a)

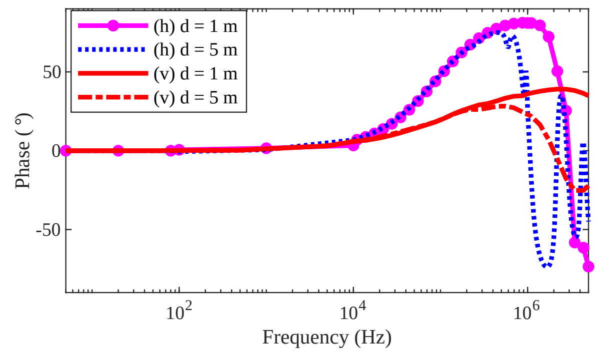


(b)

Fig. 5. Magnitude (a) and phase (b) of admittance for vertical (v) and horizontal (h) grounding electrodes using FEM.



(a)



(b)

Fig. 6. Magnitude (a) and Phase (b) of admittance for vertical (v) and horizontal (h) grounding electrodes using FEM considering the frequency dependence of soil parameters described by (5a) and (5b).

resistor from DC to 10 kHz and is equal to the static resistance. As the frequency of the injected current increases beyond this value, the grounding system assumes reactive or capacitive behavior depending on the frequency. In general, the horizontal electrodes have lower input impedance in comparison with the vertical electrodes of the same length and in order to achieve a better grounding performance, which is equivalent to having lower tower-footing grounding impedance, longer horizontal electrodes should be used.

B. Frequency-Dependent Soil

The assumption of constant parameters through the whole frequency range of interest causes considerable errors in the evaluation of the behaviour of the grounding electrodes [20], [21]. We consider the frequency dependency of conductivity and permittivity of soil in modeling the vertical and horizontal grounding electrodes using (4a) and (4b). A low frequency soil conductivity of $\sigma_0 = 0.001$ (S/m) has been considered here similar to the previous section. Figure 6 shows the magnitude (a) and phase (b) of the vertical and horizontal grounding electrodes with varying lengths, taking the frequency dependent effects into account.

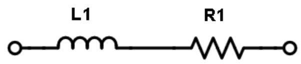
In all of the considered cases, the frequency dependent effects decrease the impedance of the grounding system, and this effect is more pronounced at higher frequencies. It can be implied from the results of this section that the frequency dependence of soil parameters results in considerable improvement of the lightning performance of transmission lines. Given the frequency domain behavior of the grounding electrodes, they can be included in the lightning studies of transmission lines.

V. Circuit Implementation

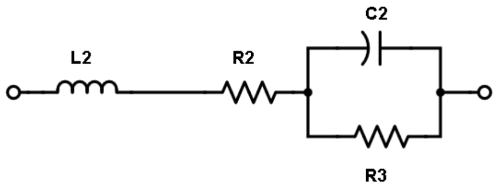
A circuit equivalent for vertical grounding electrode with length of 5 m is obtained considering constant and frequency dependent soil parameters. This is achieved in two steps: (A) the calculated frequency domain responses are fitted with rational function approximations, and (B) EMT-compatible broadband electrical circuits of the approximated frequency domain responses are developed. The circuits obtained using this procedure can be interfaced with EMT simulators to have an accurate representation of the grounding system. Once the arrangement of the grounding electrode is known, one can accurately model the grounding electrodes, calculate the transmission line surge impedance, and predict the surge overvoltages when a lightning discharge occur in the system.

A. Rational Function Approximation

One of the challenges in the accurate modeling of the grounding systems is the inclusion of frequency dependent effects in the time domain simulations. Vector Fitting is used to implement the simulated frequency-domain admittance in the time domain by approximating it as a rational function given by [22]



(a)



(b)

Fig. 7. Series equivalent circuit for (a) real pole, (b) complex pole pair.

$$Y(s) = \sum_{n=1}^{N} \frac{c_n}{s - a_n} + d + sh. \quad (5)$$

In (5), s is the complex frequency in rad/s, d and h are constants, and a_n and c_n are the n^{th} pole and residue, respectively. Given the rational function approximation of the frequency-dependent admittance, the equivalent electric circuit for the grounding electrodes is obtained in the following section.

B. Equivalent Circuit Implementation

The method presented in [23] is used to generate an RLC circuit equivalent for the simulated grounding electrodes which can be incorporated in EMT simulators. The constant term d and the proportional term h in (5) can be represented with a conductance and a capacitance of values d and h, respectively. The equivalent circuit for real poles is a series RL circuit as shown in Fig. 7-a and the equivalent circuit for a pair of complex-conjugate poles is a series RLC circuit given in Fig. 7-b [23]. The value of components in Fig. 7-a are given by

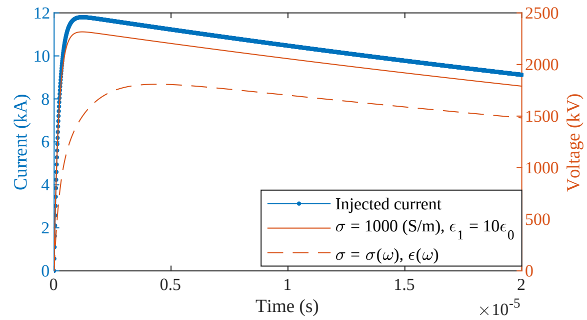
$$L_1 = 1/c_n, \quad R_1 = -a_n/c_n \quad (6)$$

and for Fig. 7-b, the value of the elements can be calculated using

$$\begin{aligned} L_2 &= 1/(c_{n1} + c_{n2}) \quad (7) \\ R_2 &= (-a_{n1}c_{n1} - a_{n2}c_{n2})/(c_{n1} + c_{n2})^2 \\ C_2 &= -(c_{n1} + c_{n2})^3/(c_{n1}c_{n2}(a_{n1} + a_{n2})^2) \\ R_3 &= -(c_{n1} + c_{n2})/[(c_{n1}a_{n2} + c_{n2}a_{n1})C_2]. \end{aligned}$$

The current wave shown in Fig. 8 were used to represent the typical lightning current of subsequent return strokes in the simulations [24]. Figure 8 shows the simulated GPR in response to the current waves of $1.2/50$ μs subsequent stroke on the vertical electrode of length 5 m considering constant and frequency-dependent soil parameters.

In can be seen that the frequency dependence has decreased the impulse impedance of the grounding electrode. This implies significant improvement of the lightning performance of the transmission lines due to frequency dependence of electrical parameters of soil. The obtained circuits can be interfaced with EMT circuit simulators using a Norton equivalent with a fixed conductance matrix and controlled current sources [25] to accurately calculate the effect of grounding electrodes and the frequency dependent effects in the lightning studies of transmission lines.



(a)

Fig. 8. Simulated GPR of the 5 m vertical grounding electrode under the assumption of constant and frequency-dependent soil parameters.

VI. CONCLUSIONS

A wideband EMT-compatible equivalent circuit of vertical and horizontal grounding electrodes was presented in this paper and the effect of frequency dependence of soil parameters was assessed using full wave approach implemented by FEM. It was shown that the frequency domain behaviour of vertical and horizontal electrodes is resistive up to 10 kHz and then it assumes inductive or capacitive behaviour depending on the frequency. Generally, the impedance of horizontal electrodes is lower than the vertical electrodes of the same length. Inclusion of the frequency-dependent effects results in a reduction of the magnitude of the grounding impedance and increases the capacitive behaviour when the frequency of the injected current goes beyond a certain value. This is because of the continuous decrease of permittivity and resistivity of soil as the frequency increases. The results show that the frequency dependent behaviour of soil implies a reduction of the ground potential rise and hence it causes a considerable improvement in the lightning performance of transmission lines and they should be considered in the lightning studies of these lines.

To include the frequency dependent effects into time domain simulations, the equivalent circuit of the grounding electrodes has been obtained by fitting the frequency-domain admittance with rational functions and expressing the poles and residues with RLC elements. The developed circuits can be interfaced with any EMT simulator to predict the surge overvoltages when a lightning discharge occurs in transmission lines.

REFERENCES

[1] F. H. Silveira, S. Visacro, A. De Conti, and C. R. De Mesquita, "Backflashovers of transmission lines due to subsequent lightning strokes," *IEEE Transactions on Electromagnetic Compatibility*, vol. 54, no. 2, pp. 316–322, 2012.

[2] *IEEE guide for Improving the lightning performance of Transmission Lines*, IEEE std.1243-1997, Dec. 1997.

[3] W. A. Chisholm, Y. L. Chow, and K. D. Srivastava, "Lightning surge response of transmission towers," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-102, no. 9, pp. 3232–3242, 1983.

[4] J. G. Anderson, "Monte Carlo Computer Calculation of Transmission line Lightning Performance," *Transactions of the American Institute of Electrical Engineers. Part III: Power Apparatus and Systems*, vol. 80, no. 3, pp. 414–419, 1961.

[5] A. Ametani and T. Kawamura, "A method of a lightning surge analysis recommended in Japan using EMTP," *IEEE Transactions on Power Delivery*, vol. 20, no. 2 I, pp. 867–875, 2005.

[6] R. L. Smith-Rose, "The Electrical Properties of Soil for Alternating Currents at Radio Frequencies," *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 140, no. 841, pp. 359–377, 1933.

[7] J. Scott, "Electrical and Magnetic Properties of Rock and Soil," *U.S. Geol. Surv., Dept. Interior*, no. Washington, D.C, 1966.

[8] A. Borghetti, C. Nucci, M. Paolone, M. Bernardi, S. Malgarotti, and I. Mastandrea, "Influence of Surge Arresters on the Statistical Evaluation of Lightning Performances of Distribution Lines," *Proc. 5th Int. Conf. on Power System Transients*, vol. 2, 2001.

[9] P. Chowdhuri, S. Li, and P. Yan, "Rigorous analysis of back-flashover outages caused by direct lightning strokes to overhead power lines," *IEE Proceedings - Generation, Transmission and Distribution*, vol. 149, no. 1, p. 58, 2002.

[10] R. Alipio and S. Visacro, "Impulse efficiency of grounding electrodes: Effect of frequency-dependent soil parameters," *IEEE Transactions on Power Delivery*, vol. 29, no. 2, pp. 716–723, 2014.

[11] P. Sumithra and D. Thiripurasundari, "A review on Computational Electromagnetics Methods," *Advanced Electromagnetics*, vol. 6, no. 1, pp. 42–55, 2017.

[12] Ansys, "User's Guide: High Frequency Structure Simulator," 2014.

[13] C. R. Paul, "Inductance: Loop and Partial," *IEEE Transactions on Microwave Theory and Techniques*, vol. I, no. 1, pp. 307–333, 2010.

[14] C. Portela, "Measurement and modeling of soil electromagnetic behavior," *IEEE Int. Symp. Electromagn. Compat.*, vol. 2, pp. 1004–1009, 1999.

[15] S. F. Visacro, F. H. Silveira, S. Xavier, and H. B. Ferreira, "Frequency dependence of soil parameters: The influence on the lightning performance of transmission lines," *Lightning Protection (ICLP), 2012 International Conference on*, vol. 57, no. 3, pp. 1–4, 2012.

[16] S. Visacro and R. Alipio, "Frequency dependence of soil parameters: Experimental results, predicting formula and influence on the lightning response of grounding electrodes," *IEEE Transactions on Power Delivery*, vol. 27, no. 2, pp. 927–935, 2012.

[17] D. Cavka, N. Mora, and F. Rachidi, "A Comparison of frequency-dependent soil models: Application to the analysis of grounding systems," *IEEE Transactions on Electromagnetic Compatibility*, vol. 56, no. 1, pp. 177–187, 2014.

[18] "Military Handbook Grounding , Bonding , and Shielding for Electronic Equipments and Facilities Volume 1 of 2 Volumes," no. December, 1987.

[19] L. Apekis, C. Christodoulides, and P. Pissis, "Dielectric properties of soils as a function of moisture content," *Dielectric Materials, Measurements and Applications, 1988., Fifth International Conference on*, pp. 97–100, 1988.

[20] S. Visacro, "Response of Grounding Electrodes to Impulsive Currents: An Experimental Evaluation," *IEEE Transactions on Electromagnetic Compatibility*, vol. 51, no. 1, pp. 161–164, 2009.

[21] S. Visacro, M. H. M. Vale, N. Miguel B Guimarães, R. Araújo, W. L. F. Pinto, and R. S. Alípio, "The response of grounding electrodes to lightning currents : the effect of frequency-dependent resistivity and permittivity of soil," *30th International Conference on Lightning Protection - ICLP 2010*, vol. 2010, pp. 1–4, 2010.

[22] B. Gustavsen, "Rational approximation of frequency domain responses by vector fitting," *IEEE Transactions on Power Delivery*, vol. 14, no. 3, pp. 1052–1061, 1999.

[23] G. Antonini, "SPICE equivalent circuits of frequency-domain responses," *IEEE Transactions on Electromagnetic Compatibility*, vol. 45, no. 3, pp. 502–512, 2003.

[24] R. B. Anderson and A. J. Eriksson, "Lightning parameters for engineering application," *Electra*, vol. 69, pp. 65–102, 1980.

[25] B. Gustavsen and H. M. De Silva, "Inclusion of rational models in an electromagnetic transients program: Y-Parameters, Z-Parameters, S-parameters, transfer functions," *IEEE Transactions on Power Delivery*, vol. 28, no. 2, pp. 1164–1174, 2013.